Anisotropic Reflection Models

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KEYWORDS: computer graphics, raster graphics, lighting models, surface mapping, texture mapping, frame mapping.

CR CATEGORIES: I.3.3, I.3.5, I.3.7

ABSTRACT. We present a new set of lighting models derived from the equations of electromagnetism. These models describe the reflection and refraction of light from surfaces which exhibit anisotropy—surfaces with preferred directions. The model allows a new mapping technique, which we call *frame mapping*. We also discuss the general relationship between geometric models, surface mapping of all types, and lighting models in the context of rendering images with extreme complexity.

§1 Introduction

A thread that runs throughout computer graphics is the quest for detail. Nature presents a nearly infinite complexity and richness of form over an enormous range of scales. In image synthesis it is our task to make convincing pictures of such natural phenomena: thus how to represent this range of scales becomes a central problem.

This paper explores the idea that along with geometric models and surface mapping, we include the use of custom lighting models to capture model complexity.

§2 Anisotropic Lighting Models

It is surprising how many surface materials in the natural world exhibit anisotropy. For example, cloth is a weave of threads. Each thread scatters light narrowly in the direction of the thread and widely in the perpendicular direction. For the same reason, hair and fur should be rendered via an anisotropic lighting model. Another example is brushed or burnished metal, as well as metal surfaces which have been treated with various chemical finishes, anodizing, alodyning, etc. These are basically surfaces with microfeatures in particular directions. Because these features have a characteristic direction their scattering profiles exhibit anisotropy.

In this paper, we will develop the basic lighting models for reflection from dielectric anisotropic surfaces. We do this more or less directly from Maxwell's basic equations of electromagnetism. For conducting surfaces the treatment may be adapted by setting the index of refraction ratio to infinity, simplifiying the ensuing integral.

Although this work develops a method of calculating a reflection model for a general surface, it is not strictly necessary to do so. It is relatively straightforward to generalize a parametric model such as the Phong or Torrance-Sparrow model to the anisotropic case. But calculating the scattering cross sections of surfaces given their actual micro-features is needed to perform the level-of-detail hierarchy calculations in section 5. Thus if one is interested in just any surface that exhibits anisotropy, a Phong-like model is sufficient. On the other hand, if one is interested in a particular surface, the treatment below is called for.

2.1 Related work

Phong(1973) introduced to computer graphics the first lighting model which went beyond the diffuse Lambert shading model, introducing highlights. Blinn (1977,1978) introduced and adapted to a form suitable for computer graphics the lighting models of Torrance and Sparrow (1967) and Trowbridge and Reitz (1975). These models used a geometrical optics model for light reflection from surfaces and were effective in simulating the reflection from surfaces with microscopic roughness. Cook and Torrance (1981) adapted models from the above sources as well as using a model based on Beckmann(1963) which is concerned with scattering of electromagnetic radiation from rough surfaces. Beckmann's models do not make the geometrical optics assumptions. Wavelength dependence was an innovation of their work.

Catmull (1975) introduced the notion of color mapping onto surfaces. Shortly thereafter, Blinn(1977) generalized the notion of surface mapping to include the case of perturbation of surface normals as well as mapping arbitrary parameters in the lighting models he introduced. In this way he was able to render images with macroscopic as well as microscopic roughness.

We also mention an image brought to our attention by one of the reviewers. In it Tomohiro Ohira has used an anisotropic reflection model.

2.2 Paper organization

Since much of what follows is a development from a simple case, we urge the reader to review the appendix describing how light reflects from a smooth plane surface. This appendix presents a review of all the equations relevant to the model we pursue.

The case for a general surface model, following Beckmann, is treated in section 3. We show how these equations can be used in a rendering algorithm to generate custom lighting models. Section 4 discusses how parameters in the model may be mapped and how this extends Blinn's well known technique. Section 5 treats in a general way the overall relationship between lighting models, surface mapping, and model geometry. It suggests a strategy for rendering complex images. Section 6 presents results of the new model. Section 7 discusses further work. The appendix reviews light reflection from a smooth surface.

§3 Reflection from a rough surface

In this section we rederive Beckmann's (1963) general formula for scattering from a rough surface in a form more suitable for computer graphics. We then present the new lighting model for anisotropic surfaces. The notation for this section is developed in the appendix.

3.1 Kirchhoff approximation

The Kirchhoff approximation for reflection from a rough surface approximates the field at any point of the surface by the field which would occur if we replaced the surface by its tangent plane. See figure 1.

If the surface is of low curvature in relation to the wavelength of light then the approximation is valid. Specifically, Brekhovskikh (Beckmann 1963 p.29) gives the criterion

$$4\pi r_c \cos(k_1, n) \gg \lambda.$$

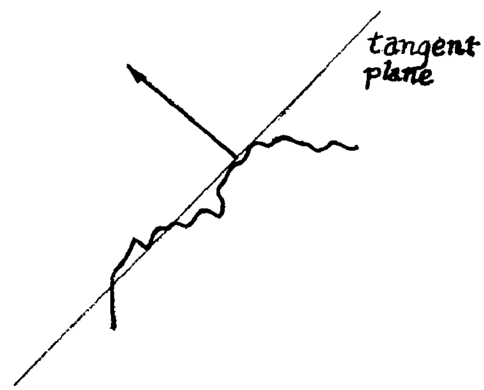

Figure 1. The Kirchhoff approximation

Where r_c is the radius of curvature of the surface, $\cos(k_1, n)$ is the cosine of the angle of incidence, and λ is the wavelength of our wave.

For lighting model calculations we assume that the incident field is always a plane wave ψ_1 with wave vector k_1. Thus for the field at the boundary we make the approximation

$$\psi|_S = (1+R)\exp(ik_1 \cdot s) \qquad (1)$$

$$\left.\frac{\partial\psi}{\partial n}\right|_s = (1-R)\exp(ik_1 \cdot s)ik_1 \cdot n \qquad (2)$$

That is, the field at the boundary $\psi\ |_S$ is simply the sum of the incident field ψ_1 and the reflected field $R\psi_1$ calculated by the Fresnel formula.

When equations (A1),(1), and (2) are substituted into equation (A7) we have a formula suitable for computing a lighting model from a specified surface microstructure.

$$\psi(x) = -\frac{i\exp(i|k||r_0|)}{4\pi|r_0|}\int_S n \cdot [R(k_1 - k_2) - (k_1 + k_2)] \times \exp(i(k_1 - k_2)\cdot s)dS. \qquad (3)$$

Following Beckmann, we calculate the reflection coefficient rather than the reflected field by normalizing by $\psi_0(x)$ the field of a smooth, perfectly conducting plane in the specular direction for the same angle of incidence and same distance. In this case, the mirror equation implies $n_0 \cdot (k_1 + k_2) = 0$ and $n_0 \cdot (k_1 - k_2) = 2n_0 \cdot k_1$. Because the surface is smooth s lies in the plane $z = 0$ and $n_0 \cdot (k_1 - k_2) = 0$. The assumption of perfect conductivity implies $R = 1$. Thus

$$\psi_0(x) = -\frac{i|k|\exp(i|k||r_0|)}{2\pi|r_0|} n_0 \cdot k_1 A \qquad (4)$$

Where A is the area of the surface we integrate over. Dividing equation (4) into (3) obtains the reflection coefficient formula:

$$\rho(x) = \frac{1}{2n_0 \cdot k_1 A}\int_S n \cdot [R(k_1 - k_2) - (k_1 + k_2)] \times \exp(i(k_1 - k_2)\cdot s)dS. \qquad (5)$$

Note that the reflection coefficient is not just a function of x, but also a function of the incidence and emittance vectors k_1, k_2, as well as the normal n. The integral is over the entire surface, but since only the first few fresnel zones are required in practice a very small surface element will suffice. The above derivation is essentially Beckmann's general solution for a surface rough in both dimensions but we have expressed it in vector form.

3.2 Using the lighting model

Using the equation we can compute a lighting model which gives the average field power scattered from an arbitrary surface. To use equation 5 there are two things to be specified. First, we must specify the micro-surface displacement function. Second, we must calculate the integral.

Specifying the surface may be done is several ways. One way is to expand the height function into a set of Fourier coefficients. These coefficients may be specified as a $1/f^{\alpha}$ function to give fractal behavior. A second way is to use bump maps from a higher scale level. Once a bump map perturbs surfaces on a scale less than a pixel it should be included in the lighting model.

Calculating the integral should be via an asymptotic expansion using the method of stationary phase. Note that the wave vectors k_1 and k_2 are very large compared with an area to integrate over. Now, the factor and integrand outside the exponential can have this large magnitude cancelled from them, so that unit vectors may be considered. The exponential term, however, is rapidly varying. Using stationary phase, this integral is easily approximated.

To calculate the integral each time a reflection coefficient is required is prohibitvely expensive. By computing the model off line this cost may be virtually eliminated. We do this by storing the lighting model reflection function in a table, linearly interpolating the values. An index i, j into the table corresponds to a pair of small cells of solid angle for the incidence and emittance directions. We store the square of the reflectance function $S_{ij} = \rho(x, k_1, k_2)$ at each cell, where k_1 and k_2 lie in the i and j cell correpondingly. The procedure is as follows:

Perform the first 3 steps offline.

1. Divide a hemisphere centered about the surface element into a number of discrete cells (say 5 by 20 cells equally divided in lattitude and longitude).

2. For each pair of cells calculate ρ by the integral in equation (5) using the microsurface of interest.

3. The incoherent scattering function for each of the cells is calculated by $S_{ij} = |\rho|^2$ and stored away.

4. Now, when calculating the lighting model for a particular ray or pixel; use the incidence and emittance vectors, ***with respect to the surface frame***, to (linearly) interpolate the sampled specular reflectance function S_{ij}.

Note that in step 4, we do not use the absolute incidence and emittance vectors. Rather, because of anisotropy, the vectors are specified in terms of the surface frame (normal, tangent, and binormal). We discuss this further in the following section. It should also be noted that, although the offline steps may be quite time consuming for every lighting model we compute, the time critical step, step 4, is simply a table lookup and linear interpolation.

§4 Frame mapping

Blinn in (Blinn 1977 and 1978) introduced an extension of Catmull's idea of surface color mapping to mapping more general parameters of a lighting model onto a surface. In this way he obtained an ability to vary the specularity and reflectivity of surfaces as well as the much celebrated ability to map apparent bumps and wrinkles onto the surface through the mapping of surface normals. The above lighting model allows us to apply Blinn's idea to the mapping of surface frame bundles.

A ***frame bundle*** for a surface is simply a local coordinate system given by the tangent, binormal, and normal to the surface. Both the tangent and binormal lie in the tangent plane to the surface. For isotropic surfaces they have no intrinsic physical significance—any othonormal set of vectors lying in the plane can serve as a frame. For anisotropic surfaces, however, we take the tangent of the surface to orient along a reference direction, e.g. the "grain" of the surface.

The idea behind frame mapping is simply to perturb the entire frame bundle—not just the normal vector—at each point on the surface. This allows a mapping of the directionality of surface features in nature, e.g. hair, cloth, etc.

Suppose we are attempting to render a surface element. For isotropic lighting models with fixed incidence and emittance vectors, the only significant geometric features about the surface element are its position in space x_0 and its normal n_0. For anisotropic lighting models the entire frame is significant because there is a preferred direction for the surface.

The rendering of an isotropic surface with frame mapping proceeds as follows: first calculate the frame of the surface element. Then for each point x_0 on the surface look up an orthogonal matrix $M(x_0) = [t\,b\,n]$ which transforms the plain surface frame to a mapped surface frame. The columns of this matrix are simply the perturbed tangent vector t, binormal vector b, and normal vector n expressed in the coordinate system of the plain surface frame.

There are nine quantities in an orthogonal matrix but orthogonality requires that the columns form an orthonormal set. Because there are six such pairwise equations, there are

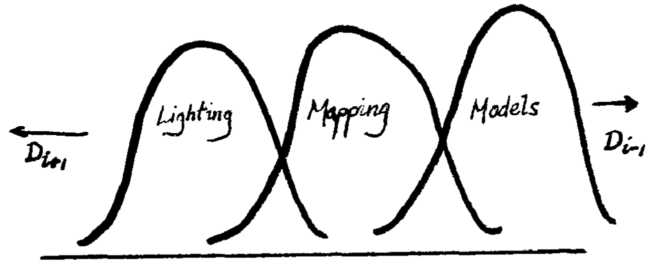

Figure 2. Hierarchy of detail

only three independent quantities. These quantities may be specified by, say, Euler angles. In what follows, its easier to use two orthonormal vectors. Two unit magnitude constraints and an inner product constraint reduce the number of independent parameters of a pair of orthonormal vectors to three.

On mapping a frame bundle, fixing the tangent and normal vectors determines the binormal. Frame mapping may be specified by the tangent and normal maps. Thus the two vectors we choose above are the normal and tangent vectors.

To map frames, we calculate the binormal by cross products. We then map the plain surface frame by multiplying by the constructed orthogonal matrix. This perturbed frame is then used to determine the incidence and emittance direction cosines used in the anisotropic lighting model. We note that the idea of frame mapping works with any anisotropic model: the general anisotropic model above as well as a parametric Phong-like model.

As an example, suppose we have calculated the scattering cross-section of a thread. Since cloth is a cross weave of threads, we set up the frame map to correspond to map the tangent of the surface to the direction of the threads according to the pattern of the weave.

§5 A hierarchy of scale

One of the striking features of images approaching the complexity of nature, is the wide range of scales represented in them. It has been suggested that the natural strategy is to use a succession of a number of geometric models, each capturing a different level of detail (Crow 1975).

We suggest that within each level of detail D_i we have three natural scales shown in figure 4.

The largest scale within D_i is the geometric model, which is the traditional scale manipulated in computer graphics. Geometric features participate in the full range of graphics manipulations: they require visibility computations, they cast shadows, etc. The next range of scale in D_i is surface mapping. In this range, the details which would be geometric at D_{i+1} are collected into surface perturbations.

The final range of scale contains the smallest detail. This is the realm of lighting models. At D_i the lighting model is derived from the surface map and lighting model of D_{i+1}.

Thus the hierarchy of detail would use several levels of geometric models, surface maps and custom lighting models for each range of scales. At any level of detail D_i, the surface maps capture geometry at a finer scale D_{i+1}. They should be derived as in (Blinn 1978). We calculate the custom lighting model for D_i using the surface map of D_{i+1} by using equation (5). The custom lighting models then capture the detail in maps at the next finer scale.

It is clear that surface maps cannot handle arbitrarily fine surface detail, for they tend to alias. Blinn has suggested filtering the surface perturbation as an expedient, but has left the antialiasing of normal maps as an open problem. Lighting models are the natural way to anti-alias normal maps.

There are a number of notable exceptions to the hierarchy of scales model outlined above. Some models use only very simple smooth surfaces. Indeed, the common practice in computer graphics is to use surfaces of this type. For this case, only geometry is necessary. No surface maps are needed and a simple, fixed lighting model is only required. Another set of exceptions to the hierarchy of scales model are the few cases in which lighting models are used directly. For example a method for rendering point light sources (Gabriel 1975), uses a trivial geometry but complex lighting models. In a real sense, the volume density ray tracing technique (Kajiya 1984) uses no geometry at all, just the calculation of a rather complex lighting model. Blinn's atmospheric model paper (Blinn 1982) is also all lighting model with very little geometry.

§6 Results

Figures 5 and 6 illustrate the anisotropic models plus the frame mapping technique in the ray tracing context. Figure 5 shows four spheres resting on a plane. The leftmost pair of spheres are simply color mapped with a stripe pattern. The right most pair are frame mapped. The tangent of the surface points in orthogonal directions on alternating stripes. Because of this, the highlights follow the stripes on one set and are transversal to the stripes on the other set. Only the tangents are mapped in this example, the normal remaining unperturbed.

Figure 6 shows a single sphere resting on a plane. The plane frame map transforms both tangent and normal fields. The anisotropic model is one for a thread. We map the surface with a cloth weave pattern which not only maps the direction of the thread but also models the threads crossing above and below each other.

Both these images were computed via ray tracing, they are 512 by 512 pixel images adaptively subsampled to a 4 by 4 subpixel grid. We have filtered the subsampled image with a simple box filter which is responsible for the moire. These images each consumed 12 hours of IBM4341 CPU time.

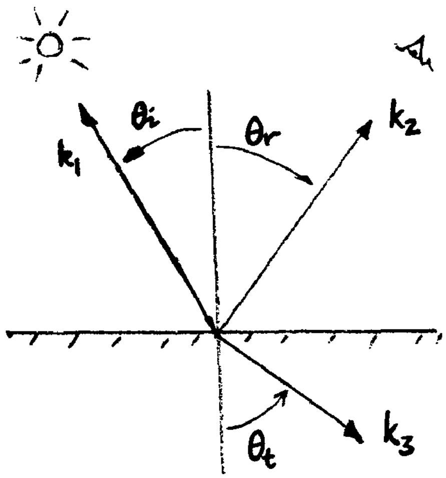

Figure 3. Reflection from a dielectric surface.

§7 Further work

It may be attractive to characterize the surface statistically. We can then compute statistical lighting models by computing the mean of the reflection coefficient intensity $\langle|\rho|^2\rangle$. This will give us a lighting model in the more conventional sense. However, for anisotropic surfaces, the ensuing integrals are extremely complex. Aside from the complexity of evaluating the integral, the ensuing complexity of the analytic formulation would make numerical evaluation of the function quite costly. Thus, precomputing tables which are then evaluated may be necessary anyway.

Appendix. Reflection from a smooth plane surface

This appendix is a brief review of the electromagnetic theory we use in this paper. This section is intended to recall and introduce notation for someone already acquainted with this subject. The reader interested in treatments of this material in full detail are encouraged to consult Jackson(1975) and Born and Wolf(1980). Knowledge of the above is essential to reading this appendix.

Plane wave solutions

Maxwell's equations in the absence of matter reduces to the wave equation which decouples the E and B fields and their spatial components:

$$\frac{1}{c}\frac{\partial\psi}{\partial t} + \Delta\psi = 0$$

Where ψ is any single component of the E or B vector. The solutions to this equation that we will be most interested in are the *plane wave* solutions:

$$U = \exp(i(\omega ct - kx))$$

Where c is the speed of light, ω is the frequency of the wave, and k is the wave vector—a 3-vector whose direction is the direction of propagation of the wave and whose magnitude is the reciprocal of the wavelength.

We first consider some kinematic constraints of a plane wave reflecting from an interface between two dielectric media, see figure 1.

At each point, the spatial and time variation of each of the components of the fields are unique. Specifically, at the boundary the spatial variation of the reflected, incident, and transmitted wave must match. This gives rise to the equation:

$$(k_1 \cdot x)_{z=0} = (k_2 \cdot x)_{z=0} = (k_3 \cdot x)_{z=0}$$

Where k_1, k_2, k_3 are the wave vectors of the incident, reflected, and transmitted waves. In many graphics papers k_1 and k_2 are known as L and E. When measuring the angle from the normal we get:

$$|k_1|\sin\theta_i = |k_2|\sin\theta_r = |k_3|\sin\theta_t.$$

Since the magnitudes of the wave vectors are

$$|k_1| = |k_2| = \frac{\omega}{c}\sqrt{\mu_1\epsilon_1}$$
$$|k_3| = \frac{\omega}{c}\sqrt{\mu_2\epsilon_2}$$

Where μ_i, ϵ_i are the magnetic permeability and dielectric constant of the two media. Thus $\sqrt{\mu_i\epsilon_i} = \eta_i$ are the indices of refraction of the media.

From the first these equations we obtain the Mirror Equation. That is, the angle of reflection equals the angle of incidence, $\theta_i = \theta_r$. From the second of these equations we obtain Snell's law, $\eta_1 \sin\theta_i = \eta_2 \sin\theta_t$.

The Fresnel formulae

A second set of kinematic constraints arise from the continuity of the tangential component of the E vector and the normal component of the B vector. These give rise to the Fresnel formulae. We skip the derivation of these equations which may be found in Jackson(1975) and Born and Wolf(1980). The importance of the Fresnel formulae is that they fix the reflection and transmission coefficients for a smooth interface between two dielectrics.

$$R_\perp = \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}$$
$$R_\parallel = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_1\cos\theta_t + \eta_2\cos\theta_i}$$
$$T_\perp = \frac{2\eta_1\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}$$
$$T_\parallel = \frac{2\eta_1\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

Where $R_\perp, R_\parallel, T_\perp, T_\parallel$ are the reflection and transmission coefficients for waves polarized in directions perpendicular and parallel to the surface.

Now, for unpolarized light, the reflection and transmission coefficients coefficients are simply the average of the two polarized coeffients. This is because unpolarized light is composed of rapidly fluctuating random polarizations whose time average is half for each of the polarizations (Born and Wolf 1980). Thus the unpolarized reflection and transmission coefficients are:

$$R = \frac{\cos^2\theta_i - \cos^2\theta_t}{\cos^2\theta_i + \cos^2\theta_t + \left(\frac{\eta_1^2+\eta_2^2}{\eta_1\eta_2}\right)\cos\theta_i\cos\theta_t} \qquad (A1)$$

$$T = \frac{\eta_1\cos\theta_i(\eta_1+\eta_2)(\cos\theta_i+\cos\theta_t)}{\cos\theta_i\cos\theta_t(\eta_1^2+\eta_2^2)+\eta_1\eta_2(\cos^2\theta_i+\cos^2\theta_t)} \qquad (A2)$$

Note that R is really a function of the angles of incidence and emittance, it should properly be written as $R(\theta_i, \theta_r)$. Note also that the reflection coefficents should be squared for incoherent illumination—because the power rather than the energy is what contributes to the total brightness of an image.

It somewhat curious that these formulae have been noted many times (Blinn 1978, Cook 1981), but have largely been ignored in Computer graphics. Their effect is quite significant. They say for example, that at normal incidence the reflection from a window is far less than the reflection from a grazing angle.

Scalar Diffraction Theory

The Kirchhoff diffraction integral is obtained from the scalar wave equation. It describes the value of any component of the E or B field, call it ψ, at a point in space x given that we know the field at an emitting surface S.

$$\psi(x) = -\frac{1}{4\pi}\int_S \psi\frac{\partial\varsigma}{\partial n} - \varsigma\frac{\partial\psi}{\partial n}\,dS. \qquad (A3)$$

Where $\frac{\partial}{\partial n}$ is the normal derivative for the emitting surface, ς is the converging spherical wave Green's function centered about the point x. We have

$$\varsigma = \frac{\exp(i|k||r|)}{|r|}$$
$$\frac{\partial\varsigma}{\partial n} = \frac{\exp(i|k||r|)}{|r|}\left[i|k| - \frac{1}{|r|}\right]\cos(n, r) \qquad (A4)$$

Note that $|k_2| = |k_1| = |k| = \frac{2\pi}{\lambda}$ is the wave number of the incident and scattered waves. And since $|r| \gg \lambda$ we ignore the term containing $1/|r|$ to give the equation

$$\frac{\partial\varsigma}{\partial n} = \frac{\exp(i|k||r|)}{|r|} i|k|\cos(n, r) \qquad (A5)$$

With this integral, once we know the value of the field and its normal derivative at all points of the surface of interest, will allow us calculate the scattered field. Now usually we will be interested in the scattered field in a *direction* given by the wave vector k_2, that is, we are interested in the field at points which are very far away compared to the wavelength

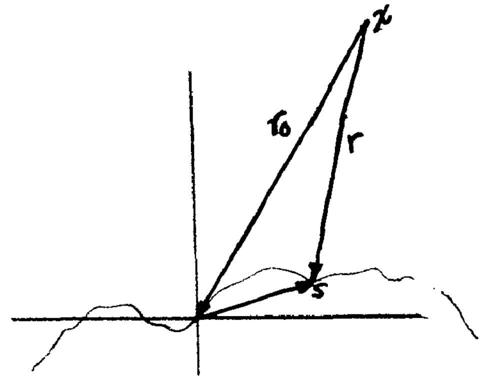

Figure 4. Scattering geometry

light and dimensions of the microstructure of the surface for which we are computing a lighting model. For this we use the Fraunhofer approximation to the Kirchhoff integral. The vector pointing from x to the surface point s we're integrating over is $r = x - s$. Let r_0 be the vector from the point x to the center of the region of scattering, see figure 2. Then,

$$r = r_0 + s$$

and

$$|r|^2 = |r_0|^2 + |s|^2 + 2|r_0||s|\cos(r_0, s).$$

But, since $|r_0| \gg |s|$ we can take square roots to obtain

$$|r| \approx |r_0| + |s|\cos(r_0, s).$$

Now, the vector r_0 and the wave vector k_2 point in opposite directions

$$\frac{k_2}{|k|} = -\frac{r_0}{|r_0|},$$

so $\cos(r_0, s) = -\cos(k_2, s)$ and we can use the cosine formula for the inner product to obtain

$$|r| \approx |r_0| - \frac{k_2 \cdot s}{|k|} \qquad (A6)$$

Substituting equations (A4),(A5), and (A6) into the Kirchhoff integral (A3) gives:

$$\psi(x) = -\frac{\exp(i|k||r_0|)}{4\pi|r_0|}\int_S\left[n\cdot k_2 i\psi - \frac{\partial\psi}{\partial n}\right] \times \exp(-ik_2\cdot s)\,dS. \qquad (A7)$$

This is the equation for computing a lighting model once we know the field at the surface of interest.

§8 References

P. Beckmann and A. Spizzichino (1963) *The scattering of electromagnetic waves from rough surfaces* Pergamon, Oxford.

J. Blinn (1977) "Models of light reflection for computer synthesized pictures", *Computer Graphics* v.11,2, pp.192-198.

J. Blinn (1978) "Simulation of wrinkled surfaces" *Computer Graphics* v.12,3, August 1978.

J. Blinn(1978) "Computer Display of Curved Surfaces", *Ph.D. thesis* University of Utah.

J. Blinn(1982) "Light reflection functions for simulation of clouds and dusty surfaces" *Computer Graphics* v.16,3, pp.21-29.

M. Born and E. Wolf(1980) *Principles of Optics* Pergamon, Oxford.

E. Catmull(1975) "Computer display of curved surfaces", *IEEE Conf. on Computer Graphics, Pattern Recognition and Data structures*May 1975, pp.11-17.

R. Cook and K. Torrance(1981) "A reflectance model for computer graphics" *Computer Graphics* v.13,3, pp.307-316.

F. Crow(1982) "A more flexible image generation environment" *Computer Graphics* v.16,3, pp.9-18.

S. Gabriel(1975), The Evans and Sutherland Lufthansa Flight simulator, *private communication.*

J. Jackson(1975) *Classical Electrodynamics* Wiley, New York.

J. Kajiya(1984) "Ray tracing volume densities", *Computer Graphics, v. 19,3.*

B. Phong(1973) "Illumination for computer generated pictures", *Comm. ACM* v.18,6 June 1975, pp.311-317.

K. Torrance and E. Sparrow(1967) "Theory for off-specular reflection from roughened surfaces", *J. Opt. Soc. Am.* v.57,9. Sept 1967, pp. 1105-1114.

T. Trowbridge and K. Reitz(1975) "Average irregularity representation for a roughened surface for ray reflection", *J. Opt. Soc. Am.* 65,5. May 1975,. pp.531-536.

Figure 5.

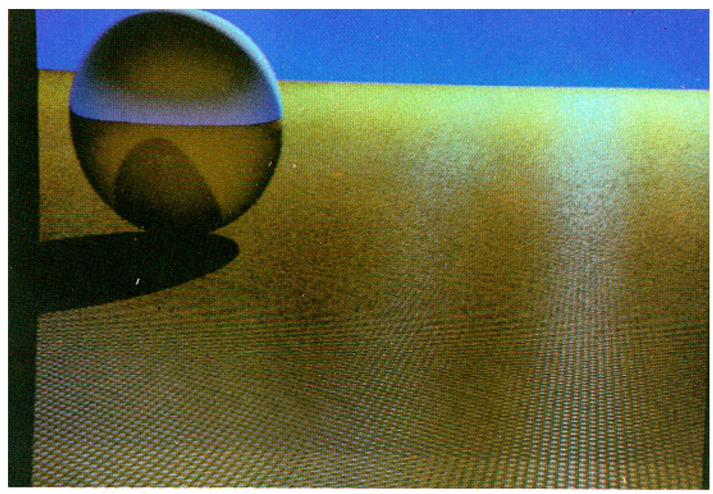

Figure 6.