

Appendix A: $\text{SOA}_{s'}(n, s^m)$ s in the tables

We arrange the columns of a saturated regular design $\text{OA}(n, (n-1)/(s'-1), s', 2)$ in Yates order and label them by $1, 2, \dots, (n-1)/(s'-1)$. For ease of expression, we represent A and B by the labels of columns.

An $\text{SOA}_2(8, 4^2)$ with $A_2(D) = 1$: $A = (5, 6)$, $B = (4, 4)$.

An $\text{SOA}_2(8, 4^3)$ with $A_2(D) = 3$: $A = (5, 6, 7)$, $B = (4, 4, 4)$.

An $\text{SOA}_2(16, 4^6)$ with $A_2(D) = 3$: $A = (2, 4, 8, 10, 14, 15)$, $B = (1, 3, 13, 6, 11, 6)$.

An $\text{SOA}_2(16, 4^7)$ with $A_2(D) = 6$: $A = (3, 4, 8, 10, 12, 14, 15)$, $B = (6, 2, 5, 11, 11, 9, 13)$.

An $\text{SOA}_2(16, 4^8)$ with $A_2(D) = 12$: $A = (1, 2, 3, 4, 8, 10, 11, 14)$, $B = (12, 5, 15, 9, 7, 5, 13, 7)$.

An $\text{SOA}_2(16, 4^9)$ with $A_2(D) = 18$: $A = (1, 2, 3, 4, 8, 10, 12, 14, 15)$, $B = (7, 11, 6, 13, 5, 7, 11, 5, 9)$.

An $\text{SOA}_2(16, 4^{10})$ with $A_2(D) = 30$: $A = (1, 2, 3, 4, 8, 10, 11, 12, 14, 15)$, $B = (7, 5, 5, 13, 5, 7, 6, 5, 9, 6)$.

An $\text{SOA}_2(32, 4^{10})$ with $A_2(D) = 1$: $A = (9, 15, 17, 18, 19, 21, 23, 25, 30, 31)$, $B = (8, 3, 13, 22, 14, 16, 29, 2, 24, 11)$.

An $\text{SOA}_2(32, 4^{11})$ with $A_2(D) = 3$: $A = (1, 2, 4, 10, 11, 14, 18, 22, 23, 24, 31)$, $B = (21, 19, 29, 12, 21, 7, 15, 19, 13, 16, 3)$.

An $\text{SOA}_2(32, 4^{12})$ with $A_2(D) = 5$: $A = (2, 3, 8, 11, 12, 14, 16, 18, 19, 20, 22, 27)$, $B = (26, 30, 21, 15, 6, 7, 9, 31, 10, 17, 1, 28)$.

An $\text{SOA}_2(32, 4^{13})$ with $A_2(D) = 8$: $A = (2, 3, 4, 10, 14, 15, 18, 22, 23, 24, 27, 28, 31)$, $B = (9, 19, 29, 12, 29, 17, 21, 8, 16, 13, 26, 25, 11)$.

An $\text{SOA}_2(32, 4^{14})$ with $A_2(D) = 11$: $A = (3, 4, 10, 11, 12, 14, 15, 16, 18, 24, 26, 27, 30, 31)$, $B = (23, 21, 8, 22, 13, 7, 25, 21, 1, 9, 28, 8, 2, 6)$.

An $\text{SOA}_2(32, 4^{15})$ with $A_2(D) = 14$: $A = (5, 6, 9, 10, 11, 17, 19, 22, 23, 25, 26, 27, 29, 30, 31)$, $B = (4, 8, 13, 18, 12, 16, 20, 20, 15, 12, 8, 21, 13, 28, 28)$.

An $\text{SOA}_2(32, 4^{16})$ with $A_2(D) = 19$: $A = (1, 2, 3, 4, 8, 11, 14, 15, 16, 19, 22, 23, 24, 27, 30, 31)$, $B = (20, 5, 6, 9, 28, 25, 28, 26, 13, 9, 17, 29, 20, 10, 18, 25)$.

An $\text{SOA}_2(32, 4^{17})$ with $A_2(D) = 26$: $A = (1, 2, 3, 4, 10, 12, 14, 15, 20, 22, 23, 24, 26, 27, 28, 30, 31)$, $B = (8, 7, 16, 21, 7, 21, 8, 18, 29, 19, 17, 11, 8, 11, 13, 25, 25)$.

An $\text{SOA}_2(32, 4^{18})$ with $A_2(D) = 33$: $A = (1, 2, 3, 4, 8, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 30, 31)$, $B = (12, 11, 6, 13, 17, 5, 5, 12, 7, 6, 29, 28, 10, 21, 11, 7, 7, 25)$.

An $\text{SOA}_2(32, 4^{19})$ with $A_2(D) = 43$: $A = (1, 3, 4, 8, 10, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30)$, $B = (6, 5, 25, 25, 7, 29, 9, 2, 21, 31, 6, 11, 11, 2, 17, 31, 25, 13, 21)$.

An $\text{SOA}_2(32, 4^{20})$ with $A_2(D) = 54$: $A = (1, 2, 3, 4, 8, 10, 11, 12, 15, 16, 18, 19, 20, 22, 24, 26, 27, 28, 30, 31)$, $B = (7, 21, 13, 13, 17, 29, 13, 17, 6, 21, 5, 6, 29, 17, 29, 23, 14, 25, 7, 14)$.

An $\text{SOA}_2(32, 4^{21})$ with $A_2(D) = 72$: $A = (1, 2, 3, 4, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30)$, $B = (6, 5, 5, 13, 17, 31, 6, 21, 17, 6, 13, 13, 6, 9, 17, 17, 29, 5, 6, 21, 25)$.

An $\text{SOA}_2(32, 4^{22})$ with $A_2(D) = 98$: $A = (1, 2, 3, 4, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31)$, $B = (7, 7, 6, 25, 13, 7, 13, 21, 7, 9, 21, 21, 6, 9, 17, 17, 9, 7, 29, 13, 7, 25)$.

An $\text{SOA}_2(64, 4^{22})$ with $A_2(D) = 7$: $A = (11, 14, 19, 22, 23, 29, 31, 34, 37, 43, 44, 45, 47, 51, 53, 55, 57, 58, 59, 60, 61, 62)$, $B = (35, 20, 3, 28, 40, 26, 39, 4, 52, 41, 36, 13, 41, 50, 48, 46, 33, 54, 21, 54, 30, 49)$.

An $\text{SOA}_2(64, 4^{23})$ with $A_2(D) = 9$: $A = (2, 3, 6, 16, 21, 27, 29, 32, 34, 35, 38, 42, 43, 44, 47, 48, 50, 51, 55, 58, 60, 61, 62)$, $B = (57, 4, 23, 28, 20, 13, 9, 49, 56, 10, 25, 31, 14, 41, 8, 39, 28, 33, 56, 30, 57, 11, 19)$.

An $\text{SOA}_2(64, 4^{24})$ with $A_2(D) = 11$: $A = (4, 8, 10, 14, 15, 18, 20, 24, 26, 30, 31, 34, 36,$

40, 42, 46, 47, 48, 50, 52, 56, 58, 62, 63), $B = (3, 13, 6, 11, 6, 49, 51, 61, 54, 59, 54, 17, 19, 29, 22, 27, 22, 16, 33, 35, 45, 38, 43, 38)$.

An $\text{SOA}_2(64, 4^{25})$ with $A_2(D) = 12$: $A = (2, 4, 8, 10, 14, 15, 18, 20, 24, 26, 30, 31, 34, 36, 40, 42, 46, 47, 48, 50, 52, 56, 58, 62, 63)$, $B = (1, 3, 13, 6, 11, 6, 49, 51, 61, 54, 59, 54, 17, 19, 29, 22, 27, 22, 16, 33, 35, 45, 38, 43, 38)$.

An $\text{SOA}_2(64, 4^{26})$ with $A_2(D) = 17$: $A = (8, 10, 12, 15, 16, 18, 19, 21, 24, 27, 28, 31, 32, 34, 36, 39, 40, 45, 51, 52, 54, 56, 58, 59, 62, 63)$, $B = (35, 44, 29, 50, 14, 60, 26, 22, 6, 2, 1, 48, 37, 41, 33, 9, 25, 7, 6, 57, 1, 60, 3, 48, 47, 50)$.

An $\text{SOA}_2(64, 4^{27})$ with $A_2(D) = 20$: $A = (13, 14, 18, 19, 21, 22, 23, 25, 27, 28, 29, 31, 33, 34, 35, 37, 41, 43, 44, 46, 47, 52, 53, 57, 58, 62, 63)$, $B = (12, 9, 10, 2, 50, 48, 30, 40, 39, 56, 55, 20, 49, 45, 5, 20, 51, 39, 8, 6, 32, 2, 48, 4, 50, 3, 59)$.

An $\text{SOA}_2(64, 4^{28})$ with $A_2(D) = 22$: $A = (4, 8, 10, 12, 14, 15, 19, 20, 24, 26, 28, 30, 31, 35, 36, 40, 42, 44, 46, 47, 48, 51, 52, 56, 58, 60, 62, 63)$, $B = (2, 5, 11, 11, 9, 13, 54, 50, 53, 59, 59, 57, 61, 22, 18, 21, 27, 27, 25, 29, 16, 38, 34, 37, 43, 43, 41, 45)$.

An $\text{SOA}_2(64, 4^{29})$ with $A_2(D) = 24$: $A = (3, 4, 8, 10, 12, 14, 15, 19, 20, 24, 26, 28, 30, 31, 35, 36, 40, 42, 44, 46, 47, 48, 51, 52, 56, 58, 60, 62, 63)$, $B = (6, 2, 5, 11, 11, 9, 13, 54, 50, 53, 59, 59, 57, 61, 22, 18, 21, 27, 27, 25, 29, 16, 38, 34, 37, 43, 43, 41, 45)$.

An $\text{SOA}_2(64, 4^{30})$ with $A_2(D) = 31$: $A = (10, 11, 13, 14, 17, 19, 21, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 41, 42, 47, 49, 50, 52, 53, 58, 59, 60, 61, 62, 63)$, $B = (45, 45, 5, 55, 7, 51, 22, 24, 44, 9, 24, 54, 23, 8, 32, 15, 3, 7, 44, 55, 20, 48, 56, 20, 40, 16, 12, 4, 40, 48)$.

An $\text{SOA}_2(64, 4^{31})$ with $A_2(D) = 35$: $A = (10, 11, 17, 18, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 33, 36, 38, 39, 41, 42, 45, 47, 51, 54, 57, 58, 59, 60, 62, 63)$, $B = (2, 2, 61, 48, 19, 6, 35, 19, 55, 46, 24, 55, 13, 50, 46, 32, 35, 3, 43, 40, 15, 5, 34, 53, 50, 9, 52, 35, 56, 15, 55)$.

An $\text{SOA}_2(64, 4^{32})$ with $A_2(D) = 40$: $A = (1, 8, 10, 11, 12, 13, 19, 21, 22, 25, 26, 28, 30, 31, 35, 37, 41, 42, 43, 45, 46, 47, 49, 51, 52, 53, 54, 55, 58, 60, 61, 63)$, $B = (38, 14, 40, 50, 36, 4, 14, 7, 17, 56, 20, 24, 5, 15, 15, 2, 16, 59, 15, 20, 16, 3, 23, 3, 38, 7, 6, 23, 27, 36, 5, 29)$.

An $\text{SOA}_2(64, 4^{33})$ with $A_2(D) = 44$: $A = (1, 11, 12, 14, 15, 16, 18, 20, 21, 23, 26, 27, 28, 29, 31, 35, 36, 37, 38, 39, 40, 42, 44, 45, 46, 51, 53, 55, 56, 58, 59, 60, 63)$, $B = (32, 57, 49, 62, 7, 8, 22, 10, 13, 33, 25, 2, 25, 24, 47, 50, 13, 17, 6, 52, 34, 47, 7, 6, 61, 2, 43, 33, 6, 3, 9, 34, 54)$.

An $\text{SOA}_2(64, 4^{34})$ with $A_2(D) = 53$: $A = (9, 10, 13, 14, 15, 17, 19, 20, 22, 23, 27, 29, 30, 33, 34, 35, 36, 37, 39, 41, 42, 47, 49, 50, 51, 52, 53, 54, 55, 57, 59, 60, 61, 62)$, $B = (11, 18, 45, 38, 8, 26, 1, 24, 16, 21, 31, 48, 26, 38, 18, 32, 28, 11, 31, 5, 12, 44, 25, 46, 63, 25, 21, 26, 63, 43, 58, 16, 7, 56)$.

An $\text{SOA}_2(64, 4^{35})$ with $A_2(D) = 59$: $A = (10, 11, 12, 13, 15, 17, 18, 19, 21, 23, 25, 27, 28, 29, 30, 31, 33, 36, 37, 38, 42, 43, 44, 45, 46, 47, 49, 50, 51, 54, 55, 57, 60, 61, 62)$, $B = (16, 34, 8, 5, 40, 24, 48, 35, 20, 35, 26, 24, 52, 58, 14, 7, 32, 6, 63, 48, 41, 63, 58, 40, 14, 40, 39, 59, 6, 56, 2, 59, 58, 2, 59)$.

An $\text{SOA}_2(64, 4^{36})$ with $A_2(D) = 65$: $A = (1, 10, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 26, 28, 29, 30, 32, 34, 35, 39, 40, 42, 43, 46, 47, 48, 50, 52, 53, 54, 55, 56, 59, 61, 62, 63)$, $B = (2, 9, 33, 49, 6, 7, 4, 2, 4, 58, 38, 60, 5, 58, 6, 25, 37, 37, 5, 44, 58, 33, 57, 58, 41, 3, 41, 25, 36, 41, 18, 33, 51, 57, 2, 36)$.

An $\text{SOA}_2(64, 4^{37})$ with $A_2(D) = 72$: $A = (10, 11, 12, 13, 15, 17, 18, 19, 20, 22, 25, 26, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 44, 46, 47, 49, 52, 53, 54, 55, 58, 59, 60, 61, 62)$, $B = (56, 2, 5, 14, 8, 24, 2, 8, 3, 16, 1, 50, 45, 45, 3, 40, 57, 27, 4, 50, 21, 32, 1, 63, 48, 43, 4, 14, 4, 51, 5, 57, 45, 43, 28, 63, 56)$.

An $\text{SOA}_2(64, 4^{38})$ with $A_2(D) = 82$: $A = (10, 12, 13, 14, 15, 17, 18, 19, 21, 22, 23, 25, 26, 27, 28, 30, 31, 33, 34, 35, 37, 42, 43, 44, 45, 46, 47, 50, 52, 53, 54, 55, 57, 58, 59, 60, 62, 63)$, $B = (9, 49, 8, 51, 4, 16, 2, 51, 51, 49, 20, 48, 2, 3, 1, 29, 7, 9, 6, 11, 32, 2, 32, 41, 4, 39, 39, 38, 16, 8, 48, 7, 16, 56, 29, 56, 6, 7)$.

An $\text{SOA}_2(64, 4^{39})$ with $A_2(D) = 93$: $A = (8, 10, 11, 12, 13, 15, 17, 18, 19, 20, 22, 23, 25, 26, 28, 29, 30, 31, 34, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 51, 52, 53, 55, 59, 60, 61, 62,$

63), $B = (1, 56, 50, 14, 3, 54, 32, 2, 6, 16, 16, 2, 33, 58, 27, 57, 36, 7, 24, 2, 4, 33, 36, 32, 49, 48, 4, 40, 54, 58, 50, 49, 5, 1, 56, 36, 5, 48, 49)$.

An $\text{SOA}_2(64, 4^{40})$ with $A_2(D) = 103$: $A = (2, 3, 4, 5, 6, 7, 8, 16, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 34, 35, 37, 38, 40, 42, 43, 44, 45, 46, 50, 51, 53, 55, 56, 58, 59, 60, 63)$, $B = (14, 61, 52, 11, 41, 49, 54, 25, 11, 26, 48, 52, 57, 54, 41, 10, 13, 57, 39, 14, 49, 11, 47, 1, 15, 15, 13, 26, 13, 9, 52, 62, 61, 52, 61, 33, 48, 15, 54, 1)$.

An $\text{SOA}_2(64, 4^{41})$ with $A_2(D) = 117$: $A = (1, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 29, 30, 32, 35, 36, 37, 40, 42, 44, 45, 46, 48, 50, 51, 52, 54, 55, 56, 58, 60, 61, 62, 63)$, $B = (38, 49, 33, 9, 21, 4, 53, 41, 25, 59, 6, 59, 39, 17, 9, 47, 57, 5, 2, 49, 34, 33, 5, 33, 3, 41, 47, 41, 41, 5, 53, 21, 43, 17, 21, 9, 17, 5, 34, 57, 38)$.

An $\text{SOA}_2(64, 4^{42})$ with $A_2(D) = 131$: $A = (1, 8, 10, 11, 12, 14, 15, 16, 18, 19, 21, 23, 26, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 42, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63)$, $B = (41, 49, 17, 2, 5, 3, 6, 20, 3, 22, 24, 20, 2, 4, 4, 27, 24, 57, 3, 58, 33, 49, 33, 22, 2, 43, 6, 40, 4, 41, 27, 58, 57, 49, 7, 22, 41, 57, 58, 22, 4, 20)$.

An $\text{SOA}_2(64, 4^{43})$ with $A_2(D) = 146$: $A = (1, 8, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 38, 39, 40, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 56, 59, 60, 62, 63)$, $B = (5, 41, 58, 25, 5, 4, 9, 9, 41, 41, 4, 17, 3, 61, 3, 58, 25, 4, 7, 58, 37, 7, 18, 48, 7, 6, 2, 41, 17, 57, 4, 18, 49, 49, 61, 20, 48, 49, 18, 42, 5, 61, 6)$.

An $\text{SOA}_2(64, 4^{44})$ with $A_2(D) = 163$: $A = (1, 8, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 24, 26, 27, 28, 29, 30, 31, 32, 34, 36, 37, 39, 40, 42, 43, 44, 45, 47, 48, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63)$, $B = (7, 10, 9, 6, 46, 9, 41, 49, 35, 25, 5, 22, 57, 41, 3, 10, 25, 25, 7, 22, 3, 38, 38, 4, 33, 46, 41, 5, 22, 46, 22, 17, 35, 9, 5, 35, 7, 17, 58, 2, 58, 57, 7, 22)$.

An $\text{SOA}_2(64, 4^{45})$ with $A_2(D) = 182$: $A = (1, 8, 10, 11, 12, 13, 15, 16, 18, 19, 20, 22, 23, 24, 26, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63)$, $B = (2, 59, 3, 14, 25, 9, 9, 5, 27, 17, 17, 3, 6, 9, 41, 27, 25, 7, 17, 25, 57, 2, 21, 33, 51, 33, 25, 27, 41, 5, 4, 21, 33, 3, 27, 51, 14, 49, 6, 33, 3, 7, 4, 5, 6)$.

An $\text{SOA}_2(64, 4^{46})$ with $A_2(D) = 207$: $A = (1, 8, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63)$, $B = (6, 32, 14, 5, 2, 9, 6, 6, 23, 16, 16, 23, 6, 33, 2, 3, 40, 5, 9, 7, 33, 23, 52, 33, 6, 32, 32, 2, 3, 52, 40, 32, 40, 33, 48, 3, 5, 23, 32, 9, 52, 3, 4, 56, 6, 7)$.

An $\text{SOA}_2(64, 4^{47})$ with $A_2(D) = 233$: $A = (1, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 39, 41, 42, 44, 45, 46, 47, 49, 50, 51, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63)$, $B = (5, 40, 3, 32, 52, 38, 40, 6, 9, 38, 3, 16, 5, 16, 16, 9, 24, 43, 24, 24, 6, 43, 38, 2, 32, 4, 38, 7, 2, 2, 52, 40, 40, 4, 9, 2, 7, 5, 48, 7, 48, 2, 56, 4, 56, 6, 7)$.

An $\text{SOA}_2(64, 4^{48})$ with $A_2(D) = 263$: $A = (1, 8, 10, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63)$, $B = (3, 40, 9, 9, 6, 11, 9, 33, 16, 24, 4, 16, 16, 7, 9, 2, 3, 4, 5, 6, 7, 3, 32, 32, 4, 33, 32, 9, 11, 11, 40, 40, 6, 40, 16, 48, 56, 48, 5, 48, 48, 9, 56, 3, 4, 5, 6, 7)$.

An $\text{SOA}_2(64, 4^{49})$ with $A_2(D) = 294$: $A = (9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63)$, $B = (1, 2, 8, 4, 5, 8, 7, 16, 16, 3, 4, 16, 16, 16, 1, 24, 24, 4, 5, 24, 24, 32, 2, 32, 32, 32, 32, 32, 1, 2, 40, 4, 40, 40, 40, 1, 2, 48, 48, 5, 48, 7, 1, 2, 3, 56, 5, 6, 7)$.

An $\text{SOA}_2(64, 4^{50})$ with $A_2(D) = 336$: $A = (1, 8, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63)$, $B = (6, 17, 3, 2, 9, 9, 9, 6, 41, 3, 2, 5, 4, 17, 6, 57, 3, 25, 25, 25, 7, 25, 17, 33, 33, 5, 33, 7, 6, 33, 3, 41, 41, 41, 7, 6, 41, 3, 2, 49, 49, 7, 49, 49, 3, 2, 57, 4, 57, 57)$.

An $\text{SOA}_3(27, 9^2)$ with $A_2(D) = 2$: $A = (4, 9)$, $B = (1, 2)$.

An $\text{SOA}_3(27, 9^3)$ with $A_2(D) = 6$: $A = (4, 9, 10)$, $B = (1, 2, 1)$.

An $\text{SOA}_3(27, 9^4)$ with $A_2(D) = 12$: $A = (4, 9, 10, 11)$, $B = (1, 2, 1, 2)$.

An $\text{SOA}_3(27, 9^5)$ with $A_2(D) = 20$: $A = (4, 9, 10, 11, 12)$, $B = (1, 2, 1, 2, 5)$.

An $\text{SOA}_3(27, 9^6)$ with $A_2(D) = 30$: $A = (4, 9, 10, 11, 12, 13)$, $B = (1, 2, 1, 2, 5, 1)$.

An $\text{SOA}_3(81, 9^{11})$ with $A_2(D) = 8$: $A = (4, 9, 10, 13, 23, 24, 26, 27, 29, 30, 34)$, $B = (14, 25, 38, 1, 37, 28, 22, 12, 8, 33, 2)$.

An $\text{SOA}_3(81, 9^{12})$ with $A_2(D) = 16$: $A = (4, 11, 12, 13, 23, 29, 31, 32, 34, 36, 38, 40)$, $B = (16, 22, 8, 35, 7, 22, 9, 28, 14, 1, 6, 2)$.

An $\text{SOA}_3(81, 9^{13})$ with $A_2(D) = 24$: $A = (9, 11, 12, 23, 29, 30, 32, 33, 34, 35, 36, 39, 40)$, $B = (2, 18, 37, 24, 31, 28, 5, 19, 22, 38, 26, 4, 13)$.

An $\text{SOA}_3(81, 9^{14})$ with $A_2(D) = 32$: $A = (4, 9, 11, 13, 18, 24, 26, 28, 29, 31, 32, 35, 36, 37)$, $B = (40, 7, 25, 25, 39, 5, 22, 27, 2, 33, 22, 27, 12, 23)$.

An $\text{SOA}_3(81, 9^{15})$ with $A_2(D) = 42$: $A = (10, 12, 23, 24, 26, 29, 30, 31, 32, 33, 35, 36, 38, 39, 40)$, $B = (4, 9, 21, 5, 7, 19, 14, 16, 34, 28, 8, 22, 2, 6, 14)$.

An $\text{SOA}_3(81, 9^{16})$ with $A_2(D) = 54$: $A = (10, 11, 13, 24, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39)$, $B = (5, 26, 9, 17, 15, 18, 16, 19, 12, 2, 21, 22, 25, 34, 3, 4)$.

An $\text{SOA}_3(81, 9^{17})$ with $A_2(D) = 68$: $A = (9, 10, 12, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 36, 39, 40)$, $B = (4, 6, 7, 37, 20, 18, 8, 11, 30, 14, 7, 38, 21, 11, 13, 22, 22)$.

An $\text{SOA}_3(81, 9^{18})$ with $A_2(D) = 84$: $A = (4, 9, 10, 11, 12, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37)$, $B = (21, 32, 32, 3, 8, 39, 19, 21, 20, 15, 16, 17, 17, 38, 40, 14, 19, 39)$.

An $\text{SOA}_3(81, 9^{19})$ with $A_2(D) = 102$: $A = (4, 9, 11, 12, 13, 18, 23, 24, 26, 27, 29, 31, 32, 33, 34, 35, 36, 37, 40)$, $B = (15, 15, 25, 3, 7, 16, 2, 6, 5, 38, 16, 25, 8, 15, 7, 39, 10, 19, 3)$.

An $\text{SOA}_3(81, 9^{20})$ with $A_2(D) = 126$: $A = (4, 9, 10, 11, 18, 23, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40)$, $B = (12, 2, 19, 13, 21, 2, 16, 5, 7, 15, 2, 17, 38, 33, 21, 19, 38, 20, 13, 3)$.

An $\text{SOA}_3(81, 9^{21})$ with $A_2(D) = 150$: $A = (4, 9, 10, 11, 12, 13, 18, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39)$, $B = (3, 6, 40, 15, 5, 15, 26, 17, 5, 15, 7, 6, 17, 17, 19, 6, 15,$

6, 20, 19, 3).

An $\text{SOA}_3(81, 9^{22})$ with $A_2(D) = 190$: $A = (4, 9, 10, 11, 12, 13, 18, 23, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40)$, $B = (2, 8, 1, 20, 27, 7, 25, 21, 20, 22, 14, 25, 17, 17, 15, 27, 7, 22, 22, 19, 22, 25)$.

An $\text{SOA}_3(81, 9^{23})$ with $A_2(D) = 234$: $A = (4, 9, 10, 11, 13, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40)$, $B = (7, 2, 21, 2, 7, 15, 2, 20, 5, 5, 22, 14, 8, 3, 17, 14, 6, 14, 12, 1, 15, 19, 1)$.

An $\text{SOA}_3(81, 9^{24})$ with $A_2(D) = 284$: $A = (4, 9, 10, 11, 12, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40)$, $B = (3, 2, 5, 13, 3, 2, 22, 15, 5, 22, 7, 1, 2, 14, 16, 5, 14, 21, 13, 16, 1, 2, 22, 21)$.

An $\text{SOA}_3(81, 9^{25})$ with $A_2(D) = 330$: $A = (4, 9, 10, 11, 12, 13, 18, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40)$, $B = (3, 2, 5, 5, 8, 8, 17, 2, 5, 21, 5, 22, 14, 16, 17, 16, 19, 14, 21, 8, 6, 20, 19, 22, 22)$.