# STAT 403 Tutorial · Week 8.

Topics of this tutorial include Latin square designs and BIBDs.

## 1. Latin square designs

Example. Suppose that we wish to compare four different manufacturing processes in four time slots on each of four days. Design an experiment and then build a model to assess whether the four manufacturing processes have the same effect.

Design (Latin square):

| days \ times | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 — 23 | 2 — 33 | 3 — 18 | 4 — 37 |
| 2 | 2 — 34 | 3 — 17 | 4 — 46 | 1 — 21 |
| 3 | 3 — 20 | 4 — 34 | 1 — 22 | 2 — 40 |
| 4 | 4 — 56 | 1 — 30 | 2 — 45 | 3 — 22 |

randomization: randomly permute $\begin{cases} \text{rows} \\ \text{columns} \\ \text{treatment labels.} \end{cases}$

Model: $Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}$, $\varepsilon_{ijk} \overset{iid}{\sim} N(0,\sigma^2)$. $i=1,\dots,4$ $j=1,\dots,4$.

where, $Y_{ijk}$ - observation on day $i$ at time $j$;

$\alpha_i$ - effect of day $i$;

$\beta_j$ - effect of time $j$;

$\gamma_k$ - effect of process $k$.

$\varepsilon_{ijk}$ - error.

Analysis: $H_0$: $\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4$ v.s. $H_a$: $\gamma_k$ not the same.

averages

| days | times | processes |
|---|---|---|
| 1: 27.75 | 1: 33.25 | 1: 24 |
| 2: 29.50 | 2: 28.5 | 2: 38 |
| 3: 29 | 3: 32.75 | 3: 19.25 |
| 4: 38.25 | 4: 30 | 4: 43.25 |

$\bar{Y}_{\dots} = 31.125$

$SS_{days} = 4\sum_{i=1}^{4}(\bar{Y}_{i..} - \bar{Y}_{\dots})^2 = 4[(27.75-31.125)^2 + \dots + (38.25-31.125)^2] = 277.25$, $df_{days} = n_{days} - 1 = 3$;

$SS_{times} = 4\sum_{j=1}^{4}(\bar{Y}_{.j.} - \bar{Y}_{\dots})^2 = 4[(33.25-31.125)^2 + \dots + (30-31.125)^2] = 61.25$, $df_{times} = n_{times} - 1 = 3$;

$SS_{trt} = 4\sum_{k=1}^{4}(\bar{Y}_{..k} - \bar{Y}_{\dots})^2 = 4[(24-31.125)^2 + \dots + (43.25-31.125)^2] = 1544.25$, $df_{trt} = n_{trt} - 1 = 3$;

$SS_{total} = \sum(Y_{ijk} - \bar{Y}_{\dots})^2 = (23-31.125)^2 + \dots + (22-31.125)^2 = 1997.75$, $df_{total} = N-1 = 15$;

$SS_{err} = 1997.75 - 277.25 - 61.25 - 1544.25 = 115$, $df_{err} = 15-3-3-3 = 6$.

ANOVA table :

| Source | Sum of squares | degrees of freedom |
|---|---|---|
| days | 277.25 | 3 |
| times | 61.25 | 3 |
| processes (trt) | 1544.25 | 3 |
| error | 115 | 15-3-3-3=6 |
| Total . | 1997.75 | 15 |

$$F = \frac{SS_{trt}/df_{trt}}{SS_{err}/df_{err}} \overset{H_0}{\sim} F_{df_{trt}, df_{err}} .$$

$$F_{obs} = \frac{1544.25/3}{115\ /6} = 26.86 . \qquad \text{p-value} = P(F \geq F_{obs}) = 0.0007 < 0.05 .$$

Therefore , we reject null hypothesis at 5% level .

2. BIBDs .

Example. Recall the experiment where we compared washing solutions on different days. ( See tutorial notes of week 6 ) Each washing solution is a treatment and each day is a block. Now suppose there are 5 washing solutions to be compared on 5 days. However, only 4 experimental runs can be carried out on each day. Design an experiment to assess whether the 5 washing solutions are the same.

Solution : Because the block size is less than the number of treatments , we need to use the BIBD, below is one possibility:

BIBD.

| | | | | |
|---|---|---|---|---|
| day 1: | 1 | 2 | 3 | 4 |
| day 2: | 1 | 2 | 3 | 5 |
| day 3: | 1 | 2 | 4 | 5 |
| day 4: | 1 | 3 | 4 | 5 |
| day 5: | 2 | 3 | 4 | 5 |

randomization :

randomly permute $\begin{cases} \text{block labels} \\ \text{trts within blocks} \\ \text{treatment labels.} \end{cases}$

Suppose this is the data observed:

| | | blk 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| | 1 | 32 | 47 | 22 | 50 | |
| | 2 | 27 | 40 | 27 | | 40 |
| trt | 3 | 41 | 66 | | 45 | 52 |
| | 4 | 40 | | 30 | 50 | 70 |
| | 5 | | 70 | 33 | 48 | 63 |

Model:

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \quad \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma^2) \qquad \begin{matrix} i = 1, \dots, 5 = t \\ j = 1, \dots, 4 = b \end{matrix}$$

where $Y_{ij}$ - obs. of solution $i$ on day $j$;

$\mu$ - grand mean;

$\alpha_i$ - effect of solution $i$;

$\beta_j$ - effect of day $j$;

$\varepsilon_{ij}$ - random error

Analysis: (See R Script.)

We still use $F = \dfrac{SStrt/dftrt}{SSerr/dferr} = \dfrac{SStrt/(t-1)}{SSerr/(N-b-t+1)} \sim F_{t-1, N-b-t+1}$.