

Hypothetical Application of The Bekenstein-Hawking Entropy to Geophysical Gravimetric Survey Data

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"God does not care about our mathematical difficulties. He integrates empirically." - Albert Einstein

Abstract

This article shows a mathematical exercise using complicated concepts from black hole physics and some quantum gravity to explore new ways to visualize geophysical gravimetric data. The purpose here is to make a "gedankenexperiment", using the human imagination, to derive new possible tools that could have applications in the real world, especially in the field of mineral exploration by geophysical prospection. Further discussions on these topics are clearly necessary, being these texts here just the thought process that could produce in the future fertile grounds for real world case scenarios. Here it will be explored a computer code that transforms Bouguer anomaly maps into gravitational entropy and spacetime information quantity maps and it will be exposed the possible interpretation of these manipulations.

Introduction

Geophysical gravimetric surveys have long been a valuable tool in Earth science, allowing us to unveil the mysteries hidden beneath the Earth's surface. These surveys measure variations in the gravitational field caused by irregularities in the distribution and density of rocks within the planet's crust. These variations, known as gravitational anomalies, are crucial for a wide range of applications, from prospecting for valuable resources like base metal mineralizations, gold deposits, oil, and gas, to understanding geological structures in sedimentary basins, and Pre-Cambrian terranes.

Traditionally, gravimetric surveys mainly result in 2D maps of Bouguer anomalies, representing variations in surface gravity caused by differences in subsurface rock densities. While these maps provide valuable insights into subsurface geology, a new approach leveraging theoretical physics could potentially offer a fresh perspective on interpreting these anomalies beforehand of a complex geophysical inversion being carried on.

In this article, we will explore the application of the Bekenstein-Hawking entropy concept to geophysical gravimetric survey data processing and analysis. To understand this approach better, we will first delve into the fundamentals of Newton's Universal Theory of Gravity, Einstein's General Theory of Relativity, the intriguing world of black holes, and finally, the concept of Bekenstein-Hawking entropy.

Newton's Universal Theory of Gravity

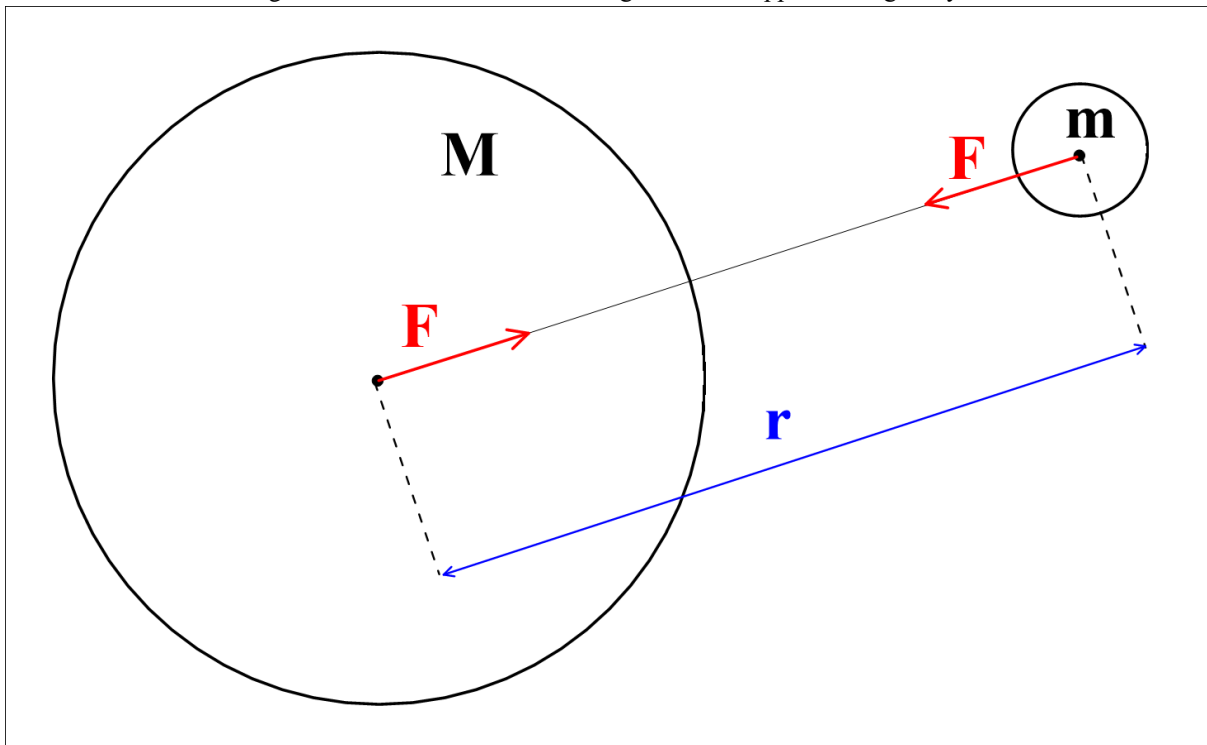
Newton's Universal Theory of Gravity, formulated in the 17th century, described gravity as an attractive force between masses, governed by the inverse square law of distance (Newton, 1687). In this classical view, gravity is mathematically treated as a force field centered on a spherical distribution of mass, with all the mass concentrated at the center of the celestial body. The Lagrangian interpretation of Newton's Gravity translates it into a gravitational potential formula, and this potential relates with the distances between test masses. Newton summarizes his theory in a simple and elegant equation that describes the gravitational force between two masses 'm₁', and 'm₂' separated by a distance 'r' (Halliday *et al.*, 2017).

$$F = \frac{m_1 m_2}{r^2} \quad (1)$$

Equation (1) states the inverse square law of gravity (Figure 1). Problems involving more than two masses require numerical calculations. In the context of geophysics, the gravimetric survey involves measuring the force of gravity and how a test mass accelerates towards the center of the planet removing all the interferences from the Moon, the Sun, and from the irregularities in the topography of Earth.

r²

Figure 1 - Newton's mathematical/geometrical approach to gravity.

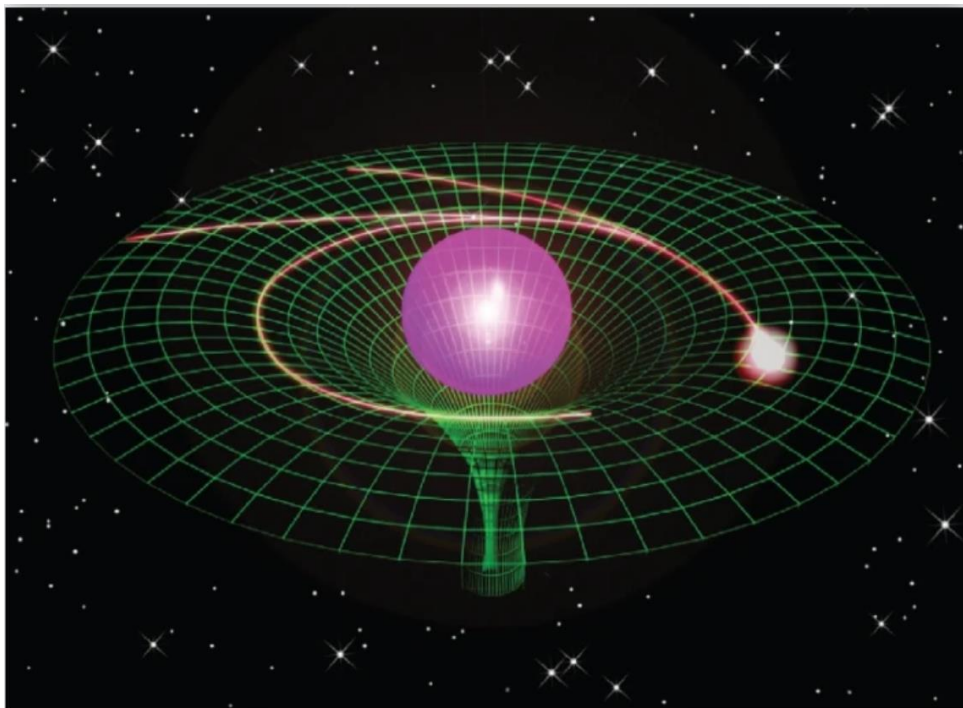


Einstein's General Theory of Relativity

Einstein's General Theory of Relativity (GR), introduced in the early 20th century, revolutionized our understanding of gravity (Einstein, 1915 (part 1 - 4)). According to his theory, gravity is not a force but rather a consequence of the curvature of spacetime caused by the presence of mass and energy distributions (Figure 2). In this view, objects move along geodesic paths in a curved spacetime, where time is intimately connected with the spatial dimensions, forming a unified entity called spacetime.

Mass and energy distributions deform spacetime, causing the straight lines of Euclidean geometry to become curved geodesics in Riemannian spaces. The Schwarzschild metric, the first exact solution of Einstein's equations, describes the geodesic trajectories of small masses around massive objects with spherical static distributions in a vacuum. It also defines a critical region known as the event horizon, where the gravitational field is so intense that not even light can escape—a defining characteristic of black holes.

Figure 2 - Oversimplified representation of the curvature of spacetime by the presence of a large mass such as a star. The spacetime curvature caused by the star's mass and energy forces the light rays and other objects to bend its trajectory.



Source: Sheng *et al.* (2013).

Mass and energy distributions in the Universe cause deformations on the fabric of spacetime and this causes straight lines to be curved in the direction of the masses (Thorne *et al.*, 2000). The Schwarzschild metric describes a mass-energy distribution that hits a critical density that creates a specific region in spacetime where the gravitational field is so intense that even light cannot escape it. This region has an escape velocity equal to the speed of light and is called the event horizon.

Using Newton's Law of the Universal Gravitation we could derive the radius of this region in which light cannot escape. The escape velocity is the minimum velocity in which a body must achieve to escape the influence of the gravitational field of a body. The formula to calculate the escape velocity is:

$$v = \sqrt{\frac{2GM}{R}} \quad (2)$$

Where in equation (2) 'v' is the escape velocity, 'G' is the universal gravitational constant, 'M' is the mass of the body, and 'R' is the radius of the body. By making 'v' equals the speed of light 'c' we obtain the maximum radius achieved by a mass distribution to become what we today call a black hole.

Black Holes: A Gravitational Enigma

Black holes are regions in spacetime where gravity is so intense that nothing, not even light, can escape their grasp. This phenomenon is intimately connected to the concept of escape velocity—the minimum velocity required for an object to break free from a celestial body's gravitational pull. The Schwarzschild radius (R_s) determines the size of a black hole, and can be derived by Newton's formula for the escape velocity of a gravitating body showed on equation (2), and it is calculated as:

$$R_s = \frac{2GM}{c^2} \quad (3)$$

Where in equation (3), 'G' is the universal gravitational constant, 'M' is the mass of the body, and 'c' is the speed of light. This radius marks the boundary of the event horizon, beyond which nothing can return. For example, for a star with the mass of the Sun to become a black hole it would need to shrink to the diameter of roughly 6000 km, about the size of a small planet. This formula is embedded in the Schwarzschild metric and represents the radius of the event horizon of the simplest type of black hole - a Schwarzschild Black Hole (Figure 3). The region where the escape velocity equals the speed of light in vacuum is called the event horizon, the point of no return. The events occurring beyond the event horizon are permanently disconnected from the outer Universe. But observers that traversed the event horizon continue to see the rest of the Universe in a very strange manner. For further material about black hole physics, one can consult Novikov (1997), Wald (2001), Frolov & Zelnikov (2011), Hawking (1965), and Bekenstein (2004).

Since the detection of powerful astronomical radio sources and the advent of more sophisticated optical, infrared, gamma, and x-ray telescopes, today we know that the Universe is abundantly filled with black holes, and the most massive ones are located at the centers of

the billions and billions of galaxies (Figure 4). It is estimated that there are trillions of stellar-mass black holes out there and the supermassive black holes can reach billions of solar masses and be as big as the entire diameter of our Solar System! A stellar-mass black hole forms when a massive star empties its hydrogen fuel and begins a process of helium fusion, then carbon fusion, all the way up to the iron element. It is through the nuclear fusion inside massive stars that the elements of the Periodic Table heavier than hydrogen, and helium, are created. Each step, as more massive elements are fused and created, the star becomes more and more depleted of energy and its self-gravity starts to crush the star over its own weight.

Once the fusion ends when element iron is created at the star's core (the iron nucleus does not generate energy in fusion, it consumes), the outer pressure of the gas generated by the fusion energy ceases and there is nothing left to counteract the crushing force of the star's own gravity. Then, the gravitational collapse begins. If the star's core is more massive than ~ 2 solar masses, then the process of collapse is unstoppable and all the mass of the star is shrunk to sizes that approaches zero radius and infinite density. This point of infinite density and zero radius is called 'singularity'. The gas envelope of the star explodes violently in a supernova. And the core becomes the singularity. Actually, the gravitational singularity is not a 'point' but rather a region where spacetime as described by GR makes no sense anymore, being necessary a more robust theory to exactly explain what happens at these exotic regions of spacetime. Incomplete theories of quantum gravity exist and must be finished to solve the mystery. A region of spacetime is formed as the process of singularity formation goes on and this region disconnects the outside Universe from the events inside the created black hole. This region of no-escape of light is called 'event horizon'.

Figure 3 - Schematic representation in space and spacetime of a Schwarzschild black hole.

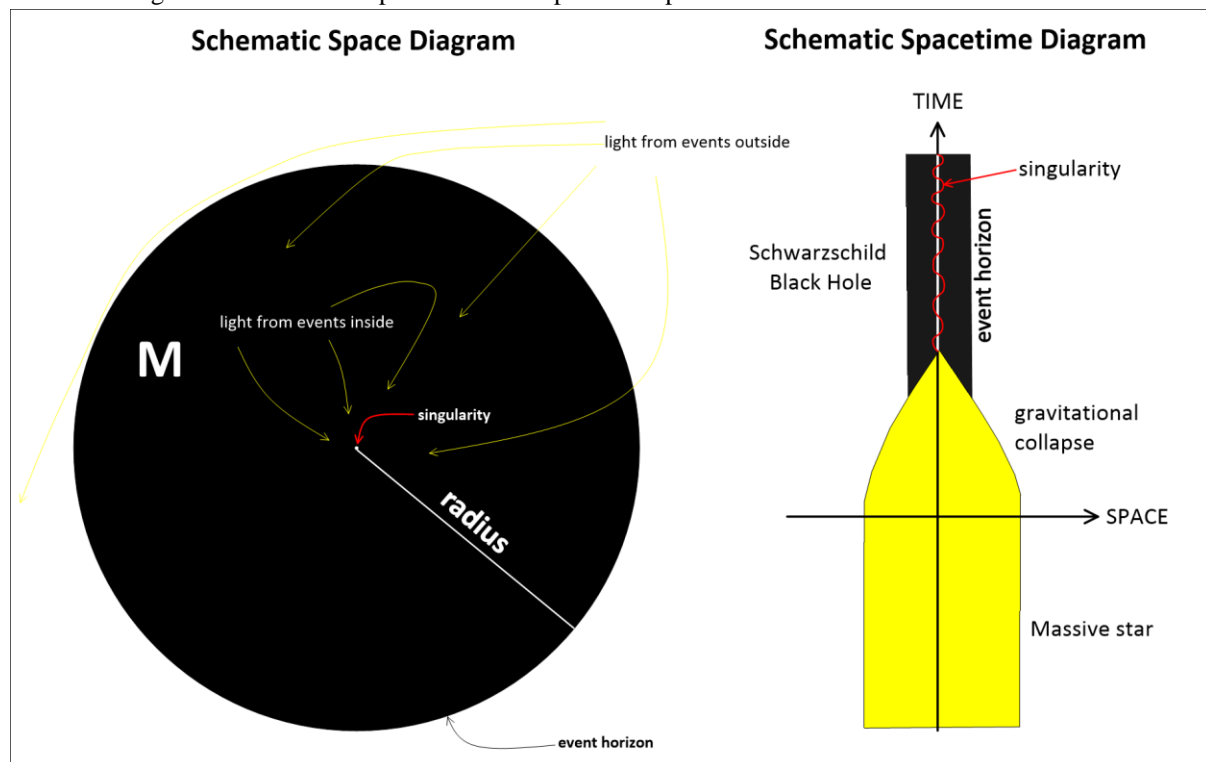
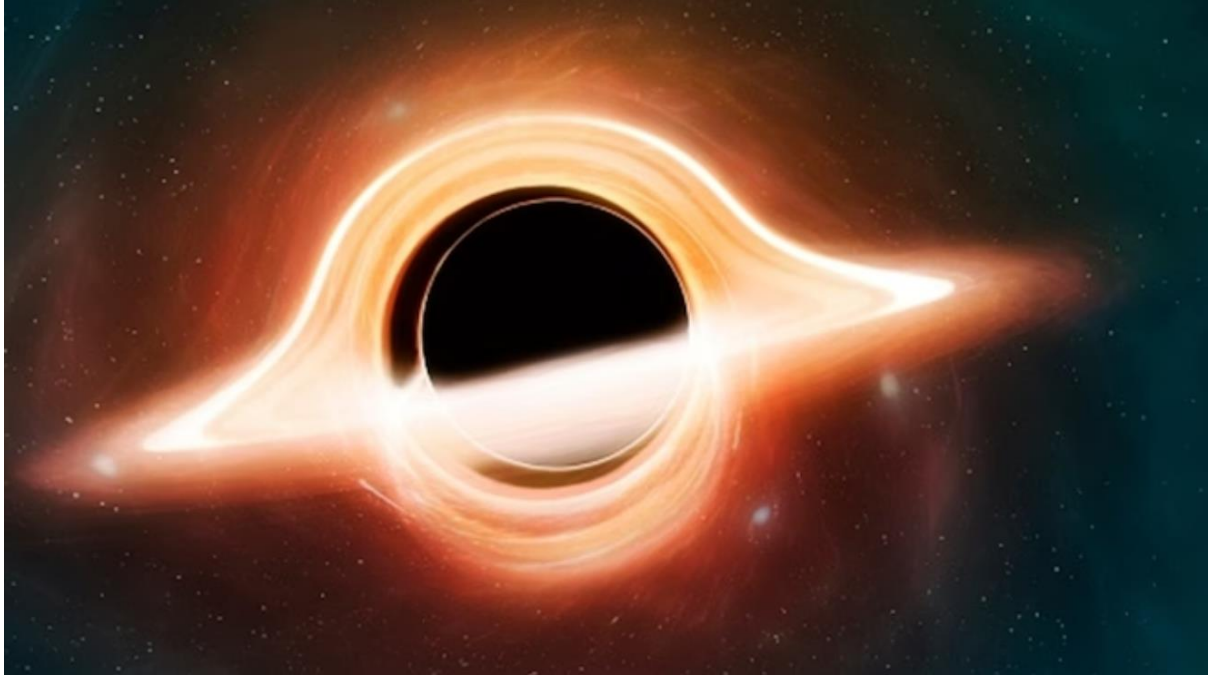


Figure 4 - Computer simulation rendering of a supermassive black hole, a common object that exists in the center of most galaxies. The dark sphere is the black hole's event horizon, a thin ring of light circulating it represent the photons trapped in a circular orbit around it. The accretion disk of gas and cosmic dust is apparently encapsulating it because of the strong gravitational lensing effect.



Source: <https://www.indiatoday.in/science/story/astronomers-discover-supermassive-black-hole-hiding-in-universe-s-most-energetic-object-1961788-2022-06-13>

The Gravimetric Method

Gravimetric surveys focus on the local variations in gravitational acceleration caused by mass irregularities within the Earth's crust. These variations are measured using devices known as gravimeters, with anomalies typically measured in milligals (1 Gal = 1 cm/s²). These variations are separated in different types of gravitational anomalies: The Bouguer anomalies, Free-Air Anomalies and the regional and residual components of the Bouguer anomaly map. The Bouguer anomaly is caused by the density gradient of the Earth's crust (Lowrie & Fichtner, 2020). It reveals indirectly the presence of dense rock bodies surrounded by less dense country rocks or vice-versa. For a crust of thickness 'H' and density 'rho', the difference in the gravitational acceleration caused by the crust is given by the integrated Bouguer formula:

$$dg = 2\pi\rho GH \quad (4)$$

Equation (4) is an approximation to understand Bouguer anomalies as the result of variations of local gravitational acceleration due to density gradients in a planetary crust. In other words, Bouguer anomalies are caused by excess or deficit of mass in the planetary crust. The Free-Air anomaly map denotes the gravitational anomalies caused by height above the reference ellipsoid in the planetary crust. In essence it reflects the variations of gravitational acceleration due to topography. As a result, Free-Air Anomaly approximately follows the geomorphology.

Common mathematical treatments employed in gravity geophysical data are the Fast Fourier Transform (FFT) to derive the Gaussian Filter that separates, using the power spectrum derivation, the gravitational field anomalies into its regional and residual components (Nussbaumer & Nussbaumer, 1982). The regional derivation of the gravity Bouguer anomalies denotes deep sources of gravitational anomalies and the residual component denotes shallow gravitational anomalies.

Geophysical inversion of Bouguer data results in a 2D and/or 3D map of the survey area showing the density variations in the crust. For the calculations of gravity inversion some geological information from drill holes or mapping are needed to input on the model to perform the inverse problem-solving processing of gravity data. Here it is proposed another kind of mathematical approach to derive other maps from gravimetric data.

The main objective is to derive an anomaly map with a new vision of gravitational geophysical anomalies. This new approach could be applied to visualize in an entirely novel and unusual perspective some geological scenarios from indirect results from pseudo-black hole entropy scenarios. The entropy, as a Boltzmann's statistical mechanics vision, represents hidden information of a system. The system is the Earth's crust in a specified survey area, and the hidden information is the rock composition and density variations in the crust.

The scenario is constructed by transforming a 2D Bouguer map into a 2D frame of mini-black holes in which in each lattice of the network we have an individual black hole with the surface gravity equals to the gravitational acceleration from Bouguer map considered in Planck units. From this transformation it is possible to calculate the Bekenstein-Hawking entropy from each individual black hole, and then interpolate all the values of the entropy into a new 2D map that will be called here Hawking Entropy Map.

We will see at the end of this article the conclusion that regions with high absolute values of Bouguer anomalies will have lower Hawking entropy in Planck units, meaning that it represents possible regions of more massive rock bodies in the crust, and vice-versa. The conclusion will be a dataframe of logarithmic gravitational entropy values in a coordinate grid. The algorithm presented here transforms Bouguer map into gravitational entropy map. The interpretations of this kind of mathematical manipulation are diverse, but the one suggested here is the Boltzmann-derived informational entropy view.

The Bekenstein-Hawking Entropy

The Bekenstein-Hawking entropy concept, rooted in black hole physics, introduces the idea that black holes possess entropy proportional to their event horizon's surface area (Bekenstein, 2004; Hawking *et al.*, 1995). Applying this concept to geophysical data analysis, we can start to visualize gravitational anomalies in a new light.

This mathematical and physical concept begins when by studying the behavior of black holes it was determined that, apparently, they violate the laws of thermodynamics. A region of spacetime in which a black hole consumes matter and energy seems to be decreasing in total entropy because according to GR a black hole cannot emit anything to compensate for the loss of entropy on the external Universe.

Hawking (1976) proposed a hybrid model that uses the concepts of quantum fields in the presence of a black hole. His calculations consider several quantum fields in the vacuum near a curved spacetime. In quantum field theory, vacuum is not empty but it is made of a swarm of emerging pairs of particles and antiparticles that appears and disappears by mutual annihilation at any given moment composing the so-called zero-point energy field (Jaffe, 2005). In a flat spacetime, far from mass-energy distributions, the spacetime fabric is described by the Minkowski metric. In this situation the particle-antiparticle pairs are constantly popping in and out of existence and the net contribution to the real energy of the Universe is zero.

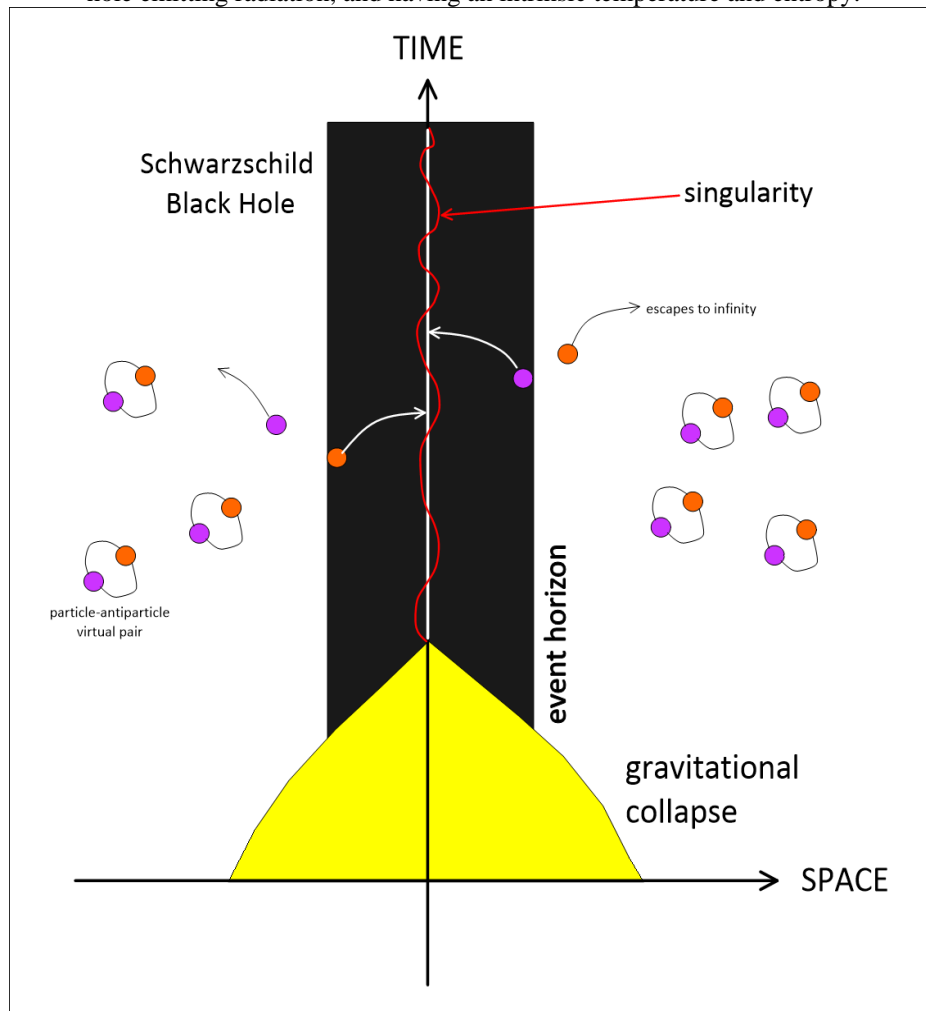
Because these particles do not survive long enough and do not contribute to the net energy of the Universe they are called virtual particles. But what Hawking discovered is that near the event horizon the quantum vacuum is disturbed by the intense gravitational field of the black hole. When a particle-antiparticle pair is created near the event horizon there is a nonzero probability of one of the particles of the pair being swallowed by the black hole's gravity and the other particle from the pair becoming a real particle and escaping to infinity (Figure 5).

For a distant observer it appears that the black hole is emitting radiation (Hajicek, 1987). The resultant scenario is that the particle emission from the vacuum subtract some tiny amount of the mass of the black hole and these particles being constantly emitted near the event horizon received the name of Hawking Radiation. This radiation has the distribution of a black body radiation and obeys Wien's displacement law and Planck statistical distribution (Ranganath, 2008). Then, Hawking could calculate that there is a nonzero temperature of the event horizon.

If black holes emit Hawking radiation and have an intrinsic temperature, therefore they have entropy. The formula to calculate this entropy is presented on the following equation (5). The black hole entropy arose from the laws of now called black hole thermodynamics. This is a quantum phenomenon and it is not permitted by GR. Then this overwhelming theoretical and mathematical breakthrough became the gateway to quantum gravity, a glimpse of the future theory that will unify General Relativity and Quantum Mechanics.

$$S = \frac{c^3 A}{4G\hbar} \quad (5)$$

Figure 5 - Hawking's discovery. A black hole interacts with quantum vacuum and eventually engulfs a virtual particle and its companion of the virtual pair becomes real, escaping to infinity. This is equivalent to a black hole emitting radiation, and having an intrinsic temperature and entropy.



Where in equation (5), ‘ S ’ is the black hole entropy, ‘ c ’ is the speed of light in vacuum, ‘ G ’ is the gravitational constant, ‘ \hbar ’ is Planck’s reduced constant, and ‘ A ’ is the area of the event horizon. According to Hawking calculations black holes will emit radiation as a black body and become increasingly hot until it will completely evaporate in a burst of pure energy. By this scenario the black hole is not eternal and the time of existence of a black hole is inversely proportional to its mass. If a supermassive black hole eventually evaporates after a mind-blowing time frame, then all the information stored in it will be lost forever, violating the fundamental principle of conservation of information (Hawking, 2005). Some interpretations from Susskind *et al.* (1993) have considered that black holes will survive at the last minute of evaporation transforming into stable tiny black holes. These are called quantum black holes.

These scenarios for the resolution of the Information Paradox can be further explored in t’Hooft (1965), Susskind *et al.* (2004), Almheiri *et al.* (2013), and Polchinski (2015). In the realm of quantum mechanics black holes behave as fundamental particles and Hawking radiation is no longer emitted by quantum black holes according to some interpretations of some theories of quantum gravity that predicts the smallest black hole possible that would be stable conserving

the residual holographic information about the pre-collapse phase of the black hole before Hawking evaporation. For further information on quantum gravity one can consult Weinstein & Rickles (2005), Smolin (2008), Rovelli (2004), and Rovelli (2008). Normally, the quantum gravity manipulations are made in Planck units to simplify calculations and visualization of the final results. That is what is done here.

The meaning of Planck units is to reduce all the measurements in Planck units of length, mass, and time (Gaarder, 2016; Humpherys, 2021). To simplify the calculations, it is common to reduce the fundamental constants to unity. Therefore, in Planck units all equations and values reduce by making $G = c = \hbar = 1$. By definition, making these assumptions, the length, mass, and time in Planck units will reduce to $l_P = m_P = t_P = 1$. By rescaling all the physical constants to unity, all other physical units become dimensionless which is much easier to analyze in extreme conditions like tiny black holes and very large numbers.

There is a mathematical relation between the Schwarzschild radius of a black hole and its surface gravity. In Planck units this relation is $R_s = 1/2g$. Therefore, the radius of black hole with $g = 10$ in Planck units will be $0.05 l_P$. Then the mass will obey the relation $R = 2M$, then $M = 0.025 m_P$. Where l_P is the Planck length and m_P is the Planck mass. Because fundamental particles have masses much lower than the Planck mass, we can conclude by assumption that quantum black holes could have masses below $1 m_P$. For further information on quantum black holes one can consult Bekenstein (1997), Carr & Giddings (2005), Dabholkar & Nampuri (2012), and Calmet *et al.* (2014).

Imagining a grid of little black holes with different surface gravities, we construct a scenario where we have a network of little black holes that will emulate the bidimensional function that describes gravitational acceleration variations of a Bouguer map. Then we perform a computation that will interpolate the different surface gravities of the individual black holes to become a hypothetical continuum mimicking a 2D grid of variable gravitational potential. The variation of Bekenstein-Hawking entropy in the 2D grid will obey black hole thermodynamics in which gravitational entropy means hidden information stored in a given region of spacetime where the total entropy of that region is dependent on the area of an individual black hole event horizon.

All the information of a gravitationally collapsed system is stored inside the black hole, behind the event horizon lying deep in the singularity. Information about the singularity is coded on the black hole entropy. One of the interpretations is that the total sum of all the information of the particles and interactions that existed in the system before gravitational collapse is scrambled and flattened in the area of the black hole horizon as a holographic projection of what is inside the black hole composing the unknown entity called gravitational singularity.

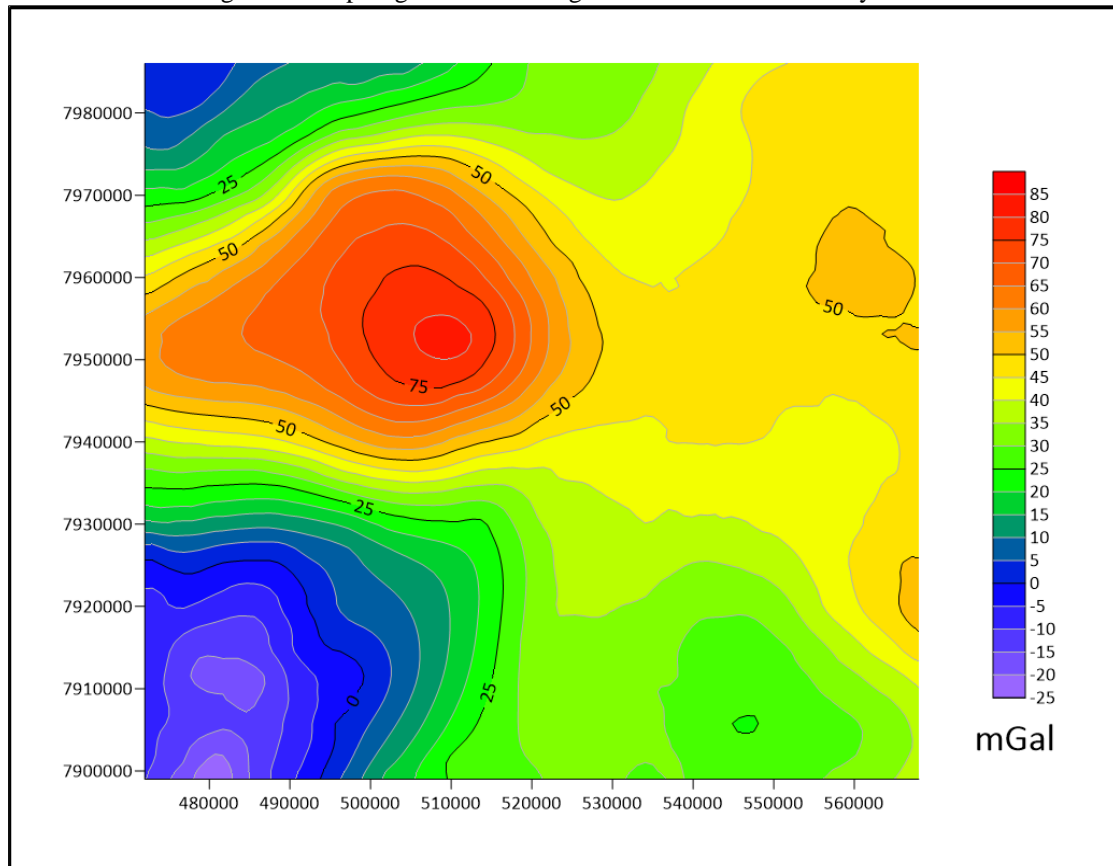
Results and Discussions

Here it will be used as an example the Bouguer gravimetric data from an area from the Brazilian coast available in the public internet gravimetric database – BNDG (*Banco Nacional de Dados Gravimétricos*) (Figure 6). The data will be transformed into a Hawking Entropy Map by using a developed python script that applies the equations for calculating entropy in Planck units. The program will read the data as a ‘csv’ file, and transform it to a new data-frame of entropy values in logarithmic scale that will be presented as a graphical map using matplotlib library for visualization.

The equation to calculate the Hawking Entropy for an individual black hole in our hypothetical mini black hole 2D grid can be derived from the Hawking entropy formula and from the relation between the surface gravity and the Schwarzschild radius formulas. By remembering that the area ‘A’ of the event horizon corresponds to the area of a spherical region where the radius of the sphere is equal to the Schwarzschild radius and to evaluate dimensionless values, to simplify our calculations, we transform this formula to contain Planck units, making $G = c = \hbar = 1$, as previously discussed. Therefore, in Planck units, substituting equation (3) into equation (5) the entropy formula becomes:

$$S = \pi R_s^2 \quad (6)$$

Figure 6 - Map of gravimetric Bouguer anomalies for the study area.



The formula to calculate the surface gravity of a black hole is derived from the Newton's gravitation equations given by:

$$g = \frac{GM}{R_s^2} \quad (7)$$

But in Planck units this equation reduces to:

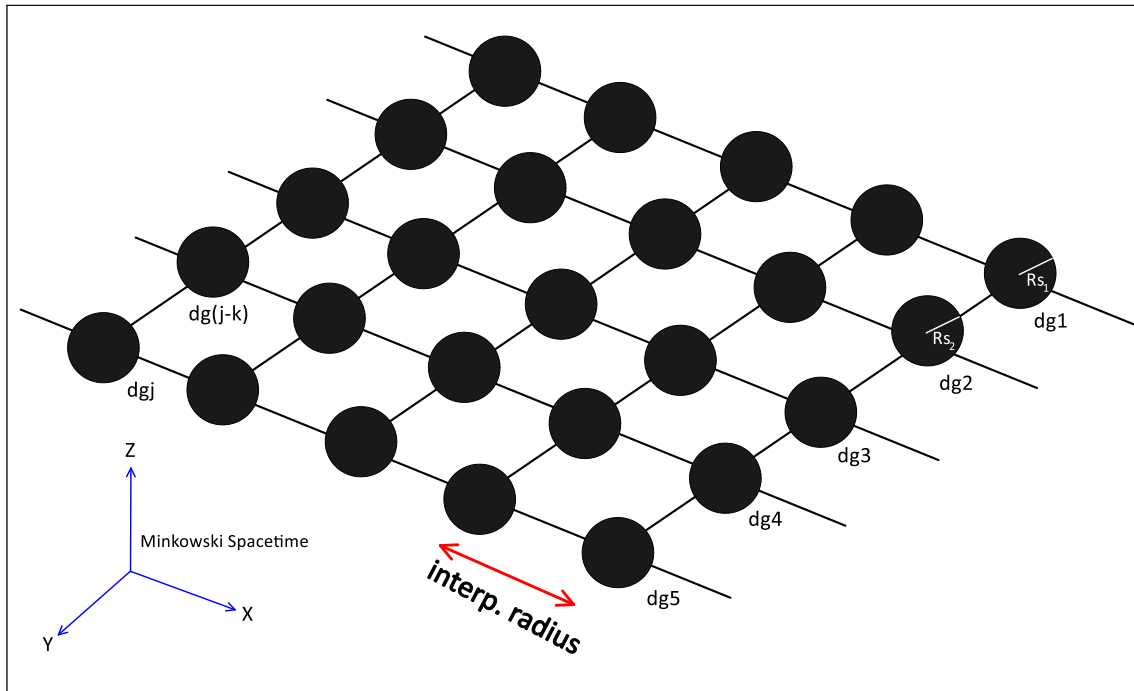
$$g = \frac{M}{R_s^2} \quad (8)$$

Remembering that $R_s = 2M$ in Planck units, the final entropy equation in relation to the surface gravity 'g' becomes:

$$S = \pi \left(\frac{1}{2g} \right)^2 \quad (9)$$

Equation (9) calculates the entropy for each individual black hole in a 2D network of black holes with different 'g' values of surface gravity. This equation will be applied to each point in the Bouguer map considering each one as a mini-black hole. The main procedure corresponds to the process of converting a 2D gravity map into a 2D network of mini-black holes to perform entropy calculations (Figure 7).

Figure 7 - The grid of Bouguer value points is transformed into a network of little black holes in a Minkowski spacetime.



The Minkowski spacetime states for the ‘normal’ spacetime that has an intrinsic flat geometry, being pseudo-Euclidian. Euclidean space represents a special case of the Minkowski spacetime. Before any further consideration, all the Bouguer values, that are represented in *milligals*, must be converted to Planck units. A simple exercise of dimensional analysis will result in the final relation:

$$\begin{aligned} 1 \text{ mGal} &= 1 \times 10^{-3} \text{ cm/s}^2 = 1 \times 10^{-5} \text{ m/s}^2 \\ 1 \text{ m} &\sim 6.2 \times 10^{34} l_P \\ 1 \text{ s} &\sim 1.8 \times 10^{43} t_P \\ 1 \text{ mGal} &= (1 \times 10^{-5}) \cdot (6.2 \times 10^{34}) / (1.8 \times 10^{43})^2 \sim 1,82 \times 10^{-57} \text{ Planck units} \end{aligned}$$

Planck units above mean ‘ l_P/t_P^2 ’. The values of entropy obtained through the Bekenstein-Hawking formula are very large because it represents a saturation of information contained in a region of spacetime of tiny black holes where it developed an event horizon. According to quantum gravity preliminary interpretations, the entropy of a black hole represents the maximum capacity of a given volume of spacetime to store information. This region is not dependent on the volume of spacetime but on the area of the surrounding region of the volume which is the area of the event horizon.

Observing again the equation (6) the entropy in Planck units can be understood as area. Then, using the definition given in some theories of quantum gravity, gravitational entropy means hidden information that can be measured in bits. As all the information inside the black hole is coded holographically in the area of the event horizon, each area unity in spacetime is proportional to a unit of entropy. The quantum of area is considered to be the Planck Area which corresponds to the Planck length squared. Each Planck area stores exactly 1 bit of information.

The value of the Planck Area in SI unit is $\sim 2.6 \times 10^{-70} \text{ m}^2$. This is a very small number, then, it will produce naturally very large numbers of bits. Notice that because we are working with Planck Units, then the entropy is proportional to the number of bits. Here it will be considered the total sum of number of bits in the map, and this number is very big indeed, being measured in ‘googolbytes’, where $1 \text{ GgB} \sim 1,6 \times 10^{100} \text{ bits}$! It even exceeds the googolbyte by 100 million factor which it could only be ‘translated’ here as “Giga-Googolbytes”. Observing the Bekenstein-Hawking formula, notice that the total area of the event horizon in equation (6) is four times the entropy. This is so, because the event horizon is a spherical region. Therefore, the relation in Planck units is:

$$A = 4S$$

After each entropy calculation at each point using a simple code, the values of the entropy are considered in logarithmic scale. This is so because the generated numbers are very large.

Then the algorithm takes all the values of Bouguer anomaly values called here ‘ dg_j ’ considered in Planck units and performs the calculation of the Hawking entropy to each point in the grid. After the computation of the entropy, all the values are converted to base 10 logarithms. Then the values are transformed into a smooth 2D function by cubic interpolation method. The result is shown as a plot of 2D Hawking Entropy Map. Each value will be considered the surface gravity of an individual black hole. The grid of black holes will have an interpolation cell size corresponding to the average of all the resultant Schwarzschild radius of the individual black holes in the grid. Then, each entropy point will be calculated using equation (9) as follows:

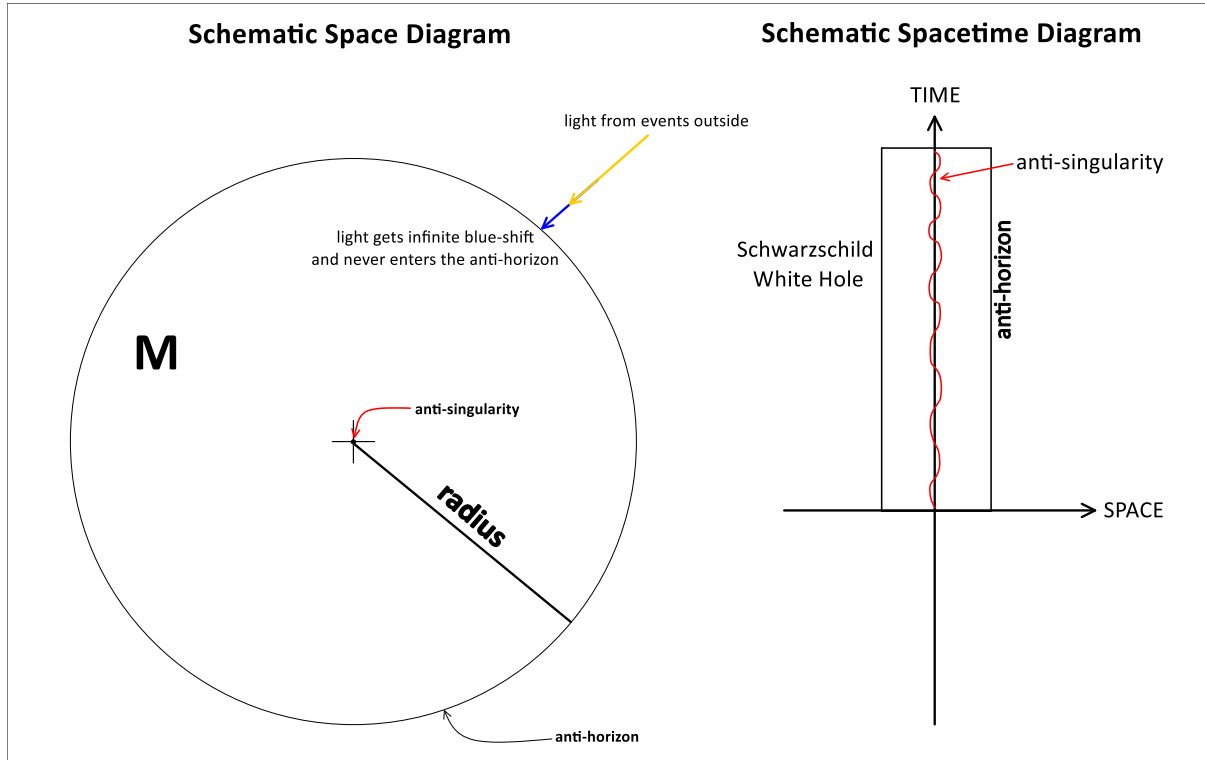
$$S_j = \pi \left(\frac{1}{2dg_j} \right)^2 \quad (10)$$

We can see by generating this maps that the pure state of the data shows that the variation of the entropy is wild and the resultant Schwarzschild radius is relatively large considering in respect to quantum scale. This could be referred here as the ‘pure state’ of the algorithm and does not need an extra layer of mathematical manipulations to ‘imitate’ the topology of the original Bouguer map.

Furthermore, the negative Bouguer values are automatically cancelled out because the gravitational entropy formula term for the surface gravity is squared, then, it does not matter if the values are negative or positive, the output is always positive. This has profound implications. That is because negative absolute gravity means repulsive gravity which is mathematically described in objects called *white holes* (Figure 8), the time-reversals of black holes. But white holes, by mathematical definition, even being gravitational repulsive, has positive total entropy. In essence, a white hole is the spacetime region where everything exits from the anti-horizon, and nothing in the Universe goes to the singularity. The instability of white holes could be dictated by the fact that their entropy is positive and then the repulsive gravitational field rapidly converts to normal gravitational field converting the white hole into a black hole.

Therefore, under this perspective, the scenario established here is equivalent to consider that our network, in its ‘initial’ state, is composed of black holes and white holes. Where the gravitational acceleration is negative, we have white holes and where the gravitational acceleration is positive, we have black holes (Figure 9). Another interesting view of this scenario is, considering that, if the network has an even number of black holes and white holes, then this is equivalent to a network of Schwarzschild wormholes which is also unstable converting itself all back into black holes.

Figure 8 – Schematic representation of a white hole.



The computer program here is available in which we could explore this fascinating scenario where we can see the natural state of the network of gravitationally collapsed objects and its behavior. In this scenario the variation of number of bits of entropy is considerably flattened depending on the original gravimetric data and, in this case, it was chosen not to include a map of bit distribution for the code represented in the file ‘Graviton_entropy.py’, otherwise, it was chosen here to represent this as a total sum of bits that is saved as ‘txt’ file. A way of visualizing graphically the spacetime governed by the Schwarzschild metric is using Kruskal Diagrams. On this diagram the region of spacetime contains the whole mathematical scenario described by this exact solution of Einstein’s Equations and allows us to visualize all three entities described here: Black hole, white hole, and the wormhole (Figure 10).

Figure 9 – The Bouguer grid transformed into a network of black holes and white holes representing its ‘pure initial state’ before unstable white holes decay into black holes.

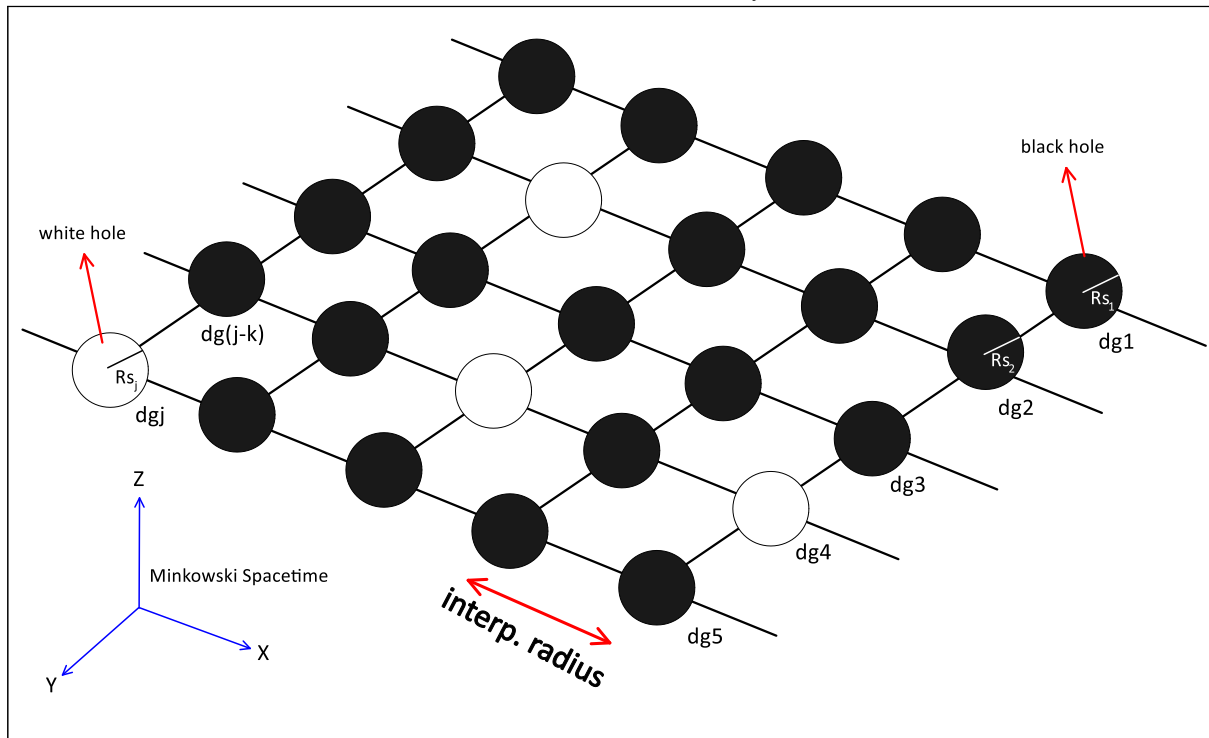
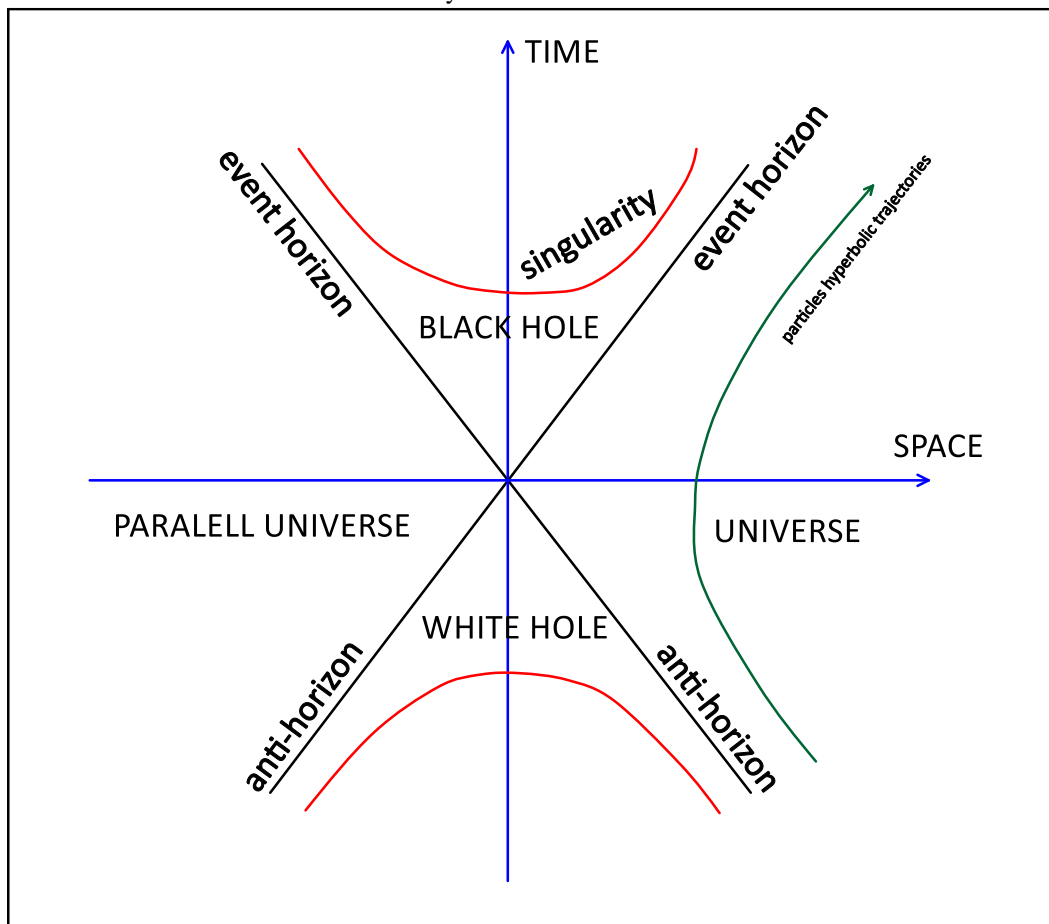


Figure 10 – Simplified representation of the Kruskal-Szekeres Diagram representing the full spacetime set described by the Schwarzschild metric.



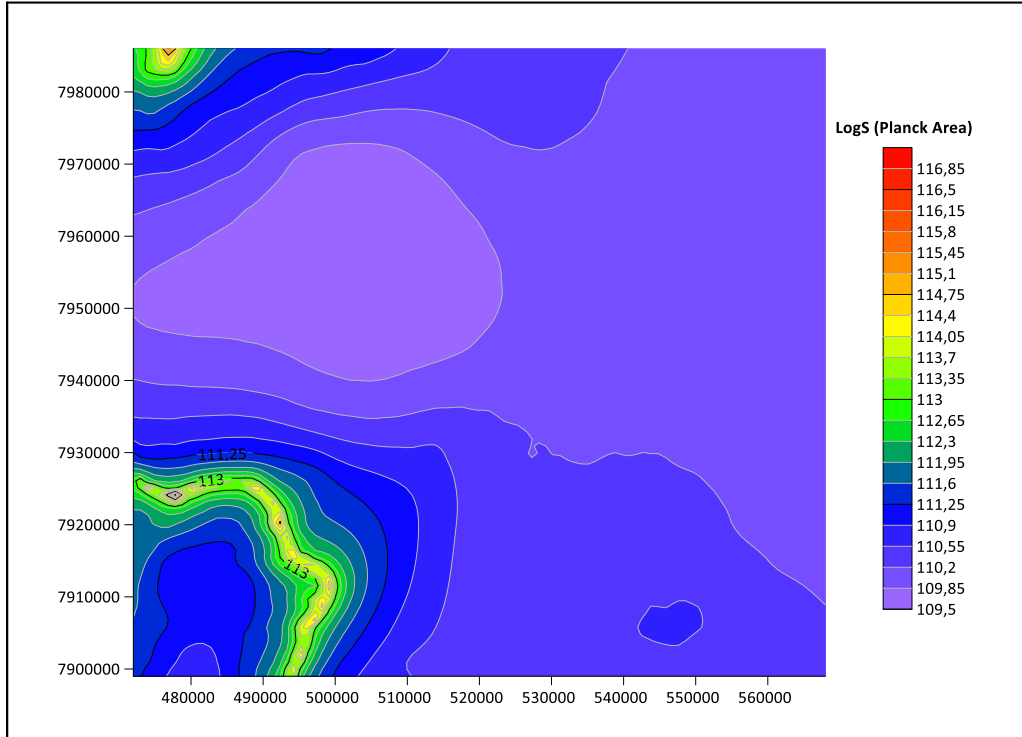
Interpretation and Further Research

There are several interpretations of the gravitational entropy, but the one that will be considered here will be the common perspective from Statistical Mechanics that treats entropy as the hidden information of the microstates of a system. The primordial hidden information encoded in the gravitational entropy on the generated Hawking map is rock density and composition. This is because what defines the surface gravity of a hypothetical collapsed mass of rocks converted into a black hole is the sum of all the microscopic states of the individual mineral grains composing the rocks and their physical properties. The main physical property responsible for rock gravitational anomalies is density.

The purpose is to produce regions or domains. Where there is a large concentration of bits or entropy then it represents regions of maximum saturation of hidden information which corresponds to high probability of dense rocks and ores. Regions with low bit concentration will represent regions of low probability to find very dense rocks. This is just some of the many interpretations, but the main one indicates the existence of a relation between entropy quantities in the pseudo-spacetime scenario and the saturation of information being translated mainly into rock density gradient, because we know what was the information before transforming it into a grid of black holes.

In a real scenario of gravimetric anomaly maps one can make these mathematical manipulations to derive a gravitational entropy map and then extract preliminary valuable information about the composition of the rocks and their nature on the region which will be considered for further geophysical inversion scenarios. From entropy maps one could deduce the geological complexity of a specific region and could infer the presence of important mineral deposits. What is done here is that a grid of numbers with geophysical meaning were transformed into a pure mathematical entity generated by gravitational entropy. Before prompting the interpolation radius, the program prints on the screen the resultant average Schwarzschild radius. Then the program prompts the user to set the interpolation radius. This radius is arbitrary, but can follow the rule that must be less than a large R_s or equal to R_s , depending on the resultant average. At the end of the program routine, it is shown the resultant value of this average. The resultant entropy map is shown below in Figure 11.

Figure 11 - Resultant Hawking entropy map derived from the Bouguer anomaly map.



Conclusions

Summarizing the scope of this article, the total area of the event horizon corresponds to the entropy of the black hole measured in Planck units. We can analyze the entropy of regions of low mass by approximating it for subdivision of small hypothetical areas collapsing to tiny black holes. This approximation makes a grid of hypothetical collapsed regions of spacetime with the surface gravity fluctuations equal to the fluctuations measured in a gravimetric map of a specific area of the planet. These fluctuations are treated as individual black holes containing the saturated information stored in a grid of spacetime.

The Hawking entropy will be calculated for this grid considering, for practical effects of visualization of values, the entropy map represented in base 10 logarithm values, because these are very large numbers. Another approach is that the Hawking formula for the entropy will be simplified to Planck units preserving only the surface gravity values of the grid measured in Planck units that were transformed from the original milligal values. Therefore, the entropy value is in Planck units, and is proportional to the Planck Area. According to Boltzmann's interpretation of thermal entropy, this measures the microstates of a system that has a degree of disorder. The entropy measures hidden information in a system. If a system has several degrees of freedom and most of them is hidden from the observer, then the entropy of this system is high.

Considering a hypothetical scenario where all the mass contained in the rocks of the Earth's crust individually collapses to form small black holes, according to the laws of black hole thermodynamics derived by Stephen Hawking, all the information contained in the rocks will be conserved and coded in the area of the event horizon of the black holes, and the total entropy of the collapsed system is the sum of the individual entropies of the black holes. The information is huge and as an example it represents the internal energies of the atoms in the crystal lattices of the minerals that compose the rocks, the distance between the atoms in the crystal lattices, the mass of the atoms and of its individual nucleons and electrons, the chemical energy stored in the ionic and covalent bonds between atoms and molecules in the minerals, the vibration of the atoms, etc. Because gravitational entropy represents values of maximum capacity of 'spacetime storage' of information, the number of bits coded in the black hole event horizon is mind-boggling, expressed here in an 'invented' unit of "Giga-Googolbyte".

The only conserved quantities that can be measured by an observer outside the black hole are the mass, electric charge and angular momentum. Considering, for simplification, that the individual black holes are Schwarzschild black holes, then only the mass is measured by an observer. The mass associated with the surface gravity of the individual black holes will derive the gravitational entropy from each one. The surface gravity is considered here at the event horizon. According to the 'no-hair theorem', a black hole has no fluctuations or irregularities in the event horizon (Israel, 1967). Then, the individual event horizons are smooth and spherically symmetric for a Schwarzschild black hole. The final result here, by using a python script, is the Hawking entropy map plot.

A further interesting analysis is that in the network of black holes derived from the grid of Bouguer anomaly values, the negative values correspond to repulsive gravity or time-reversals of black holes, called white holes. Therefore, the real network contains white holes and black holes, and because white holes are unstable, they rapidly decay to black holes, both these objects have a net positive entropy, despite different gravitational natures. Other vision of these networks is that they are equivalent to a grid of Schwarzschild wormholes, that also are highly unstable, and, again, decay back to black holes. Further research on quantum gravity and black holes applied to geophysical mathematical tools can be explored, the possibilities are infinite.

The code 'Graviton_entropy' is available at GitHub repository link:

<https://github.com/gz-plan3t/Graviton-entropy>

The original python code project 'Graviton_entropy.py' SHA-256 hash is registered in the Ethereum Blockchain!

ETH Blockchain Address: 0x5d9Bc1eA0CAda36D39ce4Ae1fcd01273d5e99fC8

Transaction Hash:

0x64394106289b3eec6c2cfb4f144759941edb027aa417839e5254e6ed3bbbf41e



Block explorer URL:

<https://etherscan.io/tx/0x64394106289b3eec6c2cfb4f144759941edb027aa417839e5254e6ed3bbbf41e>

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