Proxy-based Sliding Mode Stabilization of a Class of Second-order Nonlinear System

Guangzheng Ding[1], Jian Huang[1,*], Bin Hu[1] and Zhi-Hong Guan[1]

Abstract— **The essential feature of conventional proxy-based sliding mode control (PSMC) method is the introduction of a proxy, which is controlled by a normal sliding mode control (SMC) approach to track the desired trajectory. Both the safety problem in conventional stiff position control and the chattering problem in the SMC are overcome by the PSMC strategy. Meanwhile, the stability problem of PSMC is not well addressed for general nonlinear systems. In this paper, a new PSMC method is proposed for robust tracking control of a class of second-order nonlinear systems. A PD type virtual coupling is used and a specified sliding mode controller is designed in the proposed PSMC method. Based on the model of a class of second-order nonlinear systems, the stability of the closed-loop PSMC system is proved by Lyapunov theorem. Numerical simulations were carried out to verify the propose method.**

I. INTRODUCTION

Many physical systems can be represented by a class of second-order nonlinear models, such as magnetic levitation systems [1] [2], inertia wheel pendulums [3] [4], chaotic systems [5], robotic manipulators [6], and pneumatic muscle actuators [7]. In addition, some complicated high-order systems can also be transformed into this kind of second-order system, such as point mass planar satellites [8], permanent magnet synchronous motors (PMSMs) [9], induction motors [10], and so on. The control difficulty of these second-order nonlinear systems lies in the fact that (i) complex nonlinear dynamics of the system results in slow response time and time-varing parameters [11], (ii) imprecise mathematical models arise from parameter uncertainties or from assumptions considered for model simplification [12], and (iii) external disturbances commonly exist in practice. So far, many control strategies have been developed to address these problems such as the proportional-integral-derivative (PID) control [13], the adaptive control [14], the neural network based control [15], the sliding mode control (SMC) [16], etc.

The PID control is the most commonly-used controller in nonlinear systems and industrial applications due to its simple structure, easy implementation and robustness to noise in a wide range of operating conditions [17]. At the same time, it suffers from some problems e.g. difficulties in the stability proof and parameter tuning. What is especially considered in this paper is the safety problem of PID control. In the case of some abnormal events, such as unexpected environment contacts, temporal power failures to the actuators, etc., the tracking error will become very large which leads to an excessively large actuator force. This can do damage to the environment and even to the human body especially in the human-centered automated systems. For safety considerations, some approaches are available, including the addition of a force limiter to the control signal, or using a very high velocity feedback gain. However, these methods can deteriorate the responsiveness and control accuracy. Therefore, the tracking performance and the safety can hardly be achieved simultaneously by a conventional PID control.

The sliding mode control (SMC) has been developed for many years. It is one of the most studied control strategies and has been applied widely due to its design simplicity and robustness against matched disturbances and parameter uncertainties [18]. Theoretically, the system states can be robustly constrained to the sliding surface and be finally convergent to zero by using SMC controller, which guarantees the tracking accuracy. Besides, overdamped dynamics can be obtained by setting the magnitude limit of the control signal. Therefore, in an ideal condition, the tracking performance and the safety can be both obtained by using the SMC. However, the discontinuous function (e.g. signum-type function) need to be introduced across the sliding surface to account for the presence of modelling imprecision and disturbances and the switching of signum function can not be infinitely fast in practice. This will lead to undesired phenomenon, the "chattering", which has for a long time affected the implementation of sliding strategies in real physical systems. So far, many alternative solutions have been suggested by researchers. The boundary layer is one of the main approaches which can reduce the chattering phenomenon effectively [19]. However, this method cannot guarantee both the chattering-free and the tracking performance with large position error in the system, because the selection of boundary thickness can make a contradiction between the chattering-free and the accuracy. If the boundary layer is too narrow, the tracking performance and robustness will be deteriorated, while the chattering can not be reduced with too large value. Another solution is using higher-order sliding mode control [20]. This method requires more calculation and is not easy to implement because the observer is needed to be designed to estimate the derivative of the state variable [21].

The Proxy-based Sliding Mode Control (PSMC), which can be viewed as the combination of conventional PID

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[1]Key Laboratory of Ministry of Education for Image Processing and Intelligent Control, School of Automation, Huazhong University of Science and Technology, Wuhan, Hubei, 430074 P.R. China. huang_jan@mail.hust.edu.cn

control and SMC control, was first introduced by Kikuuwe and Fujimoto in 2006 [22]. The PSMC can achieve the chattering-free goal in essence, because the signum-type function is removed out of the closed-loop of the controller by introducing an imaginary proxy object. Thus, both the safety and the tracking accuracy can be guaranteed by using the PSMC. The PSMC has already been applied to some non-linear systems (see e.g. [23] - [29]). The experiments have been carried out to verify the validity of the PSMC. However, the theoretical stability analysis is not well addressed. As a model-free control method, the stability analysis of PSMC in [30] is based on a conjecture of the local passivity of the well-tuned PID control, which, however, remains an issue to study in the future. In this paper, we proposed a model-based PSMC for a class of second-order nonlinear systems, whose system state equation is formally similar to a class of a underactuated systems [31] after taking the proxy state in the system into consideration. After that, a special SMC design method can be applied and the closed-loop stability can be proved using the Lyapunov theory.

The rest of the paper is organized as follows. A class of second-order nonlinear system is described in section 2 and a brief introduction of the PSMC is also presented in this section. A new model-based PSMC design method with PD type virtual coupling is derived for the second-order nonlinear system in section 3. Simulation experiments are presented to demonstrate the validity of the proposed method in section 4. Finally a conclusion is given in section 5.

II. SYSTEM FORMULATION AND PRELIMINARIES

The dynamics of a class of second-order nonlinear systems is written as:

$$\ddot{q} = f(q, \dot{q}, t) + b(q, \dot{q}, t)u(t) + d(t) \tag{1}$$

where q denotes the system position. u is the control input signal. f is the nonlinear dynamic. b is the control gain and $d(t)$ is the lumped disturbance containing the model uncertainties and external disturbances which satisfy the bounded condition:

$$d(t) \leq \|D\|$$

where D is a constant.

To achieve a smooth and safe motion of the system (1), a high damping action is required when large tracking errors occur due to abnormal events. Traditionally, this action can be achieved through various methods. However, these methods can sacrifice the responsiveness and control accuracy. To overcome this problem, the PSMC was proposed as a robot control method which can achieve both the satisfactory responsiveness and the accurate tracking ability during normal operation and smooth, overdamped recovery from large position errors. The principal of PSMC is illustrated in Fig. 1(a). The essential feature of PSMC is the introduction of the proxy which is connected with the actual controlled object via a virtual coupling. The conventional SMC controller which exerts the force f_{SMC}, is designed to control the proxy to track the desired trajectory. Normally, the PID type coupling is chosen to produce an interaction force between the actual controlled object and the proxy [20]. The virtual coupling is supposed to maintain its length to be zero, which means that the error between the real system position and proxy position converges to zero.

III. CONTROLLER DESIGN

In this section, the design procedure and stability analysis of the newly designed PSMC are presented.

For the convenience of theoretical analysis, we choose a simple PD type virtual coupling which produces the force f_{PD} in this study. As shown in Fig. 1(b), the force f_{PD} is directly applied to the physically controlled object, and its reaction force is applied to the proxy. The force f_{SMC} generated by the SMC is used to control the proxy from the other side. Therefore, the dynamics of the proxy can be described as:

$$\ddot{p} = \frac{f_{SMC} - f_{PD}}{m_p}, \tag{2}$$

where m_p is the mass of proxy. Since the proxy is hypothetical, m_p can be set arbitrarily. On the other hand, the force produced by the PD virtual coupling is defined as

$$f_{PD} = P(p - q) + D(\dot{p} - \dot{q}) \tag{3}$$

which is the control input of physical system (1). P and D are the proportional and derivative gains of the PD virtual coupling. Therefore, system (1) can be rewritten as:

$$\ddot{q} = f_1(x_1, x_2, x_3, x_4), \tag{4}$$

where

$$\begin{array}{l} f_1 = \hat{f}_1(x_1, x_2, x_3, x_4) + d \\ \hat{f}_1 = f + b \cdot f_{PD} \end{array}. \tag{5}$$

Choose state variables as:

$$\begin{cases} x_1 = q \\ x_2 = \dot{q} \\ x_3 = p \\ x_4 = \dot{p} \end{cases}.$$

From (2) and (4) ,the whole system with the proxy can be transformed into the following normal form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x_1, x_2, x_3, x_4) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{u - f_c}{m_p} = f_2 + \dfrac{f_{SMC}}{m_p} \end{cases}, \tag{6}$$

where

$$f_2 = -\frac{f_{PD}}{m_p}.$$

Note that system (6) is very similar to a class of underactuated systems studied in [31], which is totally different from the usual cascade form so that the normal SMC method is not easy to be applied in it. To deal with this problem, we propose a specified SMC design procedure as follows.

First, choose the sliding mode surface as:

$$s = c_1 e_1 + c_2 e_2 + e_3, \tag{7}$$

where

$$\begin{cases} e_1 = x_1 - x_{1d} \\ e_2 = x_2 - x_{2d} \\ e_3 = \hat{f}_1 - \hat{f}_{1d} \end{cases}$$

are the error variables. Here x_{1d}, x_{2d} are the desired value, and function $\hat{f}_{1d}$ satisfies $\hat{f}_{1d} = \hat{f}_1(x_{1d}, x_{2d}, x_3, x_4)$. Differentiating the errors we have

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 - x_{2d} = e_2 \\ \dot{e}_2 = (\hat{f}_1 + d) - (\hat{f}_{1d} + d) = \hat{f}_1 - \hat{f}_{1d} = e_3 \end{cases}. \tag{8}$$

When the system state achieves around the sliding mode surface (7), it satisfies

$$s = \gamma, \tag{9}$$

where $|\gamma| \le \kappa$. Substituting (7) in (9), we have

$$e_3 = -c_1 e_1 - c_2 e_2 + \gamma. \tag{10}$$

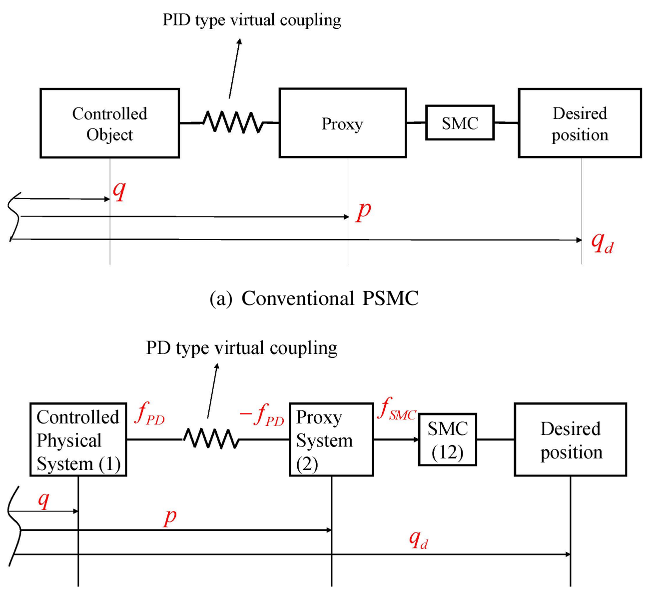
The error variables can be expressed in a vector form as $E = [e_1, e_2]^T$. It follows from (8) to (10) that

$$\dot{E} = ME + \Upsilon, \tag{11}$$

where

$$M = \begin{bmatrix} 0 & 1 \\ -c_1 & -c_2 \end{bmatrix}, \Upsilon = \begin{bmatrix} 0 \\ \gamma \end{bmatrix}.$$

We can choose proper parameters c_1, c_2 which satisfy the condition that the real parts of eigenvalues of M are negative, i.e. $c_1 > 0, c_2 > 0$. Then, according to (11), the system error is asymptotically convergent to zero if Υ vanishes as time goes by. And the system error is uniformly ultimately bounded if Υ exists all the time.



(a) Conventional PSMC

(b) Our proposed PSMC

Fig. 1. Physical interpretation of two PSMCs.

Theorem 1: Considering the second-order nonlinear system (1), the trajectory of the states will be driven to the sliding mode surface (7) which is uniformly and ultimately bounded when the PSMC (12) is applied:

$$u = -(\frac{\partial \hat{f}_1}{\partial x_4})^{-1} m_p (K + k\text{sgn}s + \lambda s), \tag{12}$$

where

$$\begin{aligned} K = {} & c_1 x_2 + c_2 \hat{f}_1 + \frac{\partial \hat{f}_1}{\partial x_1} x_2 + \frac{\partial \hat{f}_1}{\partial x_2} \hat{f}_1 + \frac{\partial \hat{f}_1}{\partial x_3} x_4 \\ & + \frac{\partial \hat{f}_1}{\partial x_4} f_2 - c_1 \dot{x}_{1d} - c_2 \dot{x}_{2d} - \dot{\hat{f}}_{1d} \end{aligned} \tag{13}$$

$$k > 0, \lambda > 0.$$

Proof: Choose the Lyapunov function as

$$V = \frac{1}{2} s^T s. \tag{14}$$

Difierentiating both sides of V and substituting (12) yield

$$\begin{aligned} \dot{V} & = s(c_1 \dot{e}_1 + c_2 \dot{e}_2 + \dot{e}_3) \\ & = s(c_1 x_2 + c_2 \hat{f}_1 + \frac{\partial \hat{f}_1}{\partial x_1} x_2 + \frac{\partial \hat{f}_1}{\partial x_2} \hat{f}_1 + \frac{\partial \hat{f}_1}{\partial x_3} x_4 + \frac{\partial \hat{f}_1}{\partial x_4} f_2 \\ & -c_1 \dot{x}_{1d} - c_2 \hat{f}_{1d} - \dot{\hat{f}}_{1d} + \frac{\partial \hat{f}_1}{\partial x_2} d_1 + \frac{\partial \hat{f}_1}{\partial x_4} \frac{u}{m_p}) \\ & = s(\frac{\partial \hat{f}_1}{\partial x_2} d_1 - k\text{sgn}s - \lambda s) \\ & = s \frac{\partial \hat{f}_1}{\partial x_2} d_1 - k|s| - \lambda s^2 \\ & \le |s| (\frac{\partial f_1}{\partial x_2} \|D\| - k - \lambda |s|) \end{aligned}. \tag{15}$$

It is easy to obtain that after a sufficiently long time, $|s|$ is bounded by

$$|s| \le \frac{\frac{\partial f_1}{\partial x_2} \|D\| - k}{\lambda}. \tag{16}$$

According to (16), the magnitude of sliding variable $|s|$ is uniformly ultimately bounded. The actuated states move around the sliding mode surface and the bounds can be lowered by selecting proper control parameters.

IV. SIMULATION EXPERIMENT

In this section, to verify the proposed control method, the simulation has been carried out for an underwater vehicle. A simplified model of the motion of the vehicle can be written as ([32]):

$$m\ddot{q} + c\dot{q}|\dot{q}| = u, \tag{17}$$

where q denotes vehicle position. u is the control input. m is the mass of the vehicle (including the so-called added-mass, associated with motion in fluid), and c is a drag coefficient. The parameters used in simulation are as follows:

$$m = 3 + 1.5\sin(|\dot{q}|\,t), c = 1.2 + 0.2\sin(|\dot{q}|\,t).$$

To test the robustness of the PSMC to parameter uncertainties and external disturbances, the estimated parameters

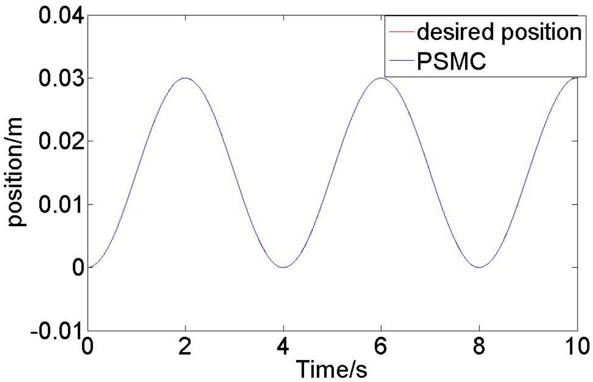


Fig. 2. Sinusoidal trajectory tracking control result of underwater vehicle using the PSMC.

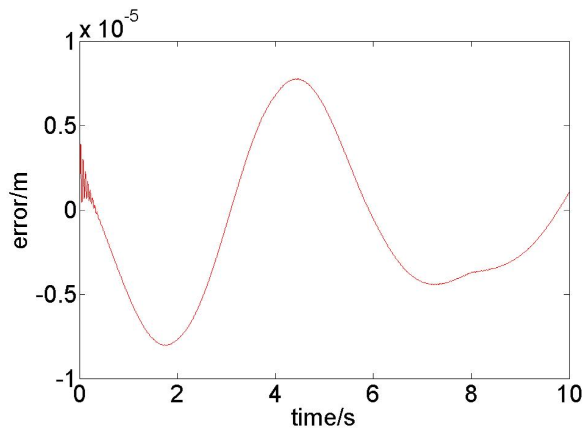


Fig. 3. Tracking error

in the controller are set as: $\hat{m} = \sqrt{5}, \hat{c} = 1$. The external disturbance $d_1 = 0.1\sin(t)$ is added into the system control input u. The vehicle is supposed to track a sinusoidal curve, given by:

$$q_d = 0.015\mathrm{sin}(0.5\pi(\mathrm{t}-1)) + 0.015.$$

The tracking result is shown in Fig. 2 (the red curve is covered by the blue curve due to the error between desired position and real position trajectory using the PSMC is too small) and the tracking error is shown in Fig. 3. As we all know, the selection of the gain factor k in front of the $sgn(\cdot)$ function is very important to the performance of the SMC. The robustness will be weakened with too small k, while the chattering problem will become more severe if the value is too large. Fig. 4 shows the comparison of the proxy position and that of the real system when a excessively large k is selected. The position of proxy, p, has large chattering while the position of the real system, q, is almost unaffected when we increase the value of the k. This is because the proxy is controlled directly by the SMC controller, which will result in chattering to p while the real system position q is controlled by a PD controller (generated by the PD virtual coupling).

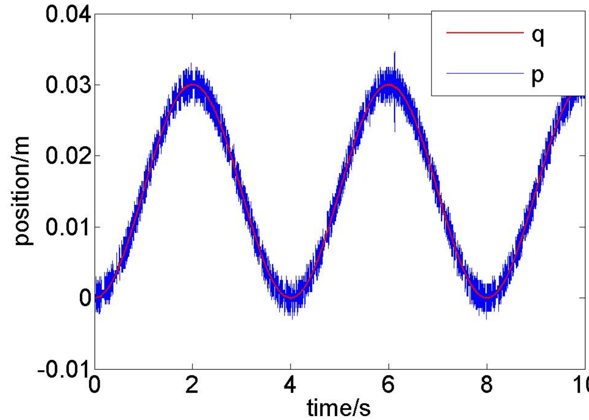


Fig. 4. Comparison between real system position q and proxy position p

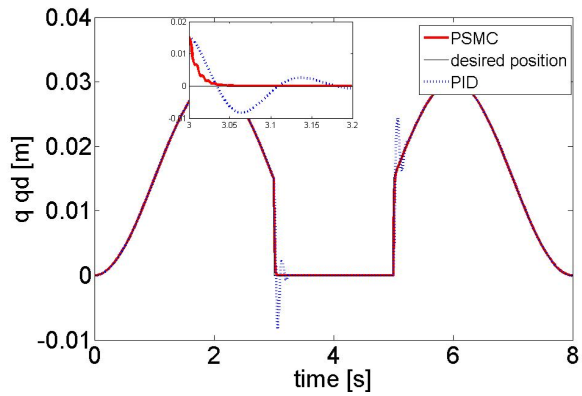


Fig. 5. System response to a desired trajectory with discontinuous change.

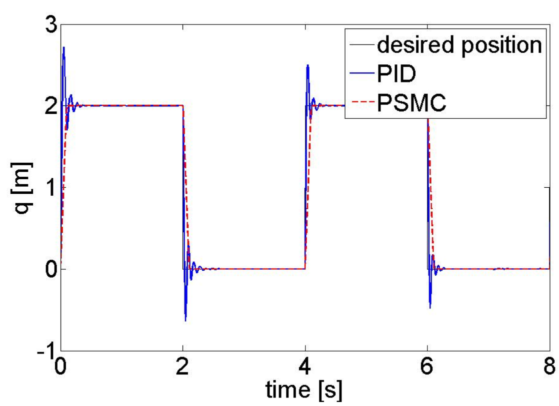


Fig. 6. Square wave trajectory tracking control results of underwater vehicle using PSMC and PID control with large gain.

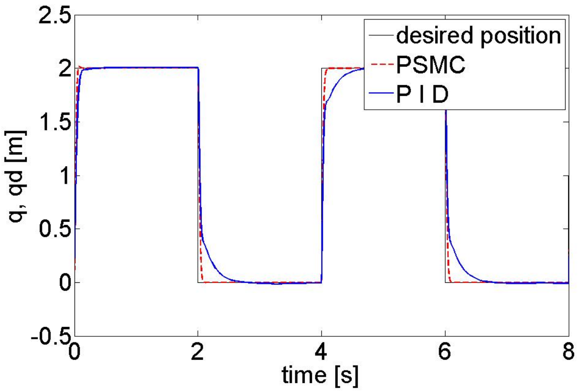


Fig. 7. Square wave trajectory tracking control results of underwater vehicle using PSMC and PID control with small gain.

The PSMC can be considered as the combination of the SMC and the PD control. To make a comparison between the PSMC and the traditional stiff position control, we tested the system response to a discontinuous desired position (18). This indicates that the large positioning error occurs using the both control strategies. The performance of the PSMC depends on the control gains of the virtual PD controller. Many tuning methods have been developed such as the trial-and-error method, the Ziegler and Nichols tuning method, the $H\infty$ constraint optimization method, and the optimum gain and phase margin tuning method. In this paper, we use the trial-and-error method. Parameters are chosen as $\mathrm{P}=10000, \mathrm{D}=1, \lambda=1, \mathrm{c}_1=10000, c_2=10, k=100000$ for the PSMC, and $\mathrm{P}=50000, \mathrm{I}=50, \mathrm{D}=100$ for the PID. It is found difficult to obtain significantly better performance for PID control by tuning parameters, and the system will become unstable when we continue to increase the gain of PID controller. Fig. 5 shows the tracking results. The overshoot occurs in the pursuit of relatively fast recovery performance when using the PID control, while the overdamped recovery can be obtained when using the PSMC control. Thus, the PSMC controller can guarantee the safety of the system by canceling the overshoot during the tracking task.

$$q_d = \begin{cases} 0, \quad 3 \le \mathrm{t} < 5 \\ 0.015\mathrm{sin}(0.5\pi(\mathrm{t}-1)) + 0.015, \quad \text{else.} \end{cases} \tag{18}$$

We also tested the system response to square wave inputs, with the magnitude and period are set to be $4m$ and $4sec$, respectively. The tracking results of underwater vehicle system using the PSMC and the PID controller are shown in Fig. 6 and Fig. 7.

It can be clearly seen from Fig. 6, that there is no chattering in the tracking curve when using the PSMC. Hence the advantage of safety and accuracy of ideal SMC can be both inherited by the proposed PSMC, while the conventional PID control can produce overshoot which is not conducive to the system safety. When we choose appropriate parameters to guarantee the overdamped dynamics of PID, the steady-state time is much longer than that of the PSMC, as it is shown in Fig. 7. Thus, the safety and good tracking performance can not be obtained by the PID control simultaneously.

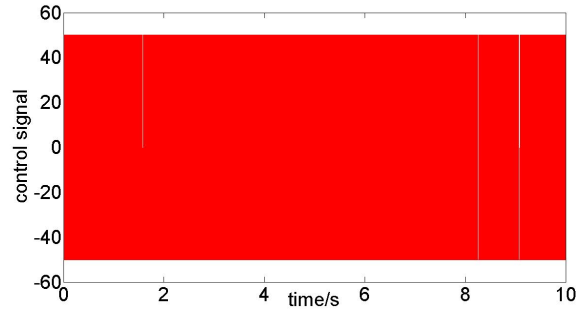


Fig. 8. Control signal using SMC.

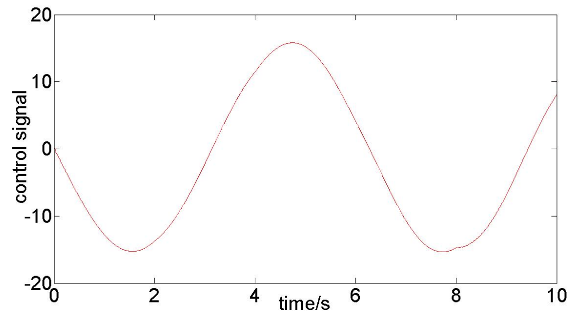


Fig. 9. Control signal using PSMC.

We also made a comparison between PSMC and SMC. Both PSMC and SMC can achieve a good tracking performance. Plotted in Fig. 8 and Fig. 9 are the control signals. Fig. 8 shows the chattering phenomenom in control signal with high frequency and magnitude by using SMC. This is unacceptable by the actuator in practical systems. By using PSMC, we achieved chatering-free target, as shown in Fig. 9.

V. CONCLUSIONS

In this paper, we proposed a novel PSMC which combines a simple virtual PD type coupling and a specified SMC control for a class of second-order nonlinear systems. The essential feature of PSMC is the introduction of the proxy which is controlled by the SMC controller, while the actual physical object is connected with the proxy via the PD type virtual coupling. No discontinuous function is included in the direct control signal of actual controlled object. Therefore, the PSMC inherits the advantages of PD control and the SMC control, while the unsafe behavior of PD control and the chattering problem of SMC can be avoided in the meanwhile. The stability of the developed PSMC strategy is also proved through Lyapunov theorem, and the effectiveness is verified by simulations of controlling an underwater vehicle system.

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