Sparse Phase Retrieval

Comparing LASSO and Generative Model-Based Approaches

Anja Kroon Giacomo Zanardini

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Delft University of Technology, The Netherlands

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Compressed Sensing

- Solving linear system with n unknowns requires n equations
- Observe natural signal sparsity
- Exploit sparsity to recover with less equations
- Compressed Sensing (CS): recovery of sparse $\mathbf{x}$ from noisy measurements $\mathbf{y}$

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta} \qquad \mathbf{y} \in \mathbb{R}^m, \mathbf{A} \in \mathbb{R}^{m \times n}, \mathbf{x} \in \mathbb{R}^n \tag{1}$$

- Decrease cost and processing time in imaging and wireless
- Imaging Applications $\rightarrow$ loss of sign information

$$\mathbf{y} = |\mathbf{A}\mathbf{x} + \boldsymbol{\eta}| \tag{2}$$

LASSO Recovery

- Applies the LASSO objective to CS [1] [2] [3]
- Recover optimal $\mathbf{x}^*$ by,

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x}\|_1 \tag{3}$$

- $\mathbf{A}$ must satisfy RIP/REC for unique recovery → Gaussian or Bernoulli matricies
- Solve via proximal gradient descent
- Sparse phase retrieval: $\mathbf{A}\mathbf{x} \to |\mathbf{A}\mathbf{x}|$

[1] A. Creswell and A. A. Bharath (2019). "Inverting the Generator of a Generative Adversarial Network". In: *IEEE Transactions on Neural Networks and Learning Systems*

[2] D. Donoho (2006). "Compressed sensing". In: *IEEE Transactions on Information Theory*

[3] J. R. Emmanuel Candès and T. Tao (2006). "Stable signal recovery from incomplete and inaccurate measurements". In: *Communications on Pure and Applied Mathematics*

Generative Prior Based Sparse Recovery

- Algorithm [4]
  1. *Get generative prior.* Train a DCGAN to obtain $G : \mathbb{R}^k \rightarrow \mathbb{R}^n$
     $$G(\mathbf{z}) \rightarrow \mathbf{x} \qquad \mathbf{z} \sim \mathcal{N}(0, \frac{1}{m})$$
  2. *Exploit generative prior to find optimal* $\mathbf{z}^*$
     $$\min_{\mathbf{z}} \|\mathbf{A}G(\mathbf{z}) - \mathbf{y}\|_2^2 + \lambda\|\mathbf{z}\|_2^2 \qquad \lambda = 1$$
  3. *Recover signal of interest.*
     $$G(\mathbf{z}^*) \rightarrow \mathbf{x}^*$$
- $\mathbf{A}$ must satisfy Set-Restricted Eigenvalue Condition (S-REC) $\rightarrow$ Guassian matricies
- Solved via gradient descent (Adam Optimizer)
- Sparse Phase Retrieval: [5] $\rightarrow \min_{\mathbf{z}} \||\mathbf{A}G(\mathbf{z})| - y\|_2^2$

[4] A. Bora, A. Jalal, E. Price, and A. G. Dimakis (2017). "Compressive Sensing using Generative Models". In: *Proceedings of the 34th International Conference on Machine Learning*

[5] P. Hand, O. Leong, and V. Voroninski (2018). "Phase Retrieval Under a Generative Prior". In: *Advances in Neural Information Processing Systems*

Experimentation

- **Goal:** Recover MNIST images using lower dim. measurements [6]
- **Data:** MNIST greyscale handwritten digits (28 x 28)
- **Algorithms:** LASSO, Generative Prior
  1. Case 1: General CS algorithms with general measurements. $\mathbf{y} = \mathbf{A}\mathbf{x} + \boldsymbol{\eta}$
  2. Case 2: General CS algorithms applied to sign-less measurements. $\mathbf{y} = |\mathbf{A}\mathbf{x} + \boldsymbol{\eta}|$
  3. Case 3: Algorithms adapted for sparse phase retrieval and applied to sign-less measurements. $\mathbf{y} = |\mathbf{A}\mathbf{x} + \boldsymbol{\eta}|$
- **Evaluation:** visual results, recovery time
- **Generative experimentation:**
  - set reconstruction error to 0 by $\mathbf{y} = \mathbf{A}G(\mathbf{z})$
  - analyze measurement error and reconstruction capability

[6] L. Deng (2012). "The mnist database of handwritten digit images for machine learning research". In: *IEEE Signal Proc. Magazine*

Results – LASSO

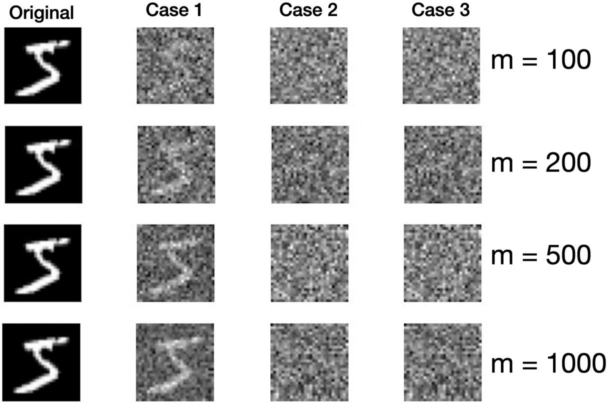


Figure: The LASSO algorithm is able to reproduce the MNIST images with noise only for Case 1.

Results – Generative Prior Based Recovery

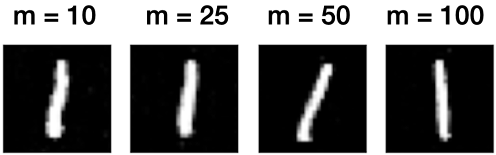
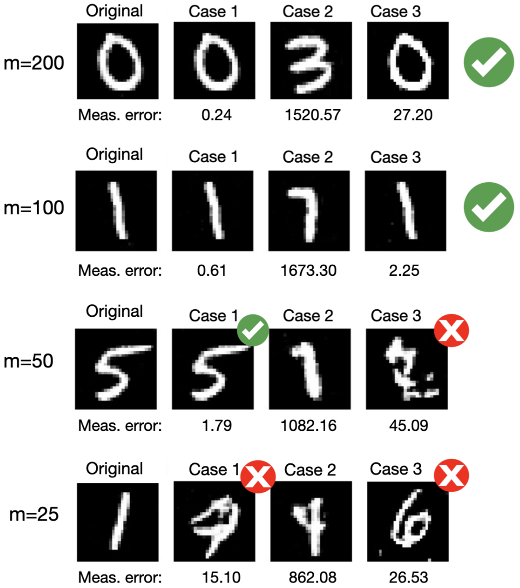


Figure: The DCGAN is able to produce samples which resemble the MNIST dataset with vector **z** of dimension m x 1 where $m = 10, 25, 50, 100$.

Results – Generative Prior Based Recovery



Results – Computation Time

- LASSO single $\mathbf{x}$ retrieval $\rightarrow$ 1 second
- Generative prior training $\rightarrow$ 1 h 10 min
- Generative prior retrieval $\rightarrow$ 4 min (step size dependent)
- Neural network training, RTX 2070 GPU
- LASSO noisiest but shortest time
- Further potential in developing generative approach

Summary

- General CS: recovery of a sparse signal
- Sparse Phase Retrieval: recovery using sign-less measurements
- LASSO – large m required, noisy results
- Generative Prior - with adequate dimension of the prior, the images are recoverable
- Issue with v1: assumed that the MNIST digits themselves fell within the range of the generator $G : \mathbb{R}^k \to \mathbb{R}^n$ [7]

[7] A. Bora, A. Jalal, E. Price, and A. G. Dimakis (2017). "Compressive Sensing using Generative Models". In: *Proceedings of the 34th International Conference on Machine Learning*

Future Work

- Implementing other generative methods
  - Sparse deviations from the range of the the generator function G [8]
  - Optimizing the latent variable during GAN training [9]
  - Faster optimal solution using ADMM [10]
  - Design loss function in GAN to maximize mutual information between input and output images [11]
  - Combine meta-learning and generative models to search for the optimal $\mathbf{z}^*$ [12]
- GAN complexity vs. number of measurements → decrease m with more complex GAN?

[8] M. Dhar, A. Grover, and S. Ermon (2018). "Modeling Sparse Deviations for Compressed Sensing using Generative Models". In: *Proc. of the 35th Intl. Conf. on ML.* PMLR

[9] M. Kabkab, P. Samangouei, and R. Chellappa (2018). "Task-aware compressed sensing with generative adversarial networks". In: *Proc. of 32 AAAI Conf.* New Orleans, Louisiana, USA

[10] S. Xu, S. Zeng, and J. Romberg (2019). "Fast Compressive Sensing Recovery Using Generative Models with Structured Latent Variables". In: *2019 IEEE ICASSP*

[11] X. Chen, Y. Duan, R. Houthooft, J. Schulman, I. Sutskever, and P. Abbeel (2016). "InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets". In: *Adv. in Neural Info. Proc. Sys.*

[12] Y. Wu, M. Rosca, and T. Lillicrap (2019). "Deep Compressed Sensing". In: *arXiv preprint arXiv:1905.06723*