Project 1

1. Show the details to form the tri-diagonal system for n = 4.

$$-u''(x) = -e^x$$

$$u''(x) = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}$$

$$u_{i-1} - 2u_i + u_{i+1} = he^{x_i}$$

$$n = 4$$

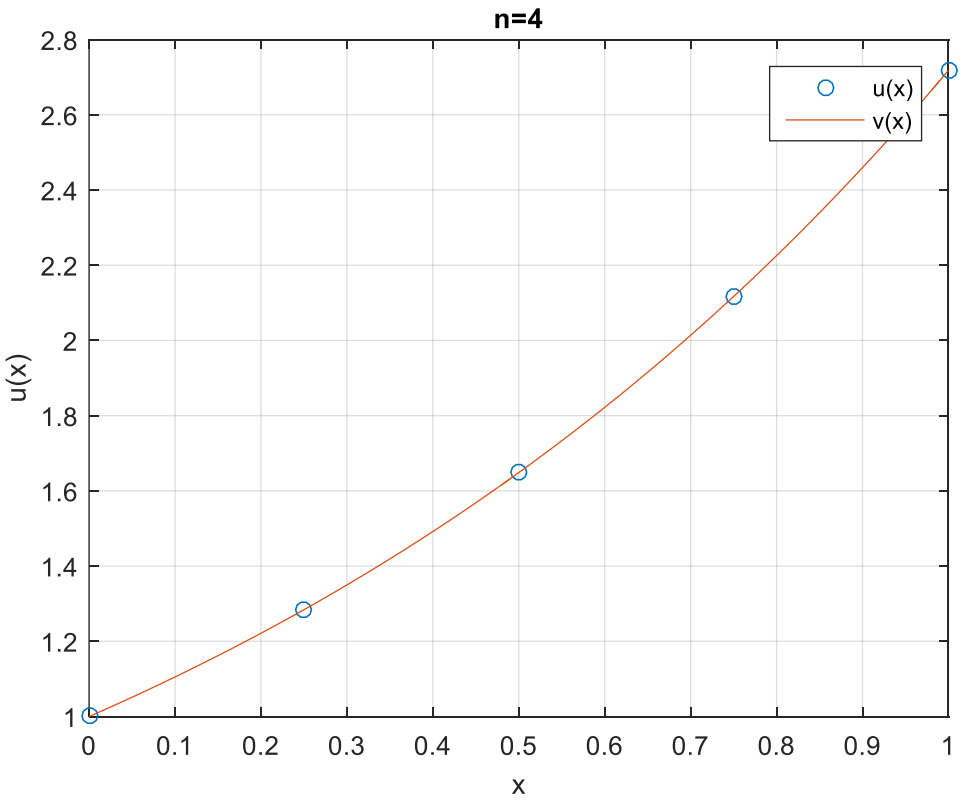
$$i = 1, 2, 3$$

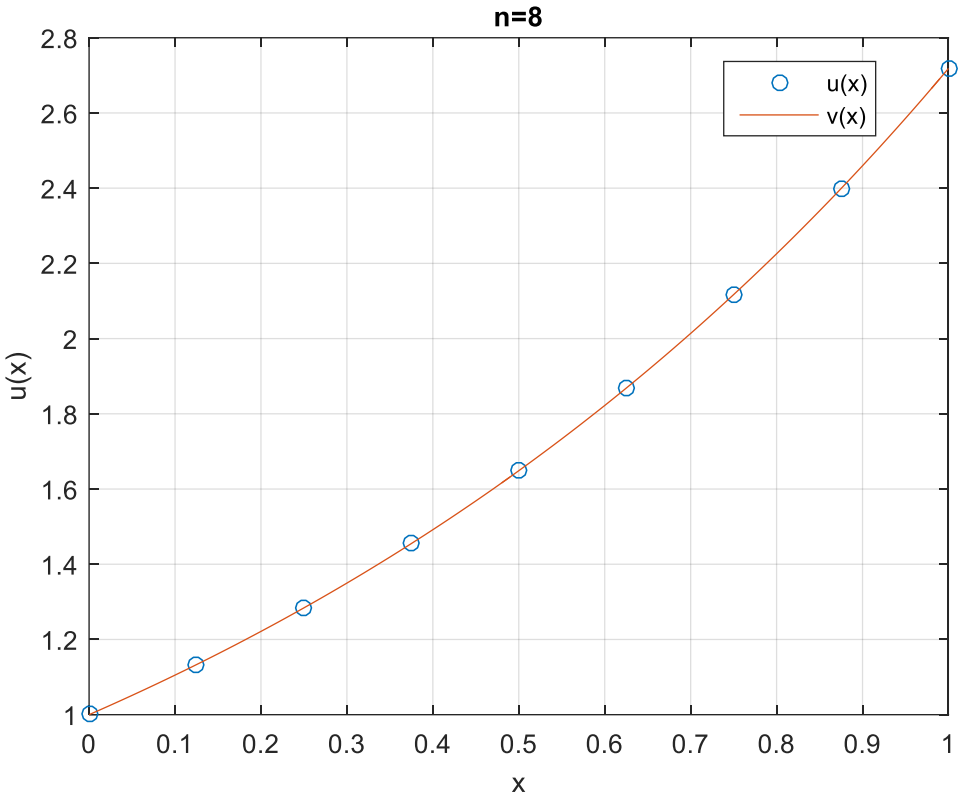
$$u_0 = u(0) = 1, u_1 = u(1) = e$$

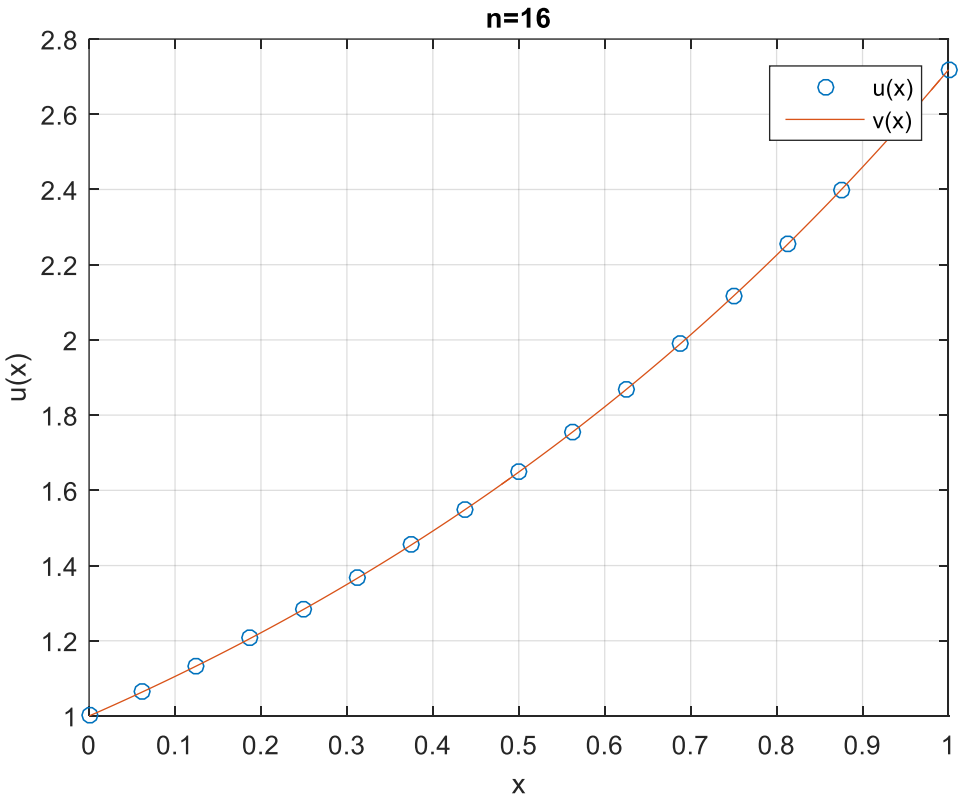
$$\begin{cases} u_0 - 2u_1 + u_2 = he^{x_1} \\ u_1 - 2u_2 + u_3 = he^{x_2} \\ u_2 - 2u_3 + u_4 = he^{x_3} \end{cases}$$

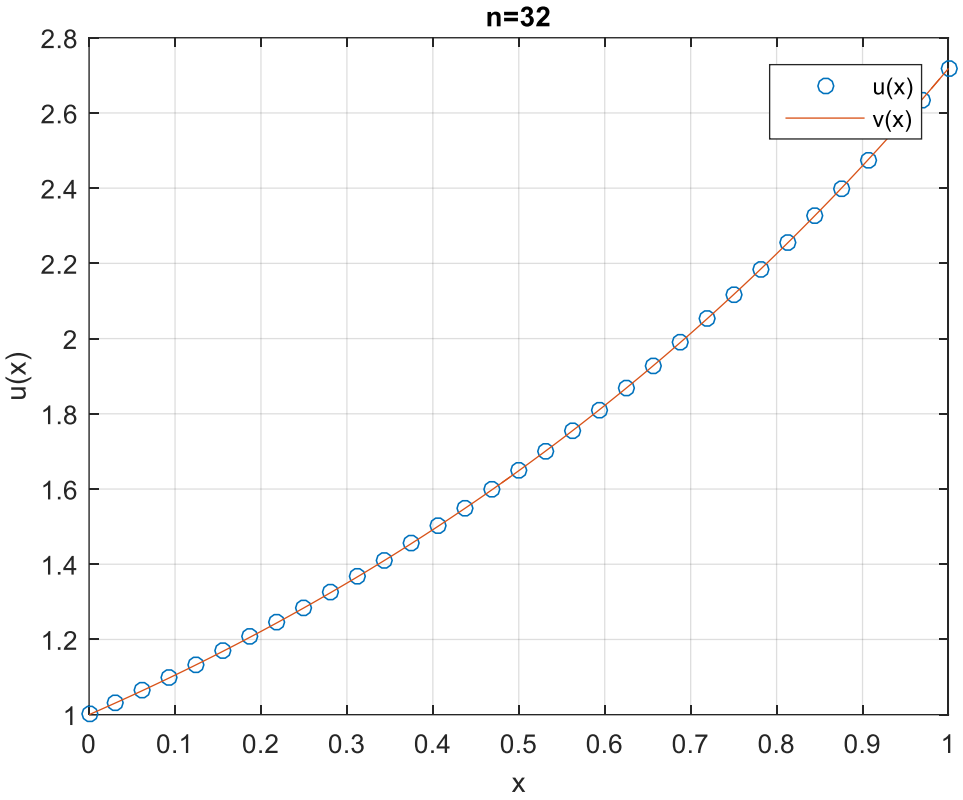
$$\therefore \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} he^{x_1} - u_0 \\ he^{x_2} \\ he^{x_3} - u_4 \end{bmatrix}$$

2. Plot the results u(x) vs. x for each n. Also plot the exact solution v(x) vs. x on the same figure for comparison purpose.




n=8
u(x)
v(x)
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
u(x)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x


n=16
u(x)
v(x)
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
u(x)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

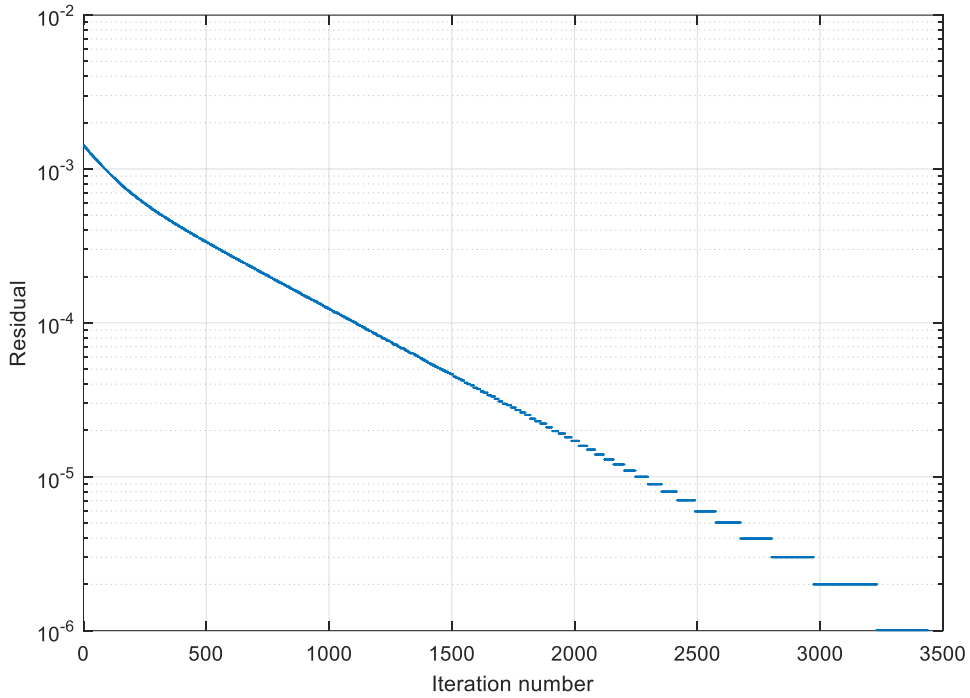


3. Compute the error for each grid using and the order of accuracy

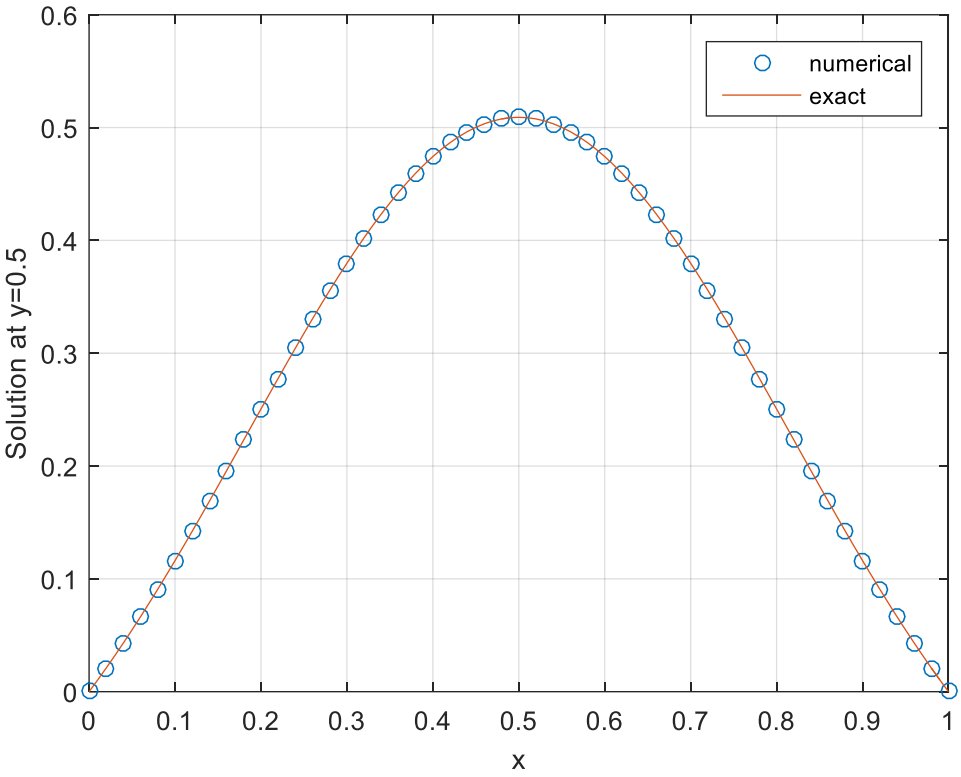
n	error	order
4	0.00079992252424295265	-
8	0.00020088630181660621	1.993481
16	5.0257888931277454e-005	1.998957
32	1.2566420588606754e-005	1.999776

Project 2

Convergence history: residual vs. iteration number

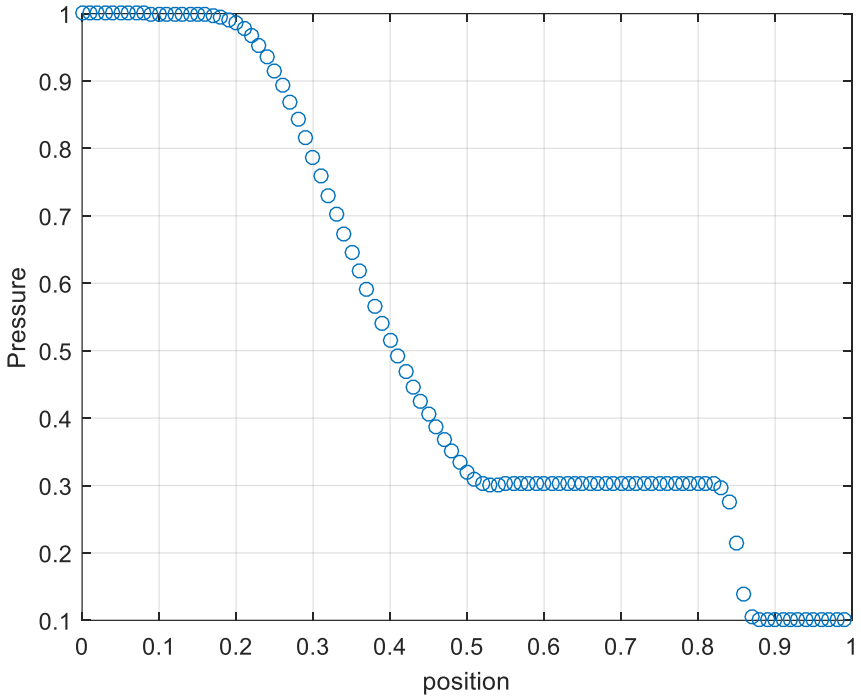


Solution along the horizontal line y = 0.5

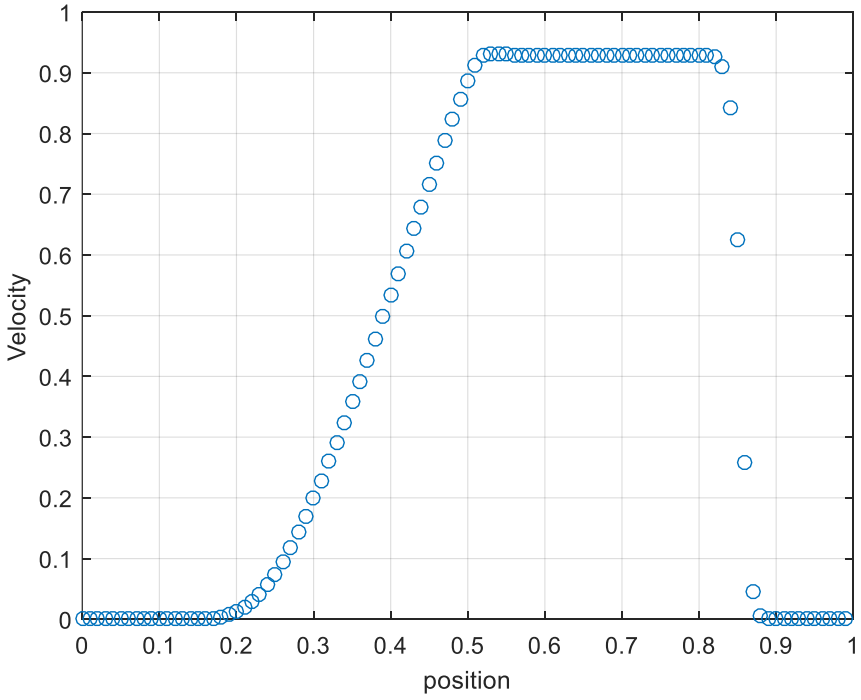

numerical
exact
Solution at y=0.5
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Project 3

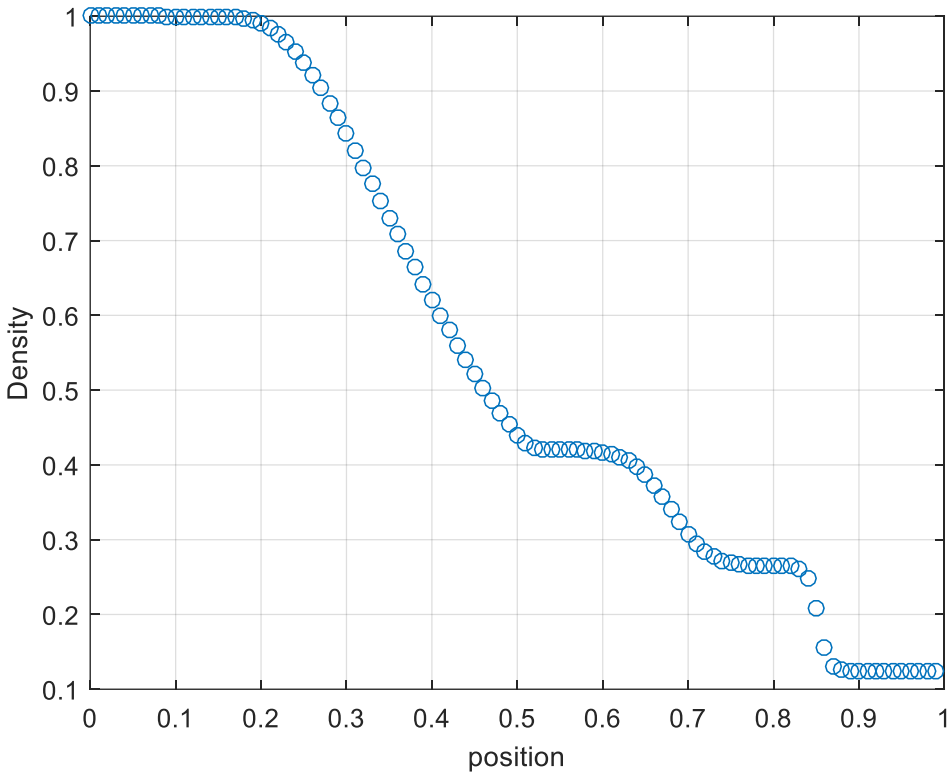
Pressure vs. x



Velocity vs. x



Density vs. x


Density
position
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Project 4

Upwind Scheme

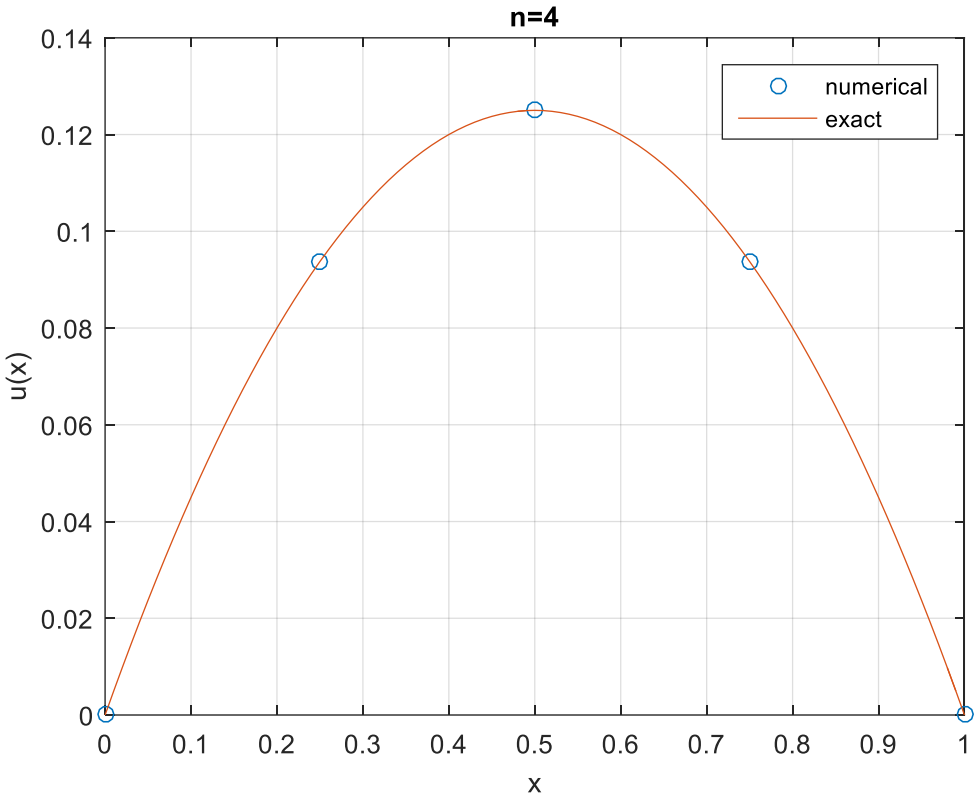
$$\begin{bmatrix} 3D+F & -D & & & \\ -(D+F) & 2D+F & -D & & \\ & -(D+F) & 2D+F & -D & \\ & & -(D+F) & 2D+F & -D \\ & & & -(D+F) & 2D+F \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} (2D+F)\phi_A \\ 0 \\ 0 \\ 0 \\ 2D\phi_B \end{bmatrix}$$

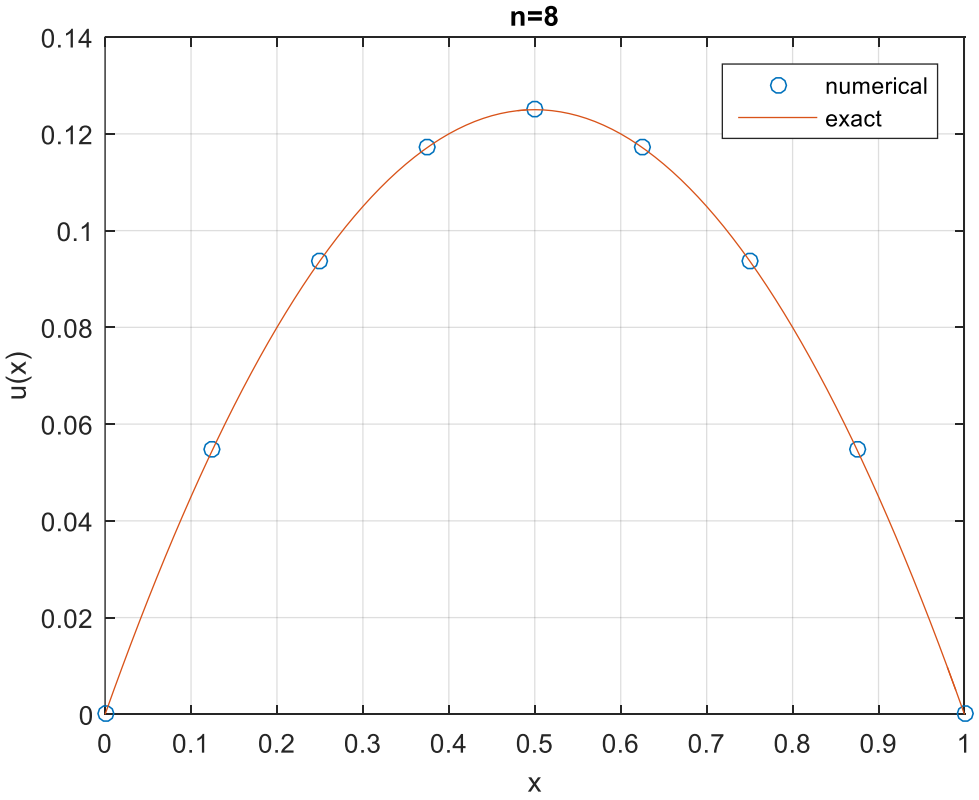
QUICK Scheme

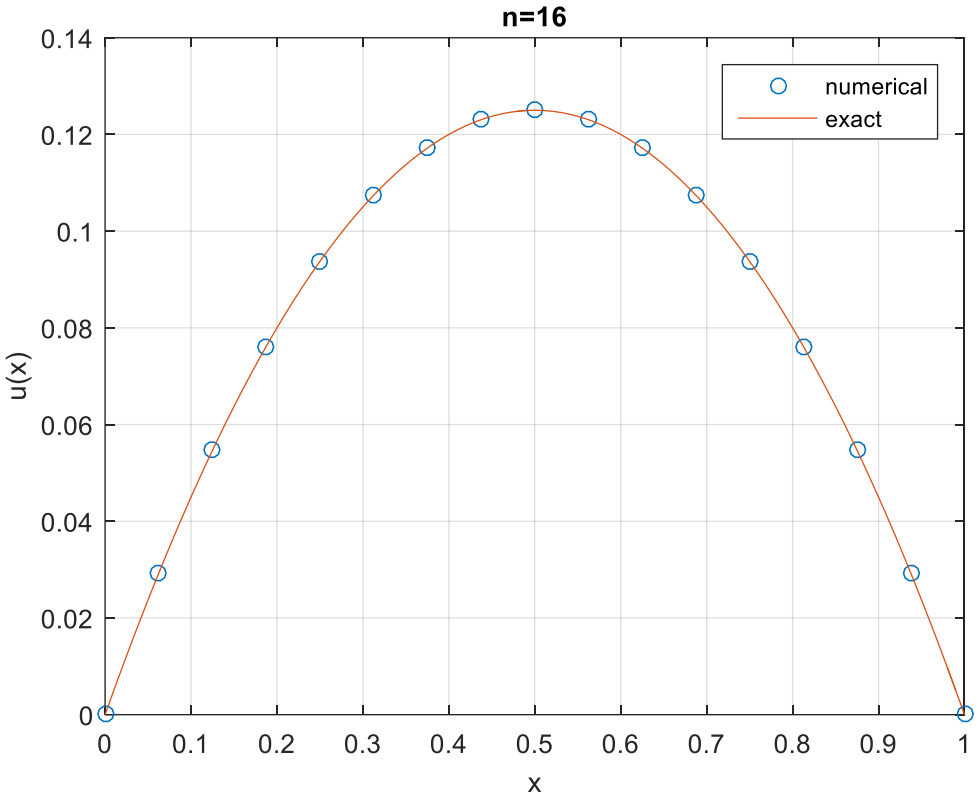
$$\begin{bmatrix} 4D+7F/8 & -4D/3+3F/8 & & & \\ -(D+F) & 2D+3F/8 & -D+3F/8 & & \\ F/8 & -(D+7F/8) & 2D+3F/8 & -D+3F/8 & \\ & F/8 & -(D+7F/8) & 2D+3F/8 & -D+3F/8 \\ & & F/8 & -(4D/3+3F/4) & 4D-3F/8 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \end{bmatrix} = \begin{bmatrix} 8D/3+5F/4 \\ F/8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

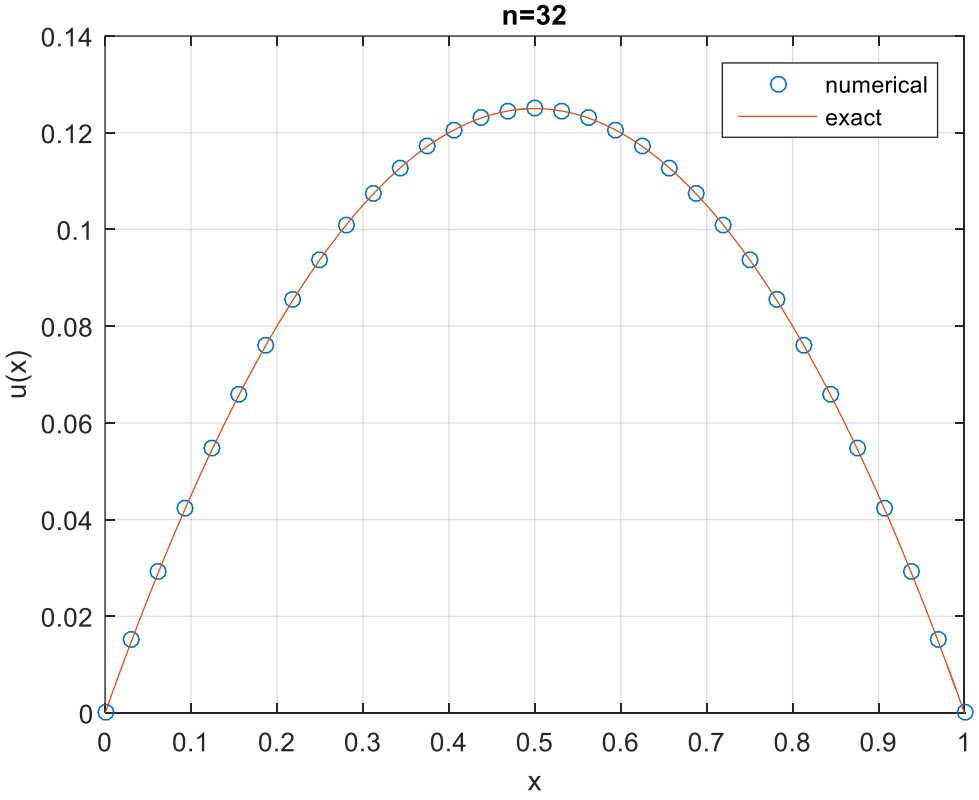
Cell	x_i	exact	upwind	error	QUICK	error
1	0.1	0.9653465621944958 8	0.9558684385194391 5	0.0094781 24	0.9648259863070423 0	0.0005205 76
2	0.3	0.8713239033026946 0	0.8499526909660928 3	0.0213712 12	0.8706981836152415 4	0.0006257 2
3	0.5	0.7310585786300049 0	0.7016706443914079 2	0.0293879 34	0.7308764687337229 2	0.0001821 1
4	0.7	0.5218073030606147 4	0.4940757791868493 1	0.0277315 24	0.5225679061335534 2	0.0007606 03
5	0.9	0.2096410821532594 9	0.2034429679004673 7	0.0061981 14	0.2122036860537555 6	0.0025626 04

Project 5


n=4
numerical
exact
u(x)
x
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


n=8
numerical
exact
u(x)
x
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


n=16
numerical
exact
u(x)
x
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


n=32
numerical
exact
u(x)
x
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1