Project #4: FVM for Advection-Diffusion Equation

A property ϕ is transported by means of convection and diffusion through the one-dimensional domain $x \in [0, L]$ according to the governing equation

$$\frac{\mathrm{d}}{\mathrm{d}x}(\rho u \phi) = \frac{\mathrm{d}}{\mathrm{d}x}\left(k\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)$$

The boundary conditions are

- $\phi_0 = 1$ at $x = 0$ and

- $\phi_L = 0$ at $x = L$.

Here $L = 1.0m$, $\rho = 1.0kg/m^3$, $u = 0.2m/s$ and $k = 0.1kg/m/s$. The exact solution of this problem is

$$\frac{\phi - \phi_0}{\phi_L - \phi_0} = \frac{\exp(\rho u x/k) - 1}{\exp(\rho u L/k) - 1}$$

Using *finite volume method* on five equally spaced cells to solve the above problem using

- method 1: upwind scheme.

- method 2: QUICK scheme.

Tabulate the solutions in the following table (you also need to submit your source code):

Cell	x_i	exact	upwind	error	QUICK	error
1	0.1					
2	0.3					
3	0.5					
4	0.7					
5	0.9					