Project #1: Finite Difference Method (1)

Use the finite difference method to solve the following 1-D Poisson equation

$$-u"(x) = f(x), \quad u(0) = 1,\ u(1) = e, \quad x \in [0, 1]$$

where $f(x) = -e^x$. The exact solution is $v(x) = e^x$ which can be used to compare with the numerical solution.

Use uniform grids

$$x_i = ih, \quad i = 0, 1, \cdots, n, \text{ where } h = \frac{1}{n}$$

and central difference formula to discretize the differential equation.

1. Show the details to form the tri-diagonal system for $n = 4$.
2. Solve the equation on four grids with $n_i = \{4, 8, 16, \text{and } 32\}$ using the Thomas Algorithm for tri-diagonal systems. Plot the results $u(x)$ vs. x for each n. Also plot the exact solution $v(x)$ vs. x on the same figure for comparison purpose.
3. Compute the error for each grid using

$$\epsilon(h) = \left(h \sum_{i=1}^{n-1} |u_i - v_i|^2 \right)^{1/2}$$

   where u_i is the numerical solution and v_i is the given exact solution at x_i.

4. Compute the order of accuracy using

$$\text{order} = \frac{\log(\epsilon(h_i)/\epsilon(h_{i-1}))}{\log(h_i/h_{i-1})}$$

   where h_i is the mesh size corresponding to n_i and tabulate your results as follows.

n_i	ϵ_i	order
4		-
8		
16		
32		