Project #3: Solving 1D Shocktube Problem

Solve the one dimensional shock tube problem.

- Computational domain is defined as $[0, 1]$.
- Governing equation is the Euler equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}}{\partial x} = \mathbf{0}$$

where the state vector and the flux vector are

$$\mathbf{u} = \begin{bmatrix} \rho \\ \rho u \\ \rho e \end{bmatrix}, \ \mathbf{f} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho h u \end{bmatrix}$$

with e the total specific energy and h the total enthalpy

$$e = e_i + \frac{1}{2}u^2, \text{ and } h = e + p/\rho$$

Here e_i is the specific internal energy related to p via

$$p = (\gamma - 1)\rho e_i$$

for ideal gases. $\gamma = 1.4$.

- Initial conditions when $t = 0$: at $x = 0.5$, a diaphragm separates the shock tube into the left high-pressure region and the right low-pressure region.
  - In the left region, $[\rho\, u\, p] = [1.0\ 0.0\ 1.0]$
  - In the right region, $[\rho\, u\, p] = [0.125\ 0.0\ 0.1]$

Requirements:

- use a grid containing 100 evenly distributed cells.
- use the following discretization for each cell of index j

$$\mathbf{u}_j^{n+1} = \mathbf{u}_j^n - \lambda[\mathbf{f}_{j+1/2}^n - \mathbf{f}_{j-1/2}^n]$$

where $\lambda = \Delta t/\Delta x$. Δx is the mesh size ($= 0.01$ if 100 cells are used in the simulation).

- use the HLLC scheme for the interface fluxes $\mathbf{f}_{j+1/2}^n$ and $\mathbf{f}_{j-1/2}^n$. Check the lecture slides for the HLLC scheme. Note:
  - for the interface $j + 1/2$, the left state $L = j$ and the right state $R = j + 1$ in the HLLC scheme.
  - for the interface $j - 1/2$, the left state $L = j - 1$ and the right state $R = j$ in the HLLC scheme.

- The resulting scheme is an explicit scheme. Therefore you need to find a stable time step Δt

$$\Delta t = CFL \frac{\Delta x}{S_{\max}}$$

  where $S_{\max} = \max_j(u-a, u, u+a)$ is the maximum wave speed in the computational domain. The speed of sound $a = \sqrt{\gamma p/\rho}$. The CFL number can take the value of 0.8.
  The time step can be computed at the beginning of the simulation.

- Run the simulation until $t = 0.2$. plot three curves:
  - ρ vs. x.
  - u vs. x.
  - p vs. x.