Project #2: FDM in 2D

Write a sequential program to solve the following Poisson equation on the domain $[0,1] \times [0,1]$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y) \quad \text{in the interior}$$
$$u(x,\, y) = 0 \quad \text{on the boundary}$$

where

- $u(x,\, y)$ is the unknown solution to be found.
- $f(x,\, y)$ is a known function which is given by

$$f(x,y) = -10(1-10(x-0.5)^2)e^{-5(x-0.5)^2}(e^{-5(y-0.5)^2} - e^{-5/4}) \\ -10(1-10(y-0.5)^2)e^{-5(y-0.5)^2}(e^{-5(x-0.5)^2} - e^{-5/4})$$

Requirements:

- use the following pseudo code as a reference:

```
k = 0
Choose an initial solution, u^0, for all (x_i, y_j).
while (not converged)
    k = k + 1
    loop over all interior (i, j)
        u_{i,j}^{k+1} = a u_{i-1,j}^k + b u_{i+1,j}^k + c u_{i,j-1}^k + d u_{i,j+1}^k - e f_{i,j}
    end loop
    check convergence.
end while
```

- the initial solution can be taken as $u^0(x,y) = 0$.
- check the convergence using the following criterion:

$$\max_{i,j} |u^k(x_i,y_j) - u^{k-1}(x_i,y_j)| < \epsilon$$

  where ϵ is a small number, say, 10^{-6}.

- use C pointers for the main data structure (*to dynamically allocate the memory based on the user provided size of the mesh*).
- you need to output two curves for the case of 51×51 mesh:
  - convergence history: residual vs. iteration number.
  - solution along the horizontal line $y = 0.5$. (Make sure the number of points in each direction is an odd number such that the center grid line lies on $y = 0.5$).