Literature Review of GO, PO, AFIM, GTD, UTD, PTD, FW, EEC, FPR and Shadowing Measurement

20190415

Duxin (D1)

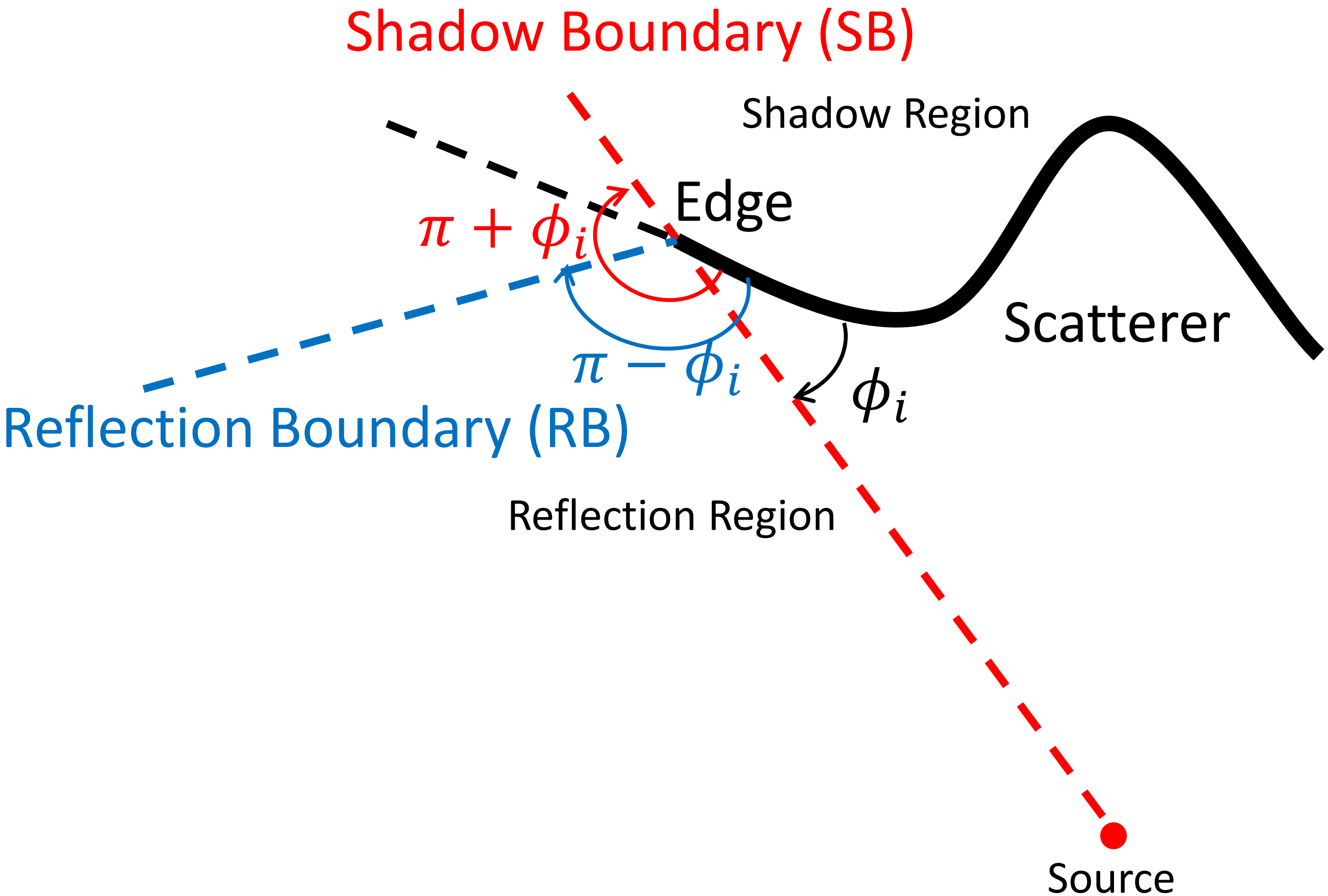
Outline of today presentation:

- Literature review
- Shadowing measurement

Terminology and Abbreviations

GO	Geometrical optics
PO	Physical optics
GTD	Geometrical theory of diffraction
UTD	Uniform geometrical theory of diffraction
AFIM	Aperture field integration method
PTD	Physical theory of diffraction
FW	Fringe wave
FPR	Fictitious penetrating rays
EEC	Equivalent edge currents

Terminology: SB and RB



EM numerical computation method

Full-wave Numerical Methods:

FDTD, MoM, FEM, MODE…

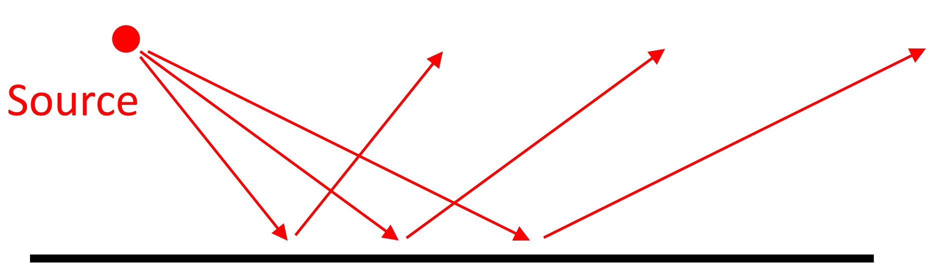
Huge calculation time

Large memory size requirement

High-Frequency Asymptotic Methods

High-Frequency Asymptotic Methods

- GO (ray-based)



✓ **Reflection**

✗ **Diffraction**

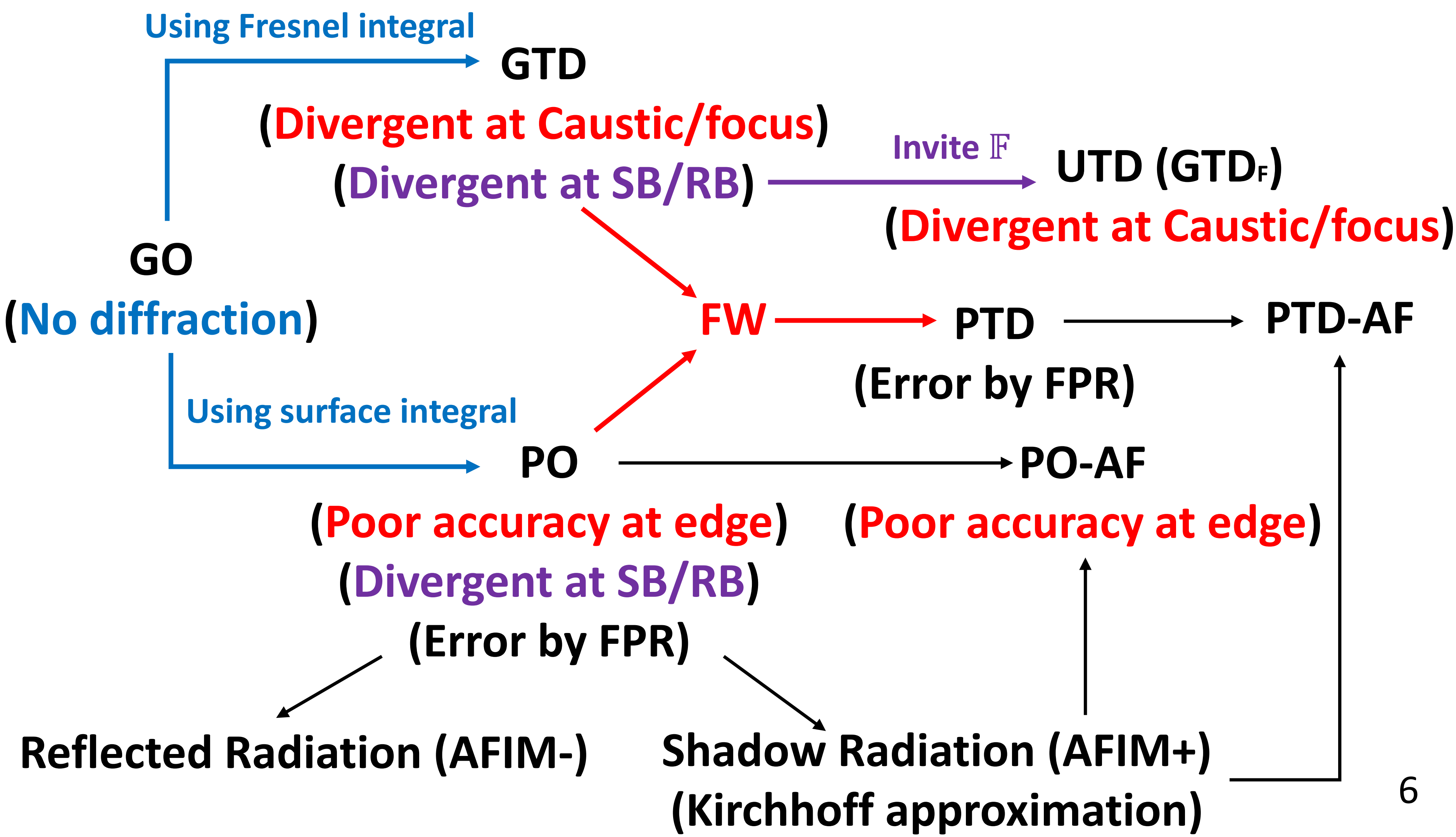
- PO (wave-based)

✓ **Diffraction**

✓ **Scattering**

- GTD, UTD, PTD…

Relation Map for GO, PO, GTD, UTD, AFIM and PTD



Outline of Literature review [1]-[3]

- GO
- GTD
- UTD
- PO
- PTD
- AFIM
- FPR
- PO-AF and PTD-AF
- EEC and diffraction coefficients

[1] Masayuki Oodo, Tsutomu Murasaki, and Makoto ANDO "Errors of Physical Optics in Shadow Region – Fictitious Penetrating Rays" IEICE Trans, Electron., Vol.E77-C, No.6, pp.995–127, 1994/06.

[2] Ken-ichi SAKINA, Suomin CUI, and Makoto ANDO "Derivation of Uniform PO Diffraction Coefficients Based on Field Equivalence Principle" IEICE, C, Vol.J83-C, No.2, pp.118–127, 2000/09.

[3] Tetsu Shijo, Makoto ANDO "Elimination of Fictitious Penetrating Rays from PO and Hybridization with AFIM" IEEJ Trans, FM, Vol.123, No.12, pp.1185–1192, 2003.

Geometrical optics (GO)

- Direct Ray

$$E^i = E_0 \frac{e^{-jkd_1}}{d_1}$$

Emitted electric field

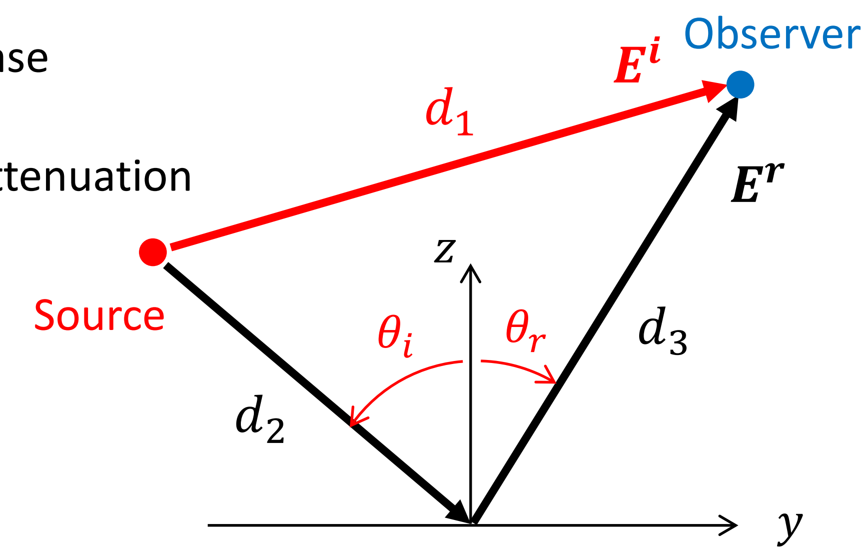
Phase

Attenuation

- Reflected Ray

$$E^r = E_0 R \frac{e^{-jk(d_2+d_3)}}{d_2 + d_3}$$

Fresnel reflection coefficient



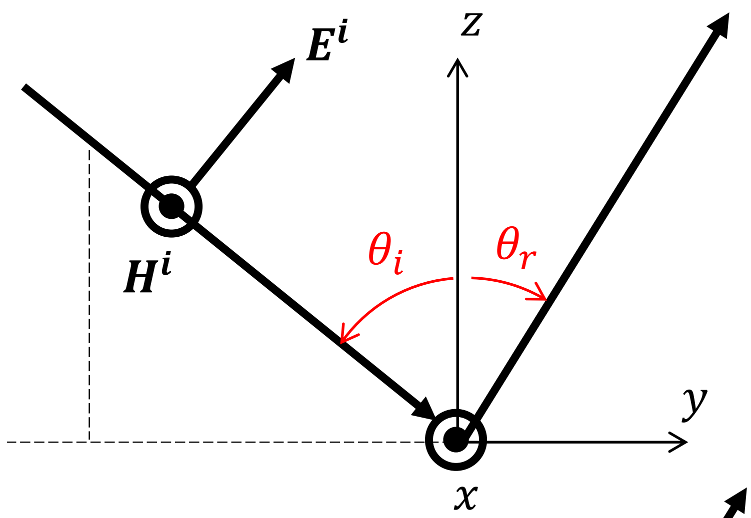
$$E^{GO} = E^i + E^r$$

Reflection Coefficient

- $\mu_1 = \mu_2$
- ε_1 – air
- ε_2 – material

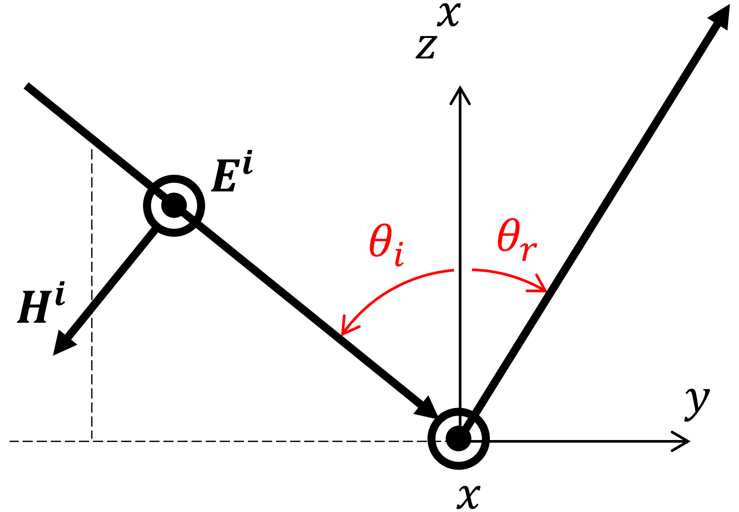
- Parallel polarization

$$R_{\parallel} = \frac{-\sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i + \sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_r}{\sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i + \sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_r}$$

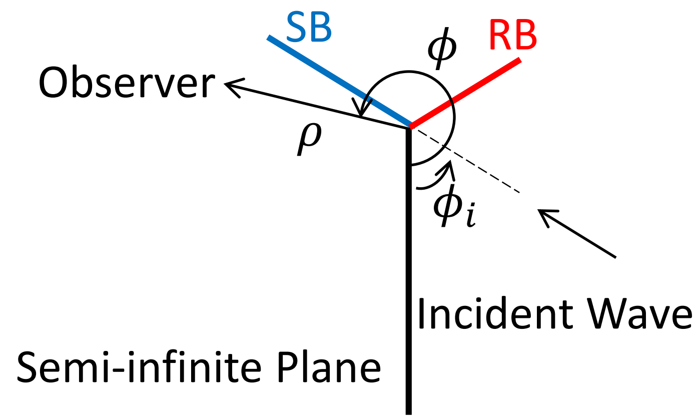


- Perpendicular polarization

$$R_{\perp} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_r}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_r}$$



Geometrical theory of diffraction (GTD)



Already presented in
2018/06/10
See appendix

$$\xi^i \equiv 2k\rho\cos^2\frac{(\phi-\phi_i)}{2}$$

$$\xi^r \equiv 2k\rho\cos^2\frac{(\phi+\phi_i)}{2}$$

$$E^s = E_0\left\{\underbrace{\sqrt{\frac{j}{\pi}}e^{j\xi^i}e^{jk\rho}\int_{-\infty}^{\sqrt{\xi^i}}e^{-jt^2}dt}_{\text{Incident (GO + Diffraction)}} - \underbrace{\sqrt{\frac{j}{\pi}}e^{j\xi^r}e^{jk\rho}\int_{-\infty}^{\sqrt{\xi^r}}e^{-jt^2}dt}_{\text{Reflection (GO + Diffraction)}}\right\} := E^{GO} + E^d$$

$$E^d = E_0\frac{e^{-\frac{j\pi}{4}}e^{-jk\rho}}{2\sqrt{2k\pi\rho}}\left\{-\left(\cos\frac{(\phi-\phi_i)}{2}\right)^{-1} + \left(\cos\frac{(\phi+\phi_i)}{2}\right)^{-1}\right\} = -E_0\frac{e^{-jk\rho-\frac{j\pi}{4}}}{\sqrt{2k\pi\rho}}D_\parallel^{GTD}$$

Diffraction coefficient of GTD: $\boxed{2D_\parallel^{GTD} = \sec\frac{\phi-\phi_i}{2} - \sec\frac{\phi+\phi_i}{2}}$

Uniform theory of diffraction (UTD)

Diffraction coefficient of GTD: $2D_{\parallel}^{GTD} = \sec\dfrac{\phi - \phi_i}{2} - \sec\dfrac{\phi + \phi_i}{2}$

Divergent at SB ($\phi = \pi + \phi_i$) and RB ($\phi = \pi - \phi_i$)

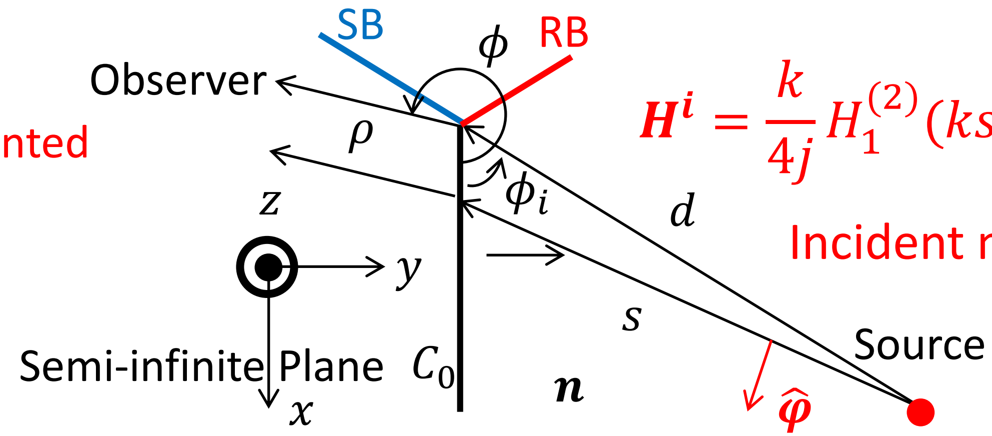
Diffraction coefficient of UTD: $\boxed{2D_{\parallel}^{UTD} = F_- \sec\dfrac{\phi - \phi_i}{2} - F_+ \sec\dfrac{\phi + \phi_i}{2}}$

(GTD$_F$)

Where, $F_\pm = F(2k\rho \cos^2 \dfrac{\phi \pm \phi_i}{2})$

$F(x) = 2j\sqrt{x}e^{jx}\int_{\sqrt{x}}^{\infty} e^{-jr^2}dr$ Modified Fresnel function

Physical optics (PO)



Already presented several times
See appendix

$$\boldsymbol{H}^{\boldsymbol{i}} = \frac{k}{4j} H_1^{(2)}(ks)\widehat{\boldsymbol{\varphi}} \approx \frac{k}{4}\sqrt{\frac{2}{\pi ks}}\, e^{-jks+j\frac{\pi}{4}}\widehat{\boldsymbol{\varphi}}$$

Incident magnetic field in far-field

2D Green function in far-field: $G \approx \frac{1}{4j}\sqrt{\frac{2}{\pi k\rho}}\, e^{-jk\rho+j\frac{\pi}{4}}$

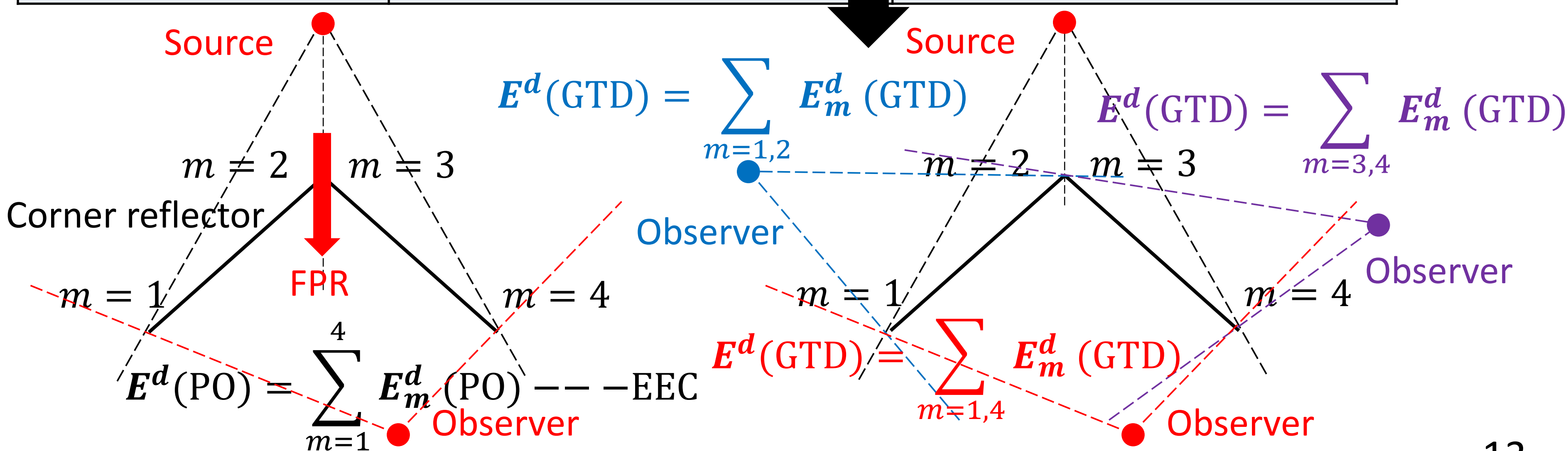
$$\boldsymbol{E}^{\boldsymbol{s}} = -j\omega\mu \int_{C_0} \boldsymbol{J}^{\boldsymbol{PO}} G\,dx = -j\omega\mu \int_0^{\infty} 2\boldsymbol{n} \times \boldsymbol{H}^{\boldsymbol{i}} G\,dx = \boldsymbol{E}^{\boldsymbol{GO}} + \boldsymbol{E}^{\boldsymbol{d}}$$

$$\boldsymbol{E}^{\boldsymbol{d}} = -\hat{\boldsymbol{z}}E_0 \frac{e^{-\frac{j\pi}{4}} e^{-jk\rho}}{2\sqrt{2k\pi\rho}} \left\{ \tan\frac{\phi-\phi_i}{2} - \tan\frac{\phi+\phi_i}{2} \right\} = -\hat{\boldsymbol{z}}E_0 \frac{e^{-jk\rho-\frac{j\pi}{4}}}{\sqrt{2k\pi\rho}} D_{\parallel}^{PO}$$

Diffraction coefficient of PO: $\boxed{2D_{\parallel}^{PO} = \tan\frac{\phi-\phi_i}{2} - \tan\frac{\phi+\phi_i}{2}}$

	PO (numerical)	GTD (analytical)
Caustic/focus	✓ Surface integral	× Divergent due to ray approximation
Edge	× Error due to infinite plane approximation	✓ Semi-infinite plane and perturbation
SB/RB	× Divergent due to diffraction coefficient	× Divergent due to diffraction coefficient
Fictitious penetrating rays	× Error due to plane assumption for scatterer with curvature/corner	✓ The ray emanating from invisible edges are omitted

PTD



Physical theory of diffraction (PTD)

$$2D_{\parallel}^{GTD}=\sec\frac{\phi-\phi_i}{2}-\sec\frac{\phi+\phi_i}{2}$$
(at visible edge)

$$2D_{\parallel}^{PO}=\tan\frac{\phi-\phi_i}{2}-\tan\frac{\phi+\phi_i}{2}$$
(at ALL edge ---- EEC)

Fringe wave: $2D_{\parallel}^{FW}=2D_{\parallel}^{GTD}-2D_{\parallel}^{PO}=\dfrac{1-\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}}-\dfrac{1-\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$

(at visible edge)

Do NOT divergent at SB/RB

$$\boldsymbol{E}^s(\mathrm{PTD})=\sum_{m=\,all\ edge}\boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}(\mathrm{PO})+\sum_{m=\,visible\ edge}\boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}(\mathrm{FW})$$

$$=\sum_{m=\,visible\ edge}\boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}(\mathrm{GTD})+\sum_{m=\,invisible\ edge}\boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}(\mathrm{PO})$$

Still has FPR error

Correct PO error at edge

Surface integral solve GTD singular like caustic

Aperture field integration method (AFIM)

- AFIM+: radiate to shadowing region (forward scattering)

  also called *Kirchhoff approximation*

- AFIM-: radiate to reflection region (backward scattering)

$$\boldsymbol{J}^{PO} = \boldsymbol{J}^{AFIM-} + \boldsymbol{J}^{AFIM+} = \boldsymbol{n} \times \boldsymbol{H}^{i} + \boldsymbol{n} \times \boldsymbol{H}^{i} = 2\boldsymbol{n} \times \boldsymbol{H}^{i}$$

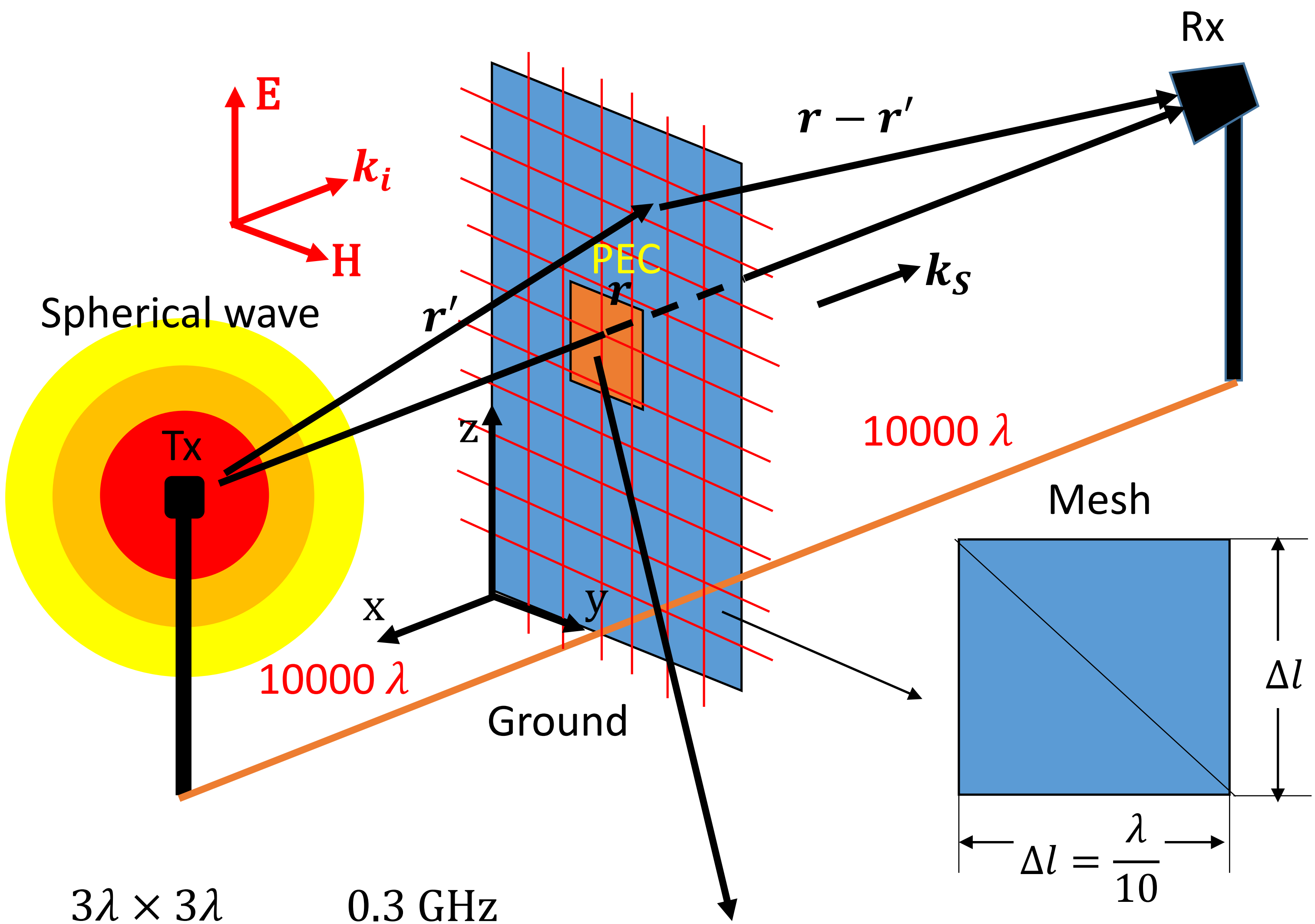
Shadow radiation source

Reflection radiation source

$$\boldsymbol{M}^{PO} = \boldsymbol{M}^{AFIM-} + \boldsymbol{M}^{AFIM+} = \boldsymbol{n} \times \boldsymbol{E}^{i} - \boldsymbol{n} \times \boldsymbol{E}^{i} = \boldsymbol{0}$$

$$\boxed{\begin{aligned} 2D_{\parallel}^{PO} &= \tan\frac{\phi - \phi_i}{2} - \tan\frac{\phi + \phi_i}{2} \\ 2D_{\parallel}^{AFIM+} &= \tan\frac{\phi - \phi_i}{2} \\ 2D_{\parallel}^{AFIM-} &= -\tan\frac{\phi + \phi_i}{2} \end{aligned}}$$

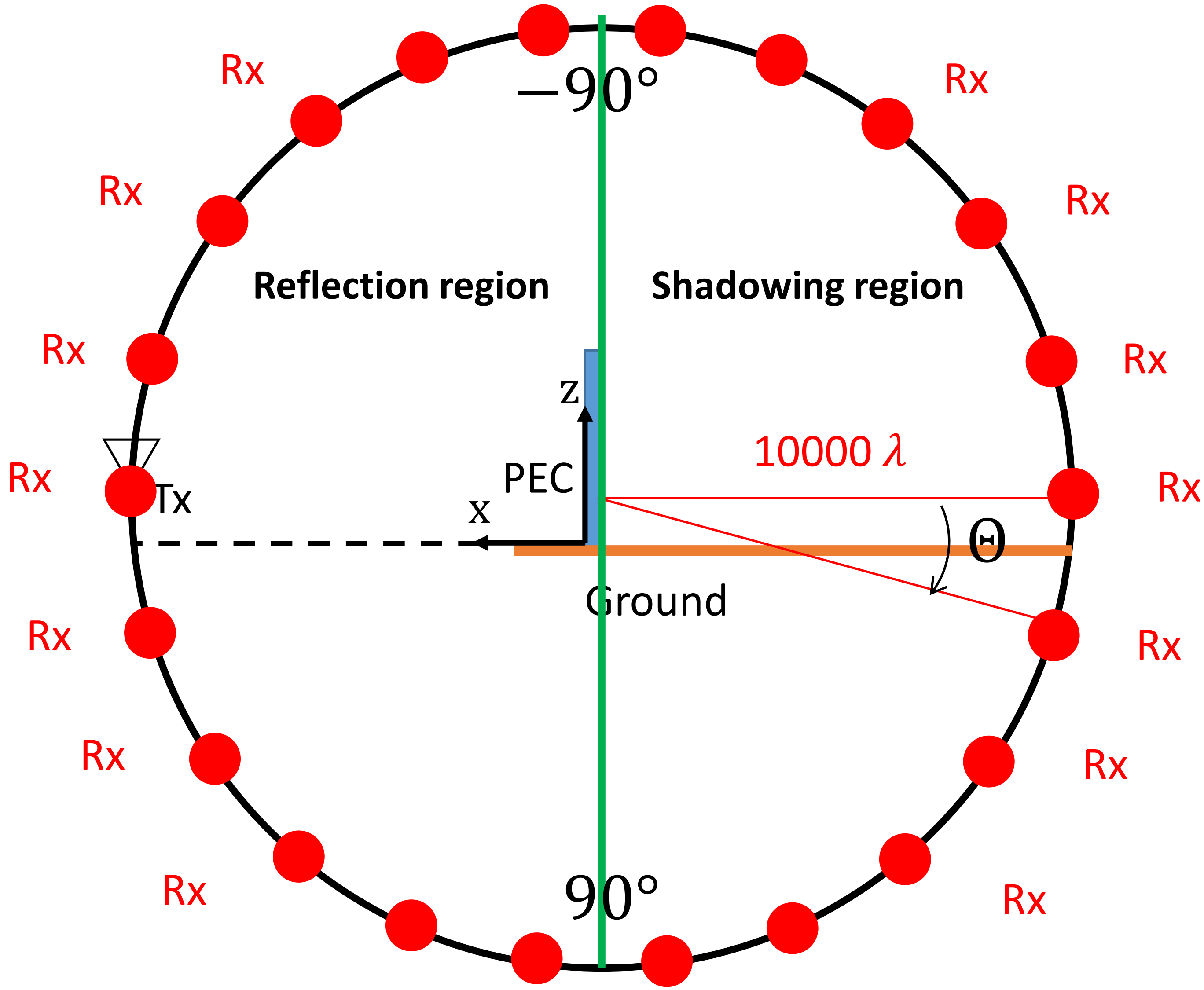
Simulation for PO, AFIM+ and AFIM-



$3\lambda \times 3\lambda$ 0.3 GHz

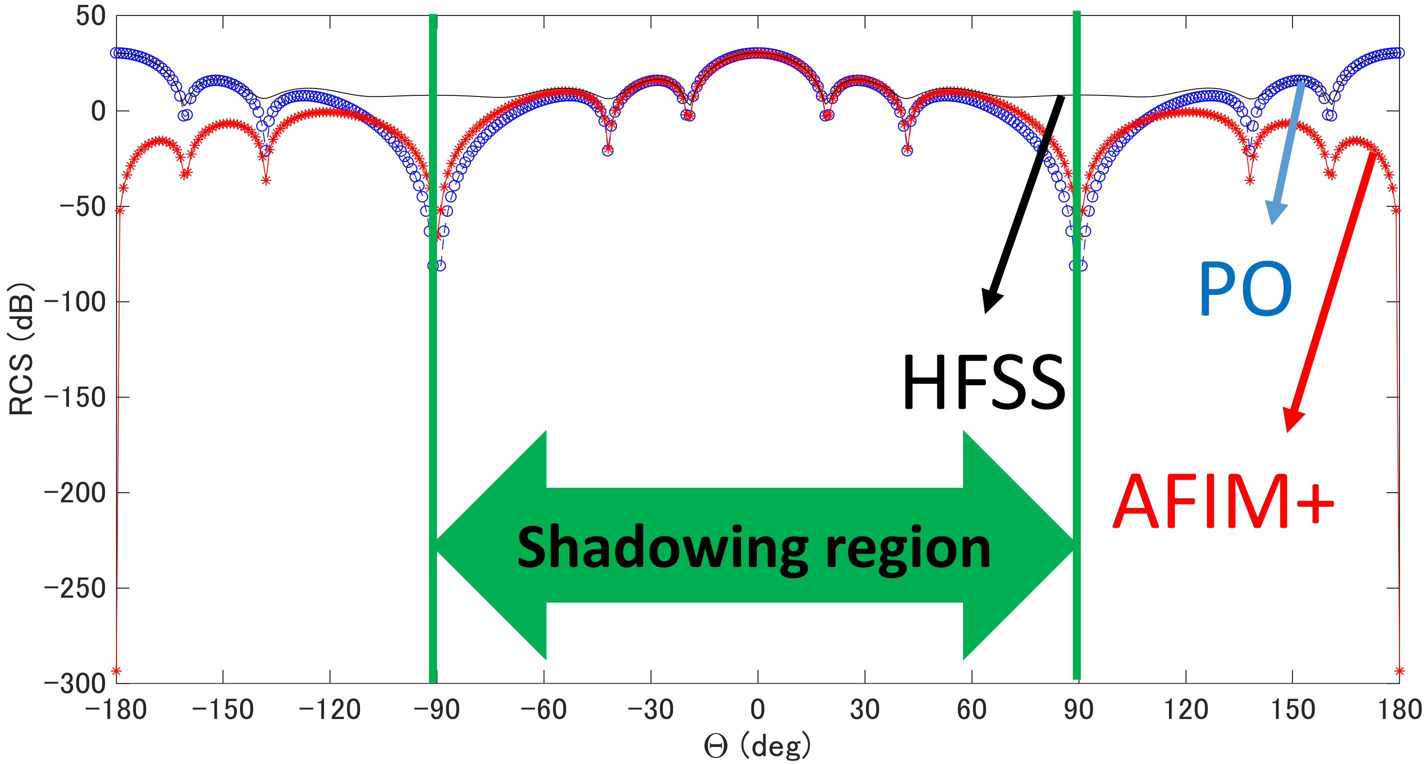
Using the same size and frequency as in the simulation from Hirano sensei

Rotating Rx for calculate RCS



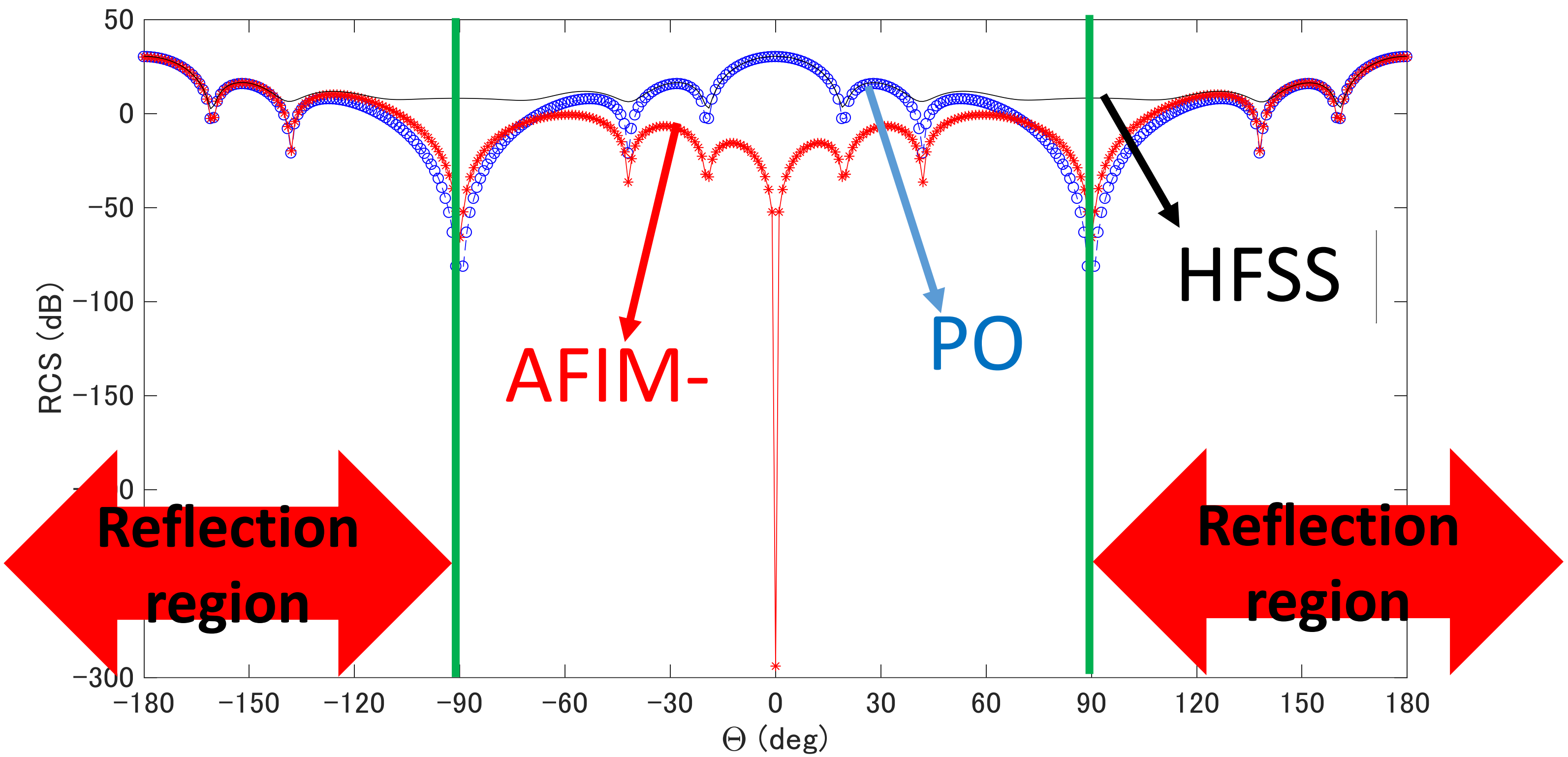
RCS for AFIM+ (shadow radiation)

From the RCS fig. we find only the reflection region (180 deg.) do not receive the scattered field.



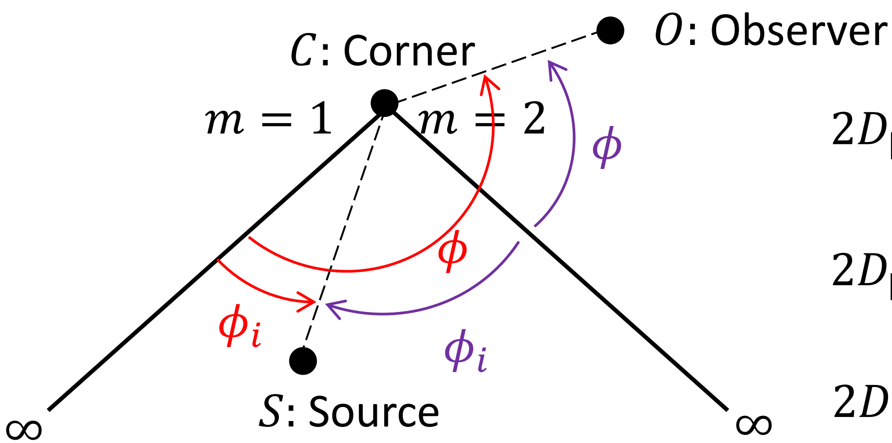
RCS for AFIM- (reflection radiation)

From the RCS fig. we find only the shadowing region (0 deg.) do not receive the scattered field.



Fictitious penetrating rays (FPR)

- FPR: PO assumes scatterer with corner is plane.
- AFIM+: No FPR
- AFIM-: Has FPR causing error



$$2D_{\parallel}^{AFIM+}(m=1) = \tan\frac{\phi-\phi_i}{2} = \tan\frac{1}{2}\angle SCO$$

$$2D_{\parallel}^{AFIM+}(m=2) = \tan\frac{\phi-\phi_i}{2} = \tan\frac{1}{2}\angle SCO$$

$$2D_{\parallel}^{AFIM+}(m=1) = 2D_{\parallel}^{AFIM+}(m=2)$$

$$\boldsymbol{E}^{\boldsymbol{d}}(AFIM+) = \sum_{m=1,2} \boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}(AFIM+) = \boldsymbol{0}$$

$$\boldsymbol{E}^{\boldsymbol{d}}(AFIM-) = \sum_{m=1,2} \boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}(AFIM-) \neq \boldsymbol{0}$$

$$\boldsymbol{E}^{\boldsymbol{d}}(PO) = \boldsymbol{E}^{\boldsymbol{d}}(AFIM-) + \boldsymbol{E}^{\boldsymbol{d}}(AFIM+)$$

FPR

PO-AF and PTD-AF

- Omit AFIM- from surface integral to omit FPR.
- Left AFIM- diffraction from visible edge.

$$\boldsymbol{E}^{\boldsymbol{s}}(\text{PO–AF}) = -j\omega\mu \int_{s_0} \boldsymbol{J}^{\boldsymbol{AFIM+}} G ds + \sum_{m=\text{ visible edge}} \boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}\ (\text{AFIM–})$$

$$\boldsymbol{E}^{\boldsymbol{s}}(\text{PTD–AF}) = -j\omega\mu \int_{s_0} \boldsymbol{J}^{\boldsymbol{AFIM+}} G ds + \sum_{m=\text{ visible edge}} \{\boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}\ (\text{AFIM–}) + \boldsymbol{E}_{\boldsymbol{m}}^{\boldsymbol{d}}\ (\text{FW})\}$$

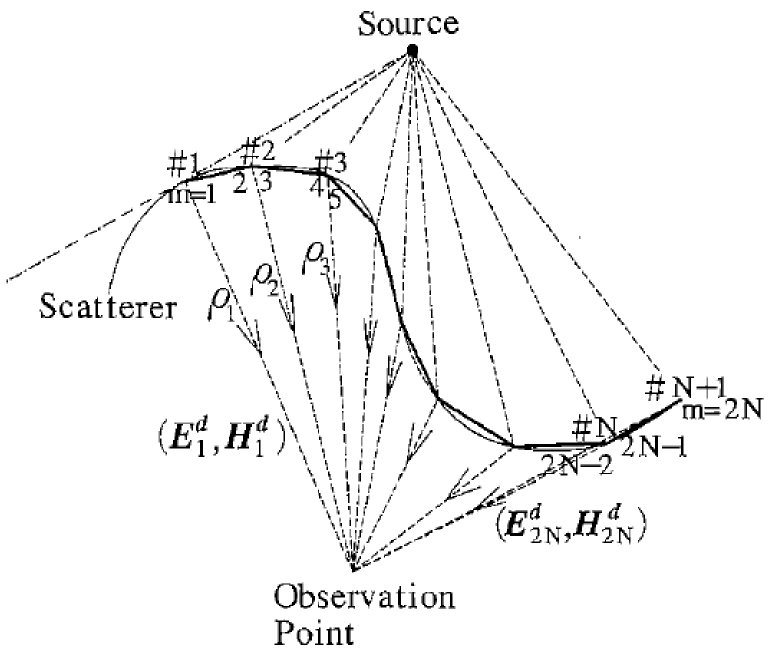
✓PO surface integral to reduce GO singularity

✓GTD diffraction coefficient to deal with edge inaccuracy

✓AFIM decomposition to reduce FPR error

✓Do NOT need Fresnel integral but still converge at SB/RB

Equivalent edge currents (EEC)



$$J^{EEC} = \frac{2E^i}{j\omega\mu} D_{\parallel}$$

$$M^{EEC} = -\frac{2H^i}{j\omega\varepsilon} D_{\perp}$$

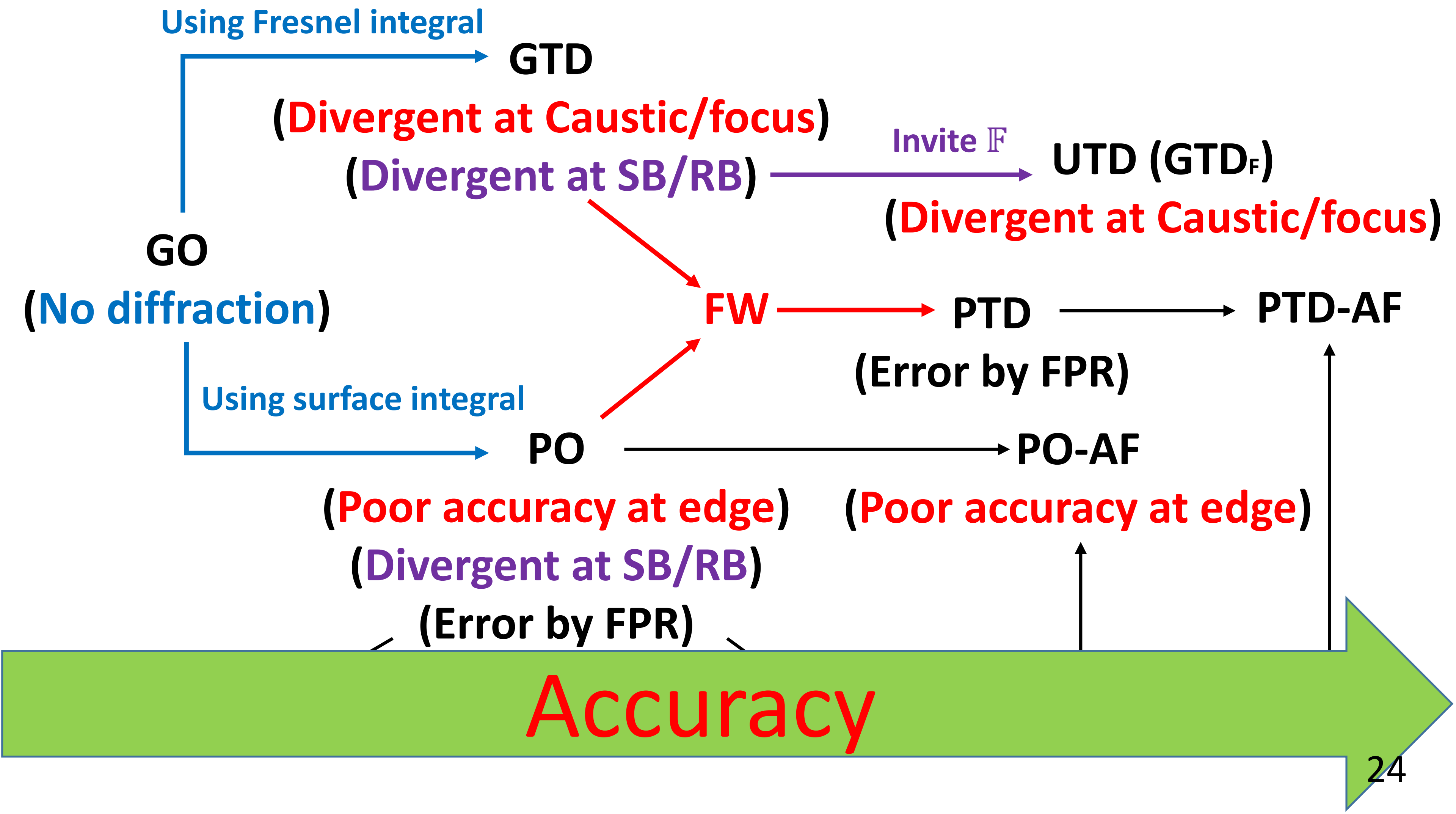
$$E^d = -\frac{\omega\mu}{4} \sum_{m=1}^{2N} \sqrt{\frac{2}{\pi k \rho_m}}\, e^{-jk\rho_m + \frac{j\pi}{4}} J^{EEC}$$

$$H^d = -\frac{\omega\varepsilon}{4} \sum_{m=1}^{2N} \sqrt{\frac{2}{\pi k \rho_m}}\, e^{-jk\rho_m + \frac{j\pi}{4}} M^{EEC}$$

Diffraction coefficient

	$2D_{\parallel}$	$2D_{\perp}$
GTD	$\sec\frac{\phi-\phi_i}{2}-\sec\frac{\phi+\phi_i}{2}$	$-\sec\frac{\phi-\phi_i}{2}-\sec\frac{\phi+\phi_i}{2}$
UTD (GTDF)	$F_-\sec\frac{\phi-\phi_i}{2}-F_+\sec\frac{\phi+\phi_i}{2}$	$-F_-\sec\frac{\phi-\phi_i}{2}-F_+\sec\frac{\phi+\phi_i}{2}$
PO	$\tan\frac{\phi-\phi_i}{2}-\tan\frac{\phi+\phi_i}{2}$	$-\tan\frac{\phi-\phi_i}{2}-\tan\frac{\phi+\phi_i}{2}$
PTD (FW)	$\frac{1-\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}}-\frac{1-\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$	$-\frac{1-\sin\frac{\phi-\phi_i}{2}}{\cos\frac{\phi-\phi_i}{2}}-\frac{1-\sin\frac{\phi+\phi_i}{2}}{\cos\frac{\phi+\phi_i}{2}}$
AFIM+	$\tan\frac{\phi-\phi_i}{2}$	$-\tan\frac{\phi-\phi_i}{2}$
AFIM-	$-\tan\frac{\phi+\phi_i}{2}$	$-\tan\frac{\phi+\phi_i}{2}$

Summary of literature review



Outline of Shadowing measurement

- Measurement objective
- Measurement equipment
- Measurement environment
- Shadowing objects
- Measurement scenarios

2019/02/19 – 2019/02/21

NTT R&D Yokosuka

Objective of Measurement

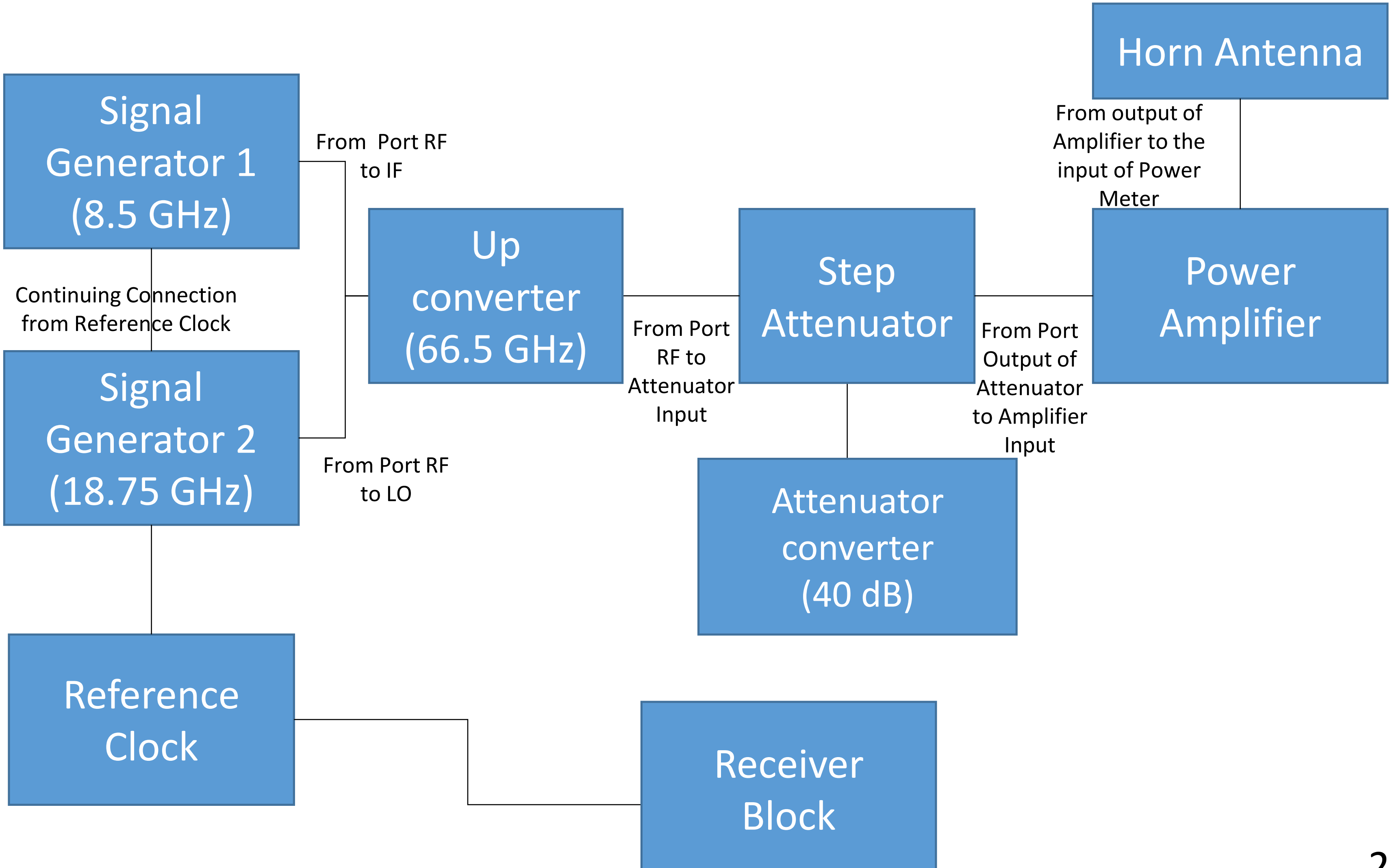
Objective：

Establishment of an accurate prediction technique of shadowing effect by 3D shadowing objects based on physical optics approach, by comparing with the measurement results.

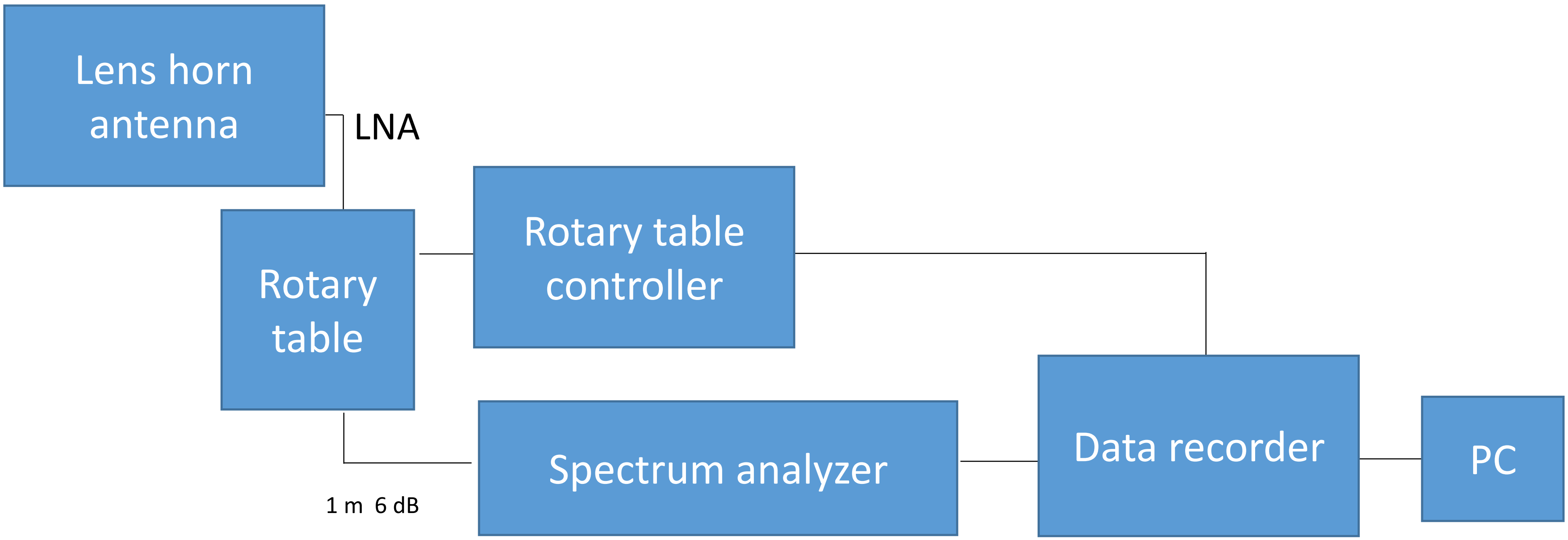
Points：

- Anechoic chamber room
- Shadowing objects with different shape and thickness
- 66.5 GHz frequency band

Measurement Equipment Tx



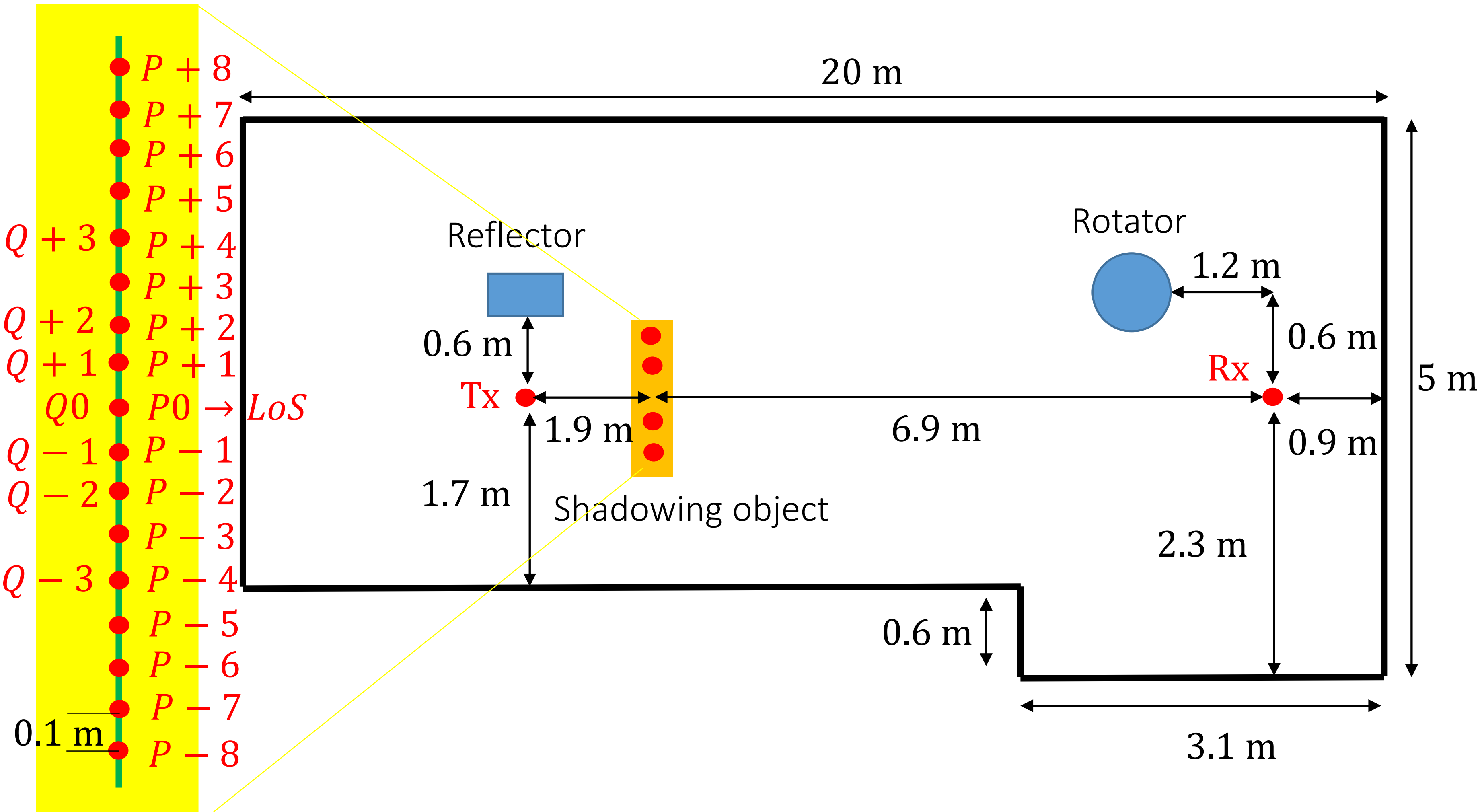
Measurement Equipment Rx



Environment

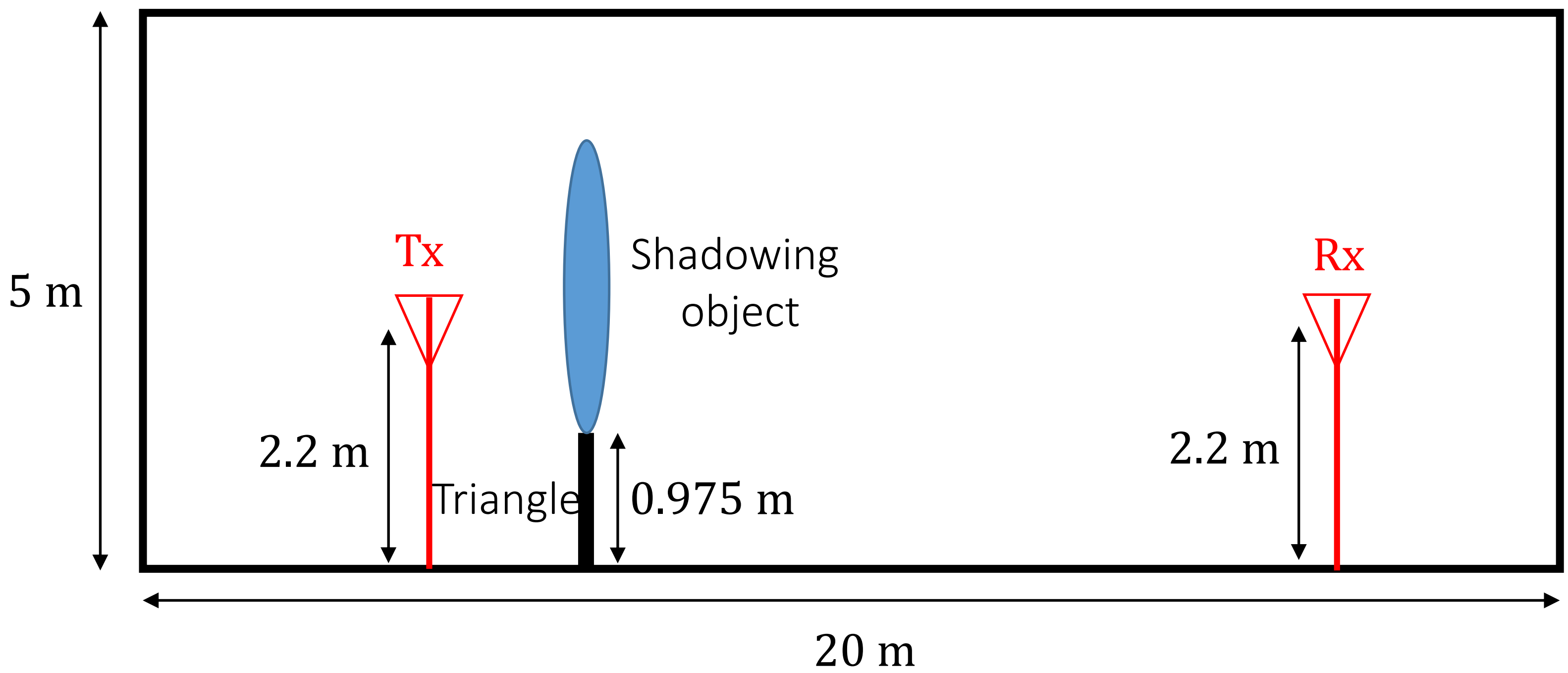
Anechoic chamber room

Vertical view of room



Cross-section view of room

- Height of antenna： 2.2m



Shadowing objects

- Rectangular metal plane (thickness 0.01 m)
- Rectangular metal box (thickness 0.01 m)
- Human-shape metal plane (thickness 0.3 m)
- Human-shape metal box (thickness 0.3 m)
- Human (thickness about 0.3 m)

Shadowing objects

Rectangular metal

Human-shape metal

Measurement scenarios

- Rectangular metal plane × [p+8 to p-8] × 4 times
- Rectangular metal box × [p+8 to p-8] × 4 times
- Human-shape metal plane × [p+8 to p-8] × 3 times
- Human-shape metal box × [p+8 to p-8] × 2 times
- Human × ([p+3 to p-3] + [p+2 to p-2])

Future work of measurement

- Wait measurement data from NTT
- Wait photograph of environment and equipment
- Do data analysis
- Do PO simulation and KEDM simulation

- Improve prediction of shadowing effect technique for shadowing objects with different shape and thickness based on measurement data and PTD-AF.

Thank you for your listening

Appendix

Maxwell's Equations

Maxwell's Eq. where $\frac{\partial}{\partial t} \equiv j\omega$

$$\begin{cases} \nabla \times \boldsymbol{H} = j\omega\varepsilon\boldsymbol{E} + \boldsymbol{J} \\ \nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H} \\ \nabla \cdot \boldsymbol{B} = 0 \quad \Rightarrow \nabla \cdot (\mu\boldsymbol{H}) = 0 \\ \nabla \cdot \boldsymbol{D} = \rho \qquad\qquad \Uparrow \end{cases}$$

For free space $\begin{cases} \boldsymbol{D} = \varepsilon\boldsymbol{E} \\ \boldsymbol{B} = \mu\boldsymbol{H} \end{cases}$

Vector formula: ① $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$ ② $\nabla \times \nabla\emptyset = 0$

$$\nabla \times \boldsymbol{E} = -j\omega(\nabla \times \boldsymbol{A}) \Rightarrow \nabla \times (\boldsymbol{E} + j\omega\boldsymbol{A}) = 0$$

$$\Uparrow \qquad\qquad \Downarrow ②$$

$$\overset{①}{\Longrightarrow} \boxed{\exists\, \boldsymbol{A}\ s.t\ \nabla \times \boldsymbol{A} = \mu\boldsymbol{H}} \qquad \boxed{\exists\, \emptyset\ s.t\ \boldsymbol{E} + j\omega\boldsymbol{A} = -\nabla\emptyset}$$

$$\Downarrow$$

$$\nabla \times \nabla \times \boldsymbol{A} = \mu(\nabla \times \boldsymbol{H}) = j\omega\varepsilon\mu\boldsymbol{E} + \mu\boldsymbol{J}$$

$$\Downarrow$$

$$\nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2\boldsymbol{A} = j\omega\varepsilon\mu(-j\omega\boldsymbol{A} - \nabla\emptyset) + \mu\boldsymbol{J}$$

$$\Downarrow$$

$$\nabla^2\boldsymbol{A} + k^2\boldsymbol{A} = \nabla(j\omega\varepsilon\mu\emptyset + \nabla \cdot \boldsymbol{A}) - \mu\boldsymbol{J} \quad \text{where } k = \omega\sqrt{\varepsilon\mu}$$

(wave number)

$$\Downarrow$$

$$\boxed{j\omega\varepsilon\mu\emptyset + \nabla \cdot \boldsymbol{A} = 0 \Rightarrow \nabla^2\boldsymbol{A} + k^2\boldsymbol{A} = -\mu\boldsymbol{J} \Rightarrow \text{Next Page}}$$

(Lorentz condition) (Vector Helmholtz equation)

$$\Downarrow$$

$$\boxed{\boldsymbol{E} = -j\omega\boldsymbol{A} - \nabla\emptyset = -j\omega\boldsymbol{A} + \frac{\nabla(\nabla \cdot \boldsymbol{A})}{j\omega\varepsilon\mu} \quad , \boldsymbol{H} = \frac{\nabla \times \boldsymbol{A}}{\mu}}$$

Terminology [8]:

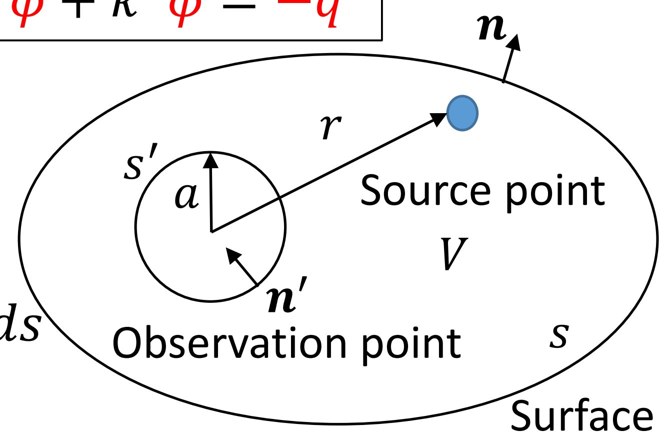
$\boldsymbol{E}$- electric field intensity
$\boldsymbol{H}$- magnetic field intensity
$\boldsymbol{B}$- magnetic flux density
$\boldsymbol{D}$- electric flux density
ε- permittivity
μ- permeability
ω- angle frequency
$\boldsymbol{J}$- impressed electric current
$\boldsymbol{A}$- vector potential
$\emptyset$- scaler potential

[8] Balanis/Advanced Engineering Electromagnetics, 2nd eds, Wiley, 2012.

Green’s Function

$$\boxed{\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J} \Rightarrow \nabla^2 A_x + k^2 A_x = -\mu J_x \Rightarrow \nabla^2 \phi + k^2 \phi = -q}$$

(Vector H. equation) (Scaler H. equation)



$$\nabla^2 \psi + k^2 \psi = 0 \Rightarrow \psi = \frac{e^{-jkr}}{r}$$ (particular solution)

$$\iiint_V \left(\phi \nabla^2 \psi - \psi \nabla^2 \phi\right) dV = \iint_s \left(\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}\right) ds$$

(Green’s theorem)

$$\iiint_V \underbrace{\left(\phi(-k^2\psi) - \psi(-q - k^2\phi)\right)}_{\psi q} dV = \lim_{a \to 0} \iint_{s+s'} \left(\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n}\right) ds$$

$$\frac{\partial \psi}{\partial n'} = -\frac{\partial \psi}{\partial r} = \frac{e^{-jkr} \overbrace{+ jkre^{-jkr}}^{0}}{r^2} \cong \frac{e^{-jka}}{a^2} \Rightarrow \lim_{a \to 0} \iint_{s'} \left(\phi \frac{\partial \psi}{\partial n'} - \overbrace{\psi \frac{\partial \phi}{\partial n'}}^{0}\right) ds \cong 4\pi\phi$$

(inward vs outward) (at $r = a$)

$$\phi = \frac{1}{4\pi} \iiint_V \frac{e^{-jkr}}{r} q \, dV + \underbrace{\frac{1}{4\pi} \iint_s \left(\frac{\partial \phi}{\partial n} \frac{e^{-jkr}}{r} - \phi \frac{\partial}{\partial n}\left(\frac{e^{-jkr}}{r}\right)\right) ds}_{0}$$

(Radiation condition)

$$\Downarrow$$

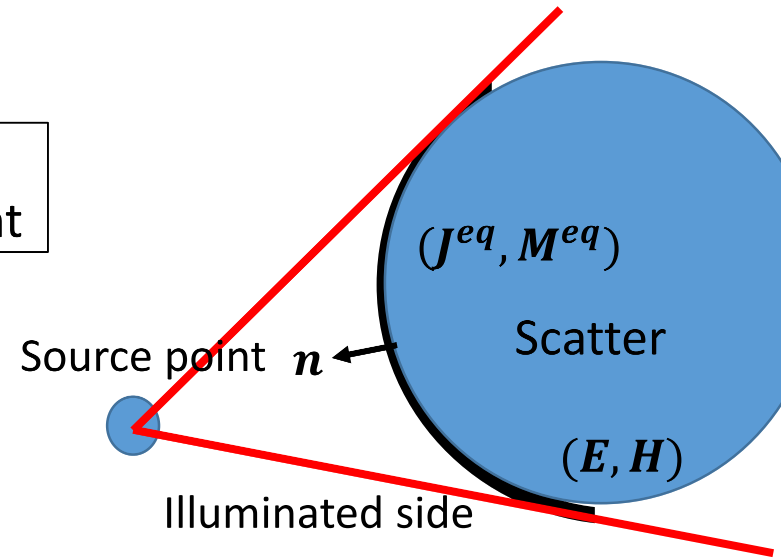
$$\boxed{\boldsymbol{A} = \frac{\mu}{4\pi} \iiint_V \frac{e^{-jkr}}{r} \boldsymbol{J} \, dV}$$

(Free space Green’s Function)

Physical Optics (PO)

- Field Equivalence Theorem:

$$\boldsymbol{J}^{eq} = \boldsymbol{n} \times \boldsymbol{H} \quad \text{equivalent electric current}$$
$$\boldsymbol{M}^{eq} = \boldsymbol{E} \times \boldsymbol{n} \quad \text{equivalent magnetic current}$$



- PO approximation:

➢ Currents are only generated on the illuminated side of the object.

➢ Integrate over the surface to give the scattered field.

➢ Infinite PEC plane.

$$\boldsymbol{J}^{PO} = 2\boldsymbol{n} \times \boldsymbol{H}$$
$$\boldsymbol{M}^{PO} = 0$$

$\Rightarrow$

$$\boldsymbol{A}^{PO} = \frac{\mu}{4\pi} \iiint_V \frac{e^{-jkr}}{r} \boldsymbol{J}^{PO}\, dV$$
$$\boldsymbol{E}^{PO} = -j\omega \boldsymbol{A}^{PO} + \frac{\nabla(\nabla \cdot \boldsymbol{A}^{PO})}{j\omega\varepsilon\mu}$$
$$\boldsymbol{H}^{PO} = \frac{\nabla \times \boldsymbol{A}^{PO}}{\mu}$$

$\Rightarrow$

Far field (kr>>1):

$$\boldsymbol{E} \cong -j\omega(\boldsymbol{A} - \hat{\boldsymbol{r}}(\boldsymbol{A} \cdot \hat{\boldsymbol{r}}))$$
$$\boldsymbol{H} \cong \sqrt{\frac{\varepsilon}{\mu}}(\hat{\boldsymbol{r}} \times \boldsymbol{E})$$

Scattering Problem in Cylindrical Coordinates

(pp. 12)

$$\boxed{\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J} \Rightarrow \nabla^2 \boldsymbol{A_c} + k^2 \boldsymbol{A_c} = 0 \Rightarrow \nabla^2 \psi_A + k^2 \psi_A = 0 \text{ where } \boldsymbol{A_c} = \hat{\boldsymbol{z}} \psi_A}$$

(Inhomogeneous D.E.) (Homogeneous D.E.) (Complementary Solution)

$$\Rightarrow \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \psi_A}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \psi_A}{\partial \phi^2} + \frac{\partial^2 \psi_A}{\partial z^2} + k^2 \psi_A = 0$$

Let $\psi_A(\rho, \phi, z) = R(\rho)\Phi(\phi)Z(z)$

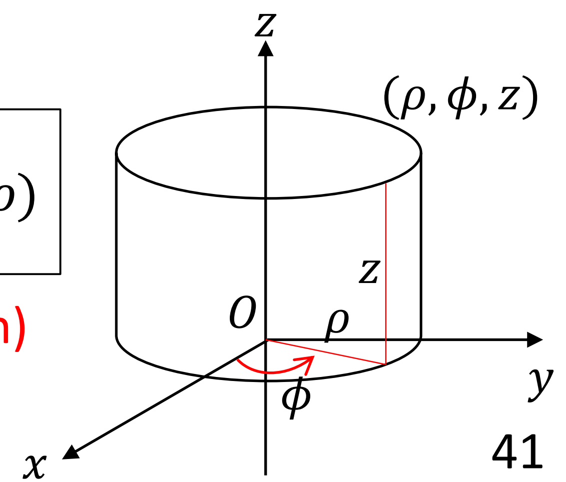
(P.D.E)

$$\Rightarrow \frac{1}{\rho R}\frac{d}{d\rho}\left(\rho \frac{\partial R}{\partial \rho}\right) + \frac{1}{\rho^2 \Phi}\frac{d^2 \Phi}{d\phi^2} + \underbrace{\frac{1}{z}\frac{d^2 Z}{\partial z^2}}_{-k_z{}^2} + k^2 = 0 \Rightarrow Z(z) = A_z e^{-jk_z z} + B_z e^{+jk_z z}$$

$$\Rightarrow \frac{\rho}{R}\frac{d}{d\rho}\left(\rho \frac{\partial R}{\partial \rho}\right) + \underbrace{\frac{1}{\Phi}\frac{d^2 \Phi}{d\phi^2}}_{-n^2} + \underbrace{(k^2 - k_z{}^2)}_{-k_\rho{}^2}\rho^2 = 0 \Rightarrow \Phi(\phi) = A_\phi \cos n\phi + B_\phi \sin n\phi$$

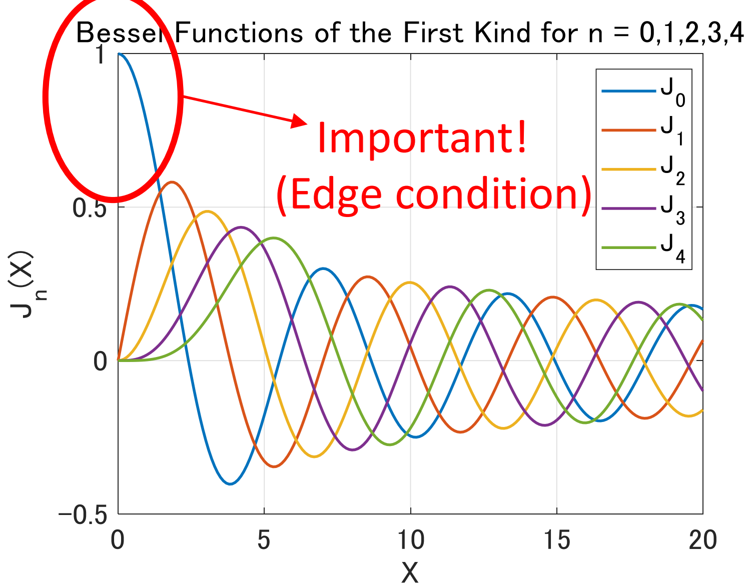
$$\boxed{\Rightarrow \rho \frac{d}{d\rho}\left(\rho \frac{\partial R}{\partial \rho}\right) + (k_\rho{}^2 \rho^2 - n^2) R = 0 \Rightarrow R(\rho) = B_n(k_\rho \rho)}$$

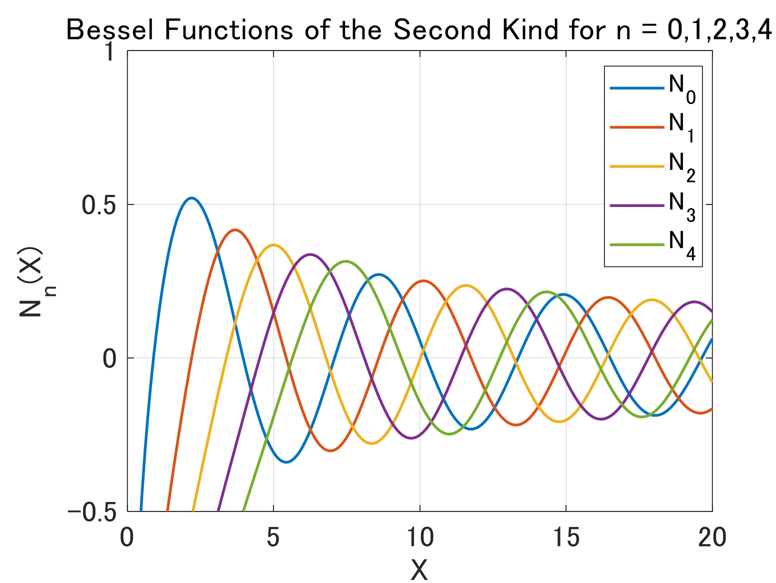
(Bessel Equation n[th] order) (Bessel Function)



Bessel Function

Harmonic Function	$B_n(x)$	Name
$\cos x$	$J_n(x)$	1st kind of Bessel function
$\sin x$	$N_n(x)$	2nd kind of Bessel function
$e^{+jx} = \cos x + j\sin x$	$H_n^{(1)}(x) = J_n(x) + jN_n(x)$	1st kind of Hankel function
$e^{-jx} = \cos x - j\sin x$	$H_n^{(2)}(x) = J_n(x) - jN_n(x)$	2nd kind of Hankel function
$e^{+x} = e^{+j(-jx)}$	$I_n(x) = j^n J_n(-jx)$	1st kind of modified Bessel function
$e^{-x} = e^{-j(-jx)}$	$K_n(x) = \dfrac{\pi}{2}(-j)^{n+1}H_n^{(2)}(-jx)$	2nd kind of modified Hankel function





Field Produced by the Line Current

$$\psi_A = (A_z e^{-jk_z z} + B_z e^{+jk_z z})(A_\phi \cos n\phi + B_\phi \sin n\phi)\left(A_\rho \overset{\text{inward}}{\boxed{H_n^{(1)}(k_\rho\rho)}} + B_\rho \overset{\text{outward}}{\boxed{H_n^{(2)}(k_\rho\rho)}}\right)$$

$\Downarrow$

Boundary condition:

$$\frac{\partial}{\partial z} = 0 \Rightarrow k_z = 0 \Rightarrow k_\rho = k$$

$$\frac{\partial}{\partial \phi} = 0 \Rightarrow n = 0\,, B_\phi = 0$$

Outgoing wave $\Rightarrow A_\rho = 0$

$$\Rightarrow \quad \boxed{\psi_A = A_0 H_0^{(2)}(k\rho)}$$

Unknown $\Downarrow$

$$\boxed{H_\phi = A_0 k H_1^{(2)}(k\rho)} \quad \Leftarrow$$

TM problem:

$$E_\rho = E_\phi = H_z = 0$$

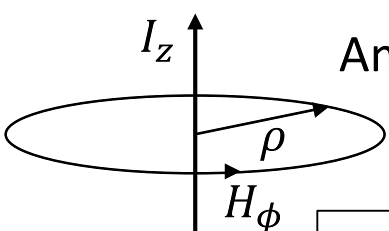
$$E_z = \frac{k^2}{j\omega\varepsilon}\psi_A$$

$$H_\phi = -\frac{\partial \psi_A}{\partial \rho}$$

$$H_\rho = -\frac{1}{\rho}\frac{\partial \psi_A}{\partial \phi}$$

$$H_1^{(2)}(k\rho) = J_1(k\rho) - jN_1(k\rho) \cong \frac{k\rho}{2} - j\left(-\frac{2}{\pi k\rho}\right) \cong \frac{2j}{\pi k\rho}$$

$(k\rho \ll 1)$



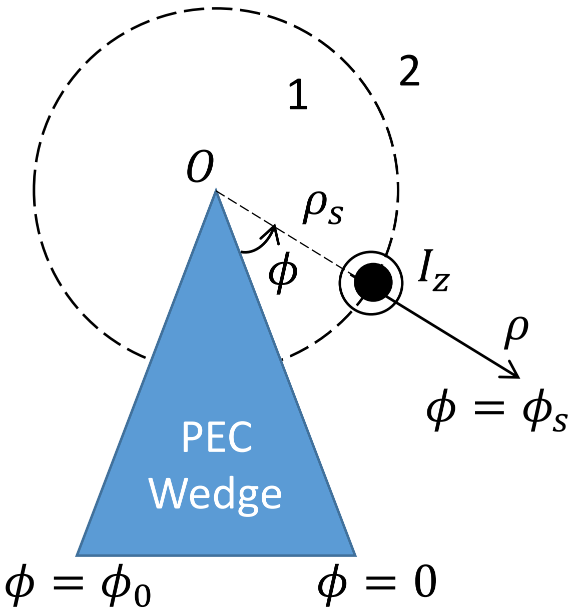
Ampere's Law:

$$I_z = H_\phi \cdot 2\pi\rho \Rightarrow A_0 = -\frac{j}{4} I_z \Rightarrow \psi_A = -\frac{j}{4} I_z H_0^{(2)}(k\rho)$$

$$\Rightarrow \quad E_z = -\frac{\mu\omega}{4} I_z H_0^{(2)}(k\rho) \xrightarrow[(\rho \to \infty)]{} -\frac{\mu\omega}{4} I_z \sqrt{\frac{2}{\pi k\rho}}\, e^{-j\left(k\rho - \frac{\pi}{4}\right)} \equiv E_0$$

(Plane wave from the infinite)

Electric Line Current Scattering by a Conductor Wedge



$$\Phi(\phi) = A_\phi \cos\alpha\phi + B_\phi \sin\alpha\phi$$

Boundary condition: $\Phi(0) = \Phi(\phi_0) = 0$

$$\Rightarrow A_\phi = 0, \sin\alpha\phi_0 = 0 \Rightarrow \alpha = \frac{n\pi}{\phi_0} \equiv U_n \notin \mathbb{Z}$$

$$\psi_1 = \sum_{all\ n} A_n \boxed{J_{U_n}(k\rho)} \sin U_n\phi$$

(Edge condition)
(pp. 14)

$$\psi_2 = \sum_{all\ n} B_n \boxed{H_{U_n}^{(2)}(k\rho)} \sin U_n\phi$$

(Radiation condition)
(pp. 15)

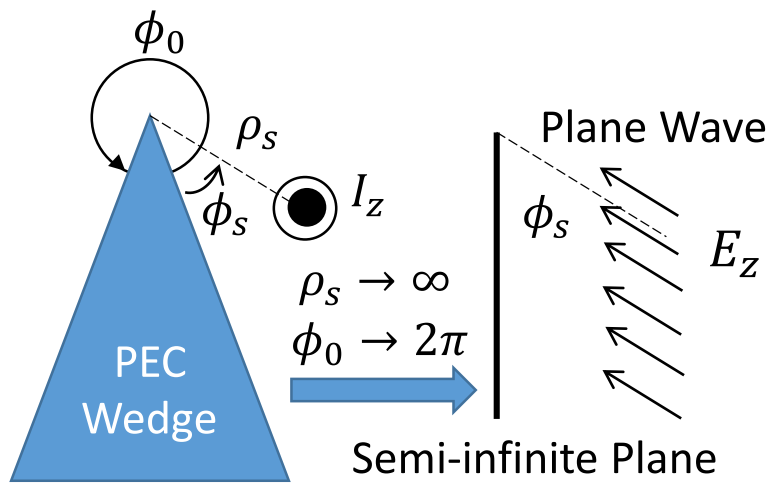
$E_{z1} = E_{z2},\ H_{\phi1} - H_{\phi2} = I_z\delta(\phi - \phi_s)$ at (ρ_s, ϕ_s) (Boundary condition at source)

$$J_{U_n}(x)H_{U_n}^{(2)\prime}(x) - J'_{U_n}(x)H_{U_n}^{(2)}(x) = -\frac{2j}{\pi x}$$

(Mathematics formula)

$$\Rightarrow \begin{aligned} E_{z1} &= -\sum_{all\ n} I_z \frac{\omega\mu\pi}{\phi_0} J_{U_n}(k\rho) H_{U_n}^{(2)}(k\rho_s) \sin U_n\phi \sin U_n\phi_s \\ E_{z2} &= -\sum_{all\ n} I_z \frac{\omega\mu\pi}{\phi_0} J_{U_n}(k\rho_s) H_{U_n}^{(2)}(k\rho) \sin U_n\phi_s \sin U_n\phi \end{aligned}$$

Diffraction by Semi-infinite Plane



$$U_n \equiv \frac{n\pi}{\phi_0} = \frac{n\pi}{2\pi} = \frac{n}{2}$$

$2E_0 e^{\frac{jn\pi}{4}}$ (pp. 16)

$$E_z = \sum_{all\ n} J_{\frac{n}{2}}(k\rho)\left(\frac{-\omega\mu\pi I_z}{2\pi}\right)H_{\frac{n}{2}}^{(2)}(k\rho_s)\sin\frac{n\phi}{2}\sin\frac{n\phi_s}{2}$$

$$\frac{1}{2}\left\{\cos\frac{n(\phi-\phi_s)}{2} - \cos\frac{n(\phi+\phi_s)}{2}\right\}$$

$$E_z \cong E_0\left\{\sqrt{\frac{j}{\pi}}e^{j\xi^i}e^{jk\rho}\int_{-\infty}^{\sqrt{\xi^i}} e^{-jt^2}dt - \sqrt{\frac{j}{\pi}}e^{j\xi^r}e^{jk\rho}\int_{-\infty}^{\sqrt{\xi^r}} e^{-jt^2}dt\right\} \equiv E_i - E_r$$

Incident (GO + Diffraction) Reflection (GO + Diffraction)

$R = -1$

(PEC)

Where,

$$\xi^i \equiv 2k\rho\cos^2\frac{(\phi-\phi_s)}{2}$$

$$\xi^r \equiv 2k\rho\cos^2\frac{(\phi+\phi_s)}{2}$$

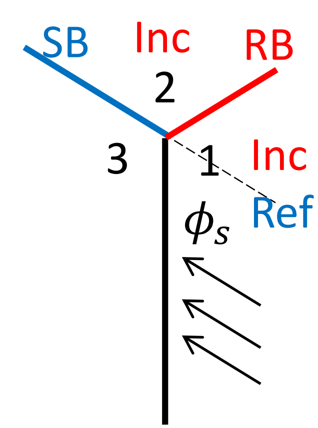
+: GO+Diff
−: Only Diff

(Next Page)

$\phi = 0$ $\phi = \pi - \phi_s$ $\phi = \pi + \phi_s$

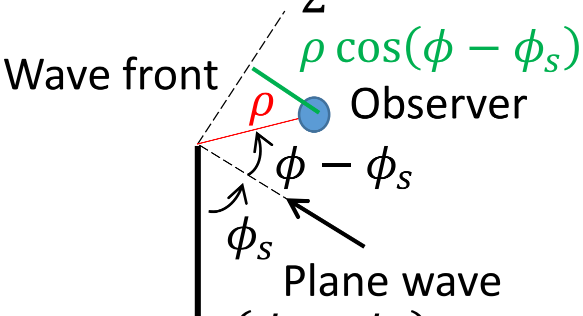
Region	1	2	3
$\cos\frac{(\phi-\phi_s)}{2}$	+	+	−
$\cos\frac{(\phi+\phi_s)}{2}$	+	−	−

Reflection Boundary Shadowing Boundary



Geometrical Theory of Diffraction (GTD)

If $\cos\frac{(\phi-\phi_s)}{2} > 0 \Rightarrow E_i = E_0\sqrt{\frac{j}{\pi}}e^{j\xi^i}e^{jk\rho}\left(\int_{-\infty}^{+\infty}e^{-jt^2}dt - \int_{\sqrt{\xi^i}}^{+\infty}e^{-jt^2}dt\right)$



$$= \underbrace{\boxed{E_0 e^{jk\rho\cos(\phi-\phi_s)}}}_{\text{Inc. GO (consider phase)}} + \underbrace{\boxed{E_0\frac{\sqrt{j}e^{-\frac{j\pi}{2}}}{2\sqrt{\pi}\sqrt{\xi^i}}e^{-jk\rho}\mathbb{F}(\xi^i)}}_{\text{Diffraction}}$$

(Fresnel integral)

If $\cos\frac{(\phi-\phi_s)}{2} < 0 \Rightarrow E_i = E_0\sqrt{\frac{j}{\pi}}e^{j\xi^i}e^{jk\rho}\int_{-\infty}^{-\sqrt{\xi^i}}e^{-jt^2}dt$ Only diffraction

Far field ($k\rho$>>1):

Inc. GO Ref. GO Diffraction

$$E_z \cong \begin{cases} E_i^{GO} - E_r^{GO} + E_0\frac{e^{-\frac{j\pi}{4}}e^{-jk\rho}}{2\sqrt{2k\pi\rho}}\left\{-\left(\cos\frac{(\phi-\phi_s)}{2}\right)^{-1} + \left(\cos\frac{(\phi+\phi_s)}{2}\right)^{-1}\right\} & \text{Region 1} \\ E_i^{GO} - 0 + E_0\frac{e^{-\frac{j\pi}{4}}e^{-jk\rho}}{2\sqrt{2k\pi\rho}}\left\{-\left(\cos\frac{(\phi-\phi_s)}{2}\right)^{-1} - \left(\cos\frac{(\phi+\phi_s)}{2}\right)^{-1}\right\} & \text{Region 2} \\ 0 - 0 + E_0\frac{e^{-\frac{j\pi}{4}}e^{-jk\rho}}{2\sqrt{2k\pi\rho}}\left\{+\left(\cos\frac{(\phi-\phi_s)}{2}\right)^{-1} + \left(\cos\frac{(\phi+\phi_s)}{2}\right)^{-1}\right\} & \text{Region 3} \end{cases}$$