The comparison of simulation results between Effective Roughness (ER) and Beckmann-Kirchhoff (B-K) models

Regular seminar

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Outline

- Background
- Review of B-K model
- B-K model simulation
- Comparison between ER and B-K model
- Conclusion
- Future plan

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Background

- To provide higher date rate and throughput, the next generation (5G) mobile communication systems will use higher frequency bands over 6 GHz.
- As radio wavelength is much smaller in high frequency bands, the diffuse scattering waves due to the roughness of building walls and complex structures of furniture affect the propagation characteristics more significantly.
- Traditional ways of treating diffuse scattering includes rigorous methods and asymptotic methods
  - rigorous methods: finite elements, method of moments and finite-difference time domain.

    → consume excessive time
  - asymptotic methods: physical optics, etc.

    → have difficulty in treating the compound scatters

Background

- One effective and simple model - "effective roughness" (ER) model
  - proposed for modeling the diffuse scattering from buildings, based on Lambertian scattering pattern [1].
  - extended to single-lobe and double-lobe directive scattering patterns in [2].
- Research plan
  - ER model simulation. Analyze the effect of different parameters on the diffuse scattering pattern.
  - Compare with B-K scattering models. Take into account the influence of correlation distance of the rough surface.
  - Measurement in anechoic chamber. By measuring the scattering of different rough surface samples, we are trying to identify the relation between ER parameters and roughness parameters.

[1] V. Degli-Esposti, A Diffuse Scattering Model for Urban Propagation Prediction, *IEEE Trans. Antennas Propag.*, vol. 49, no. 7, pp. 1111–1113, Jul. 2001.

[2] V. Degli-Esposti, F. Fuschini, M. Vitucci, E., & G. Falciasecca. (2007). Measurement and Modelling of Scattering from Buildings. *IEEE Transactions on Antennas and Propagation*, 55(1), 143-153.

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The Beckmann-Kirchhoff (B-K) model [3]

Starting from the Helmholtz integral, Beckmann and Spizzichino derived an analytical description of the electromagnetic field scattered from a rough surface in [3]. The scattering coefficient is defined as

$$\rho = \frac{E_2}{E_r},$$

where E_2 is the scattered electric field, and E_r is the field as it would be reflected in specular direction by a smooth, ideally conducting surface of the same dimensions as the rough one under the assumption of horizontal polarization. Then

$$E_r = \frac{jkA \cdot \cos(\Theta_1) \cdot e^{jkr_0}}{\pi r_0},$$

with $k = 2\pi/\lambda$ being the wave number, $A = l_x \cdot l_y$ being the rectangular surface area, $\Theta_1$being the incident angle and r_0 meaning the distance from the scattering point to the receiver.

[3] Beckmann, P., & Spizzichino, A. (1963). The scattering of electromagnetic waves from rough surfaces. *Norwood, MA, Artech House, Inc., 1963, 503 p.*

A closed solution can be found for statistically rough surfaces with a Gaussian height distribution. Such surfaces are characterized by their height standard deviation σ_h and the correlation length l_{corr}.

The average scattering coefficient of an incident wave on a rough surface of angle Θ_1, reflection and scattered angles Θ_2 and Θ_3, respectively, is determined by the following expressions

$$\mathrm{E}\{\rho\rho^*\}_\infty = \left(\rho_0^2 + \frac{\pi l_{corr}^2 F^2}{dS} \sum_{m=1}^{\infty} \frac{g^m}{m!\sqrt{m}} e^{-\frac{v_{xy}^2 l_{corr}^2}{4m}}\right) e^{-g}$$

where

Specular reflection

Diffusely scattered field

$$F = \frac{1+\cos(\Theta_1)\cos(\Theta_2)-\sin(\Theta_1)\sin(\Theta_2)\cos(\Theta_3)}{\cos(\Theta_1)(\cos(\Theta_1)+\cos(\Theta_2))},$$

$$g = k^2\sigma_h^2(\cos(\Theta_1)+\cos(\Theta_2))^2,$$

$$\rho_0 = \mathrm{sinc}(v_x l_x)\,\mathrm{sinc}(v_y l_y),$$

$$v_x = k(\sin(\Theta_1)-\sin(\Theta_2)\cos(\Theta_3)),$$

$$v_y = k(-\sin(\Theta_2)\sin(\Theta_3)),$$

$$v_{xy} = \sqrt{v_x^2+v_y^2}.$$

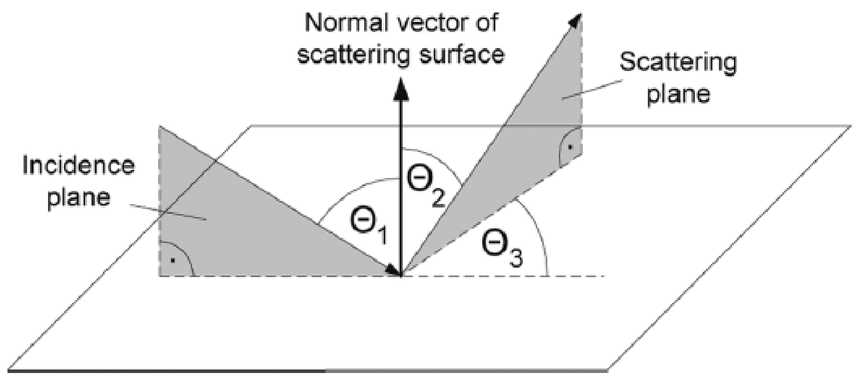

Fig 1. The basic geometry of scattering at a rough surface.

- With Γ being the respective conventional Fresnel reflection coefficient, the mean power scattered by finite conducting surfaces is

$$\mathrm{E}\{\rho\rho^*\}_{finite} = \mathrm{E}\{\Gamma\Gamma^*\} \cdot \mathrm{E}\{\rho\rho^*\}_{\infty}.$$

- Thus, the average power reflection coefficient of a surface area dS, specifying the scattered field in a distance r_2 from surface to the observation point *P* relative to the incident power leading to

$$\mathrm{E}\{R_{power}\} = \frac{4dS^2\cos^2(\Theta_1)}{\lambda^2 r_2^2}\mathrm{E}\{\rho\rho^*\}_{finite}.$$

- The Kirchhoff solution to the scattering problem assumes that there are no sharp edges or corners, which is fulfilled when the correlation length is larger than the wavelength ($l_{corr} > \lambda$). Furthermore, l_x and l_y have to be much larger than the correlation length ($\lambda < l_{corr} < l_x$).

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- In order to illustrate the angular dependent scattering behavior of a rough surface, the simulation result is shown in Fig 2. In the simulation, we set $\sigma_h = 0.13mm,\ l_{corr} = 2.3mm,\ l_x = l_y = 10 \cdot l_{corr},\ \theta_1 = 45°,\ \Gamma = 1,\ r_0 = 2m$ and $f = 300GHz$.
- At $\theta_1 = \theta_2 = 45°$, $\theta_3 = 0°$ the specular component can be seen.

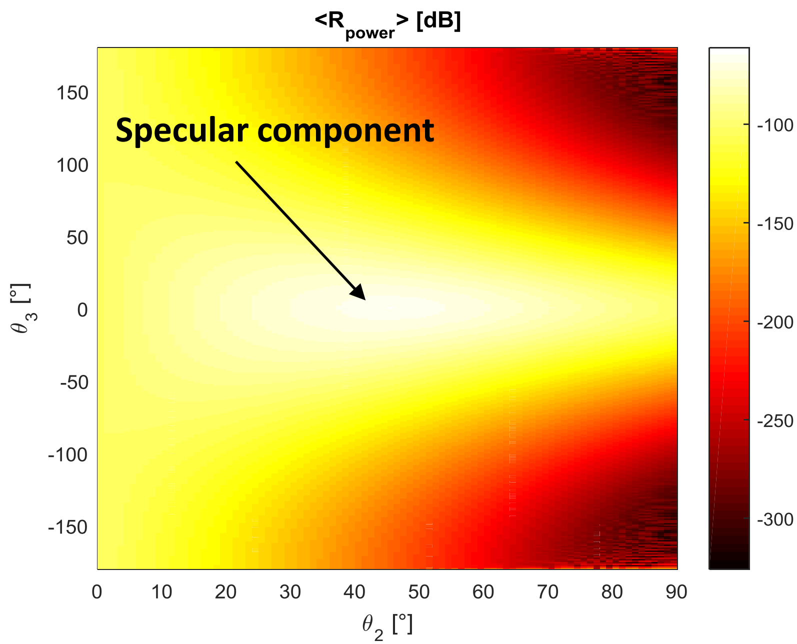


Fig 2. angular dependent power reflection factor.

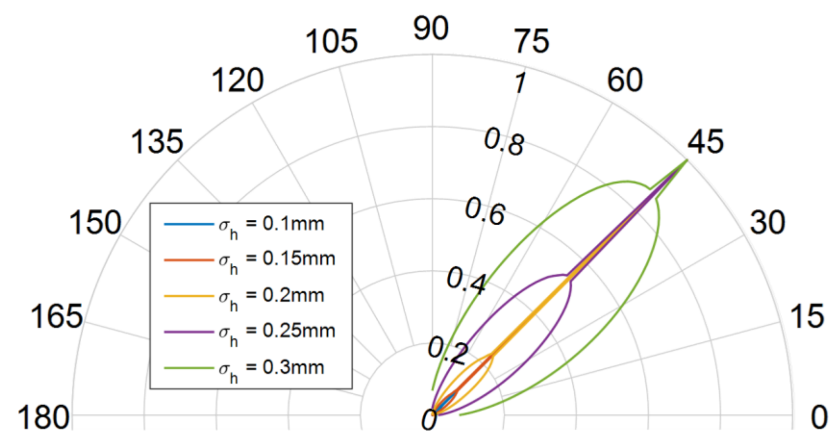
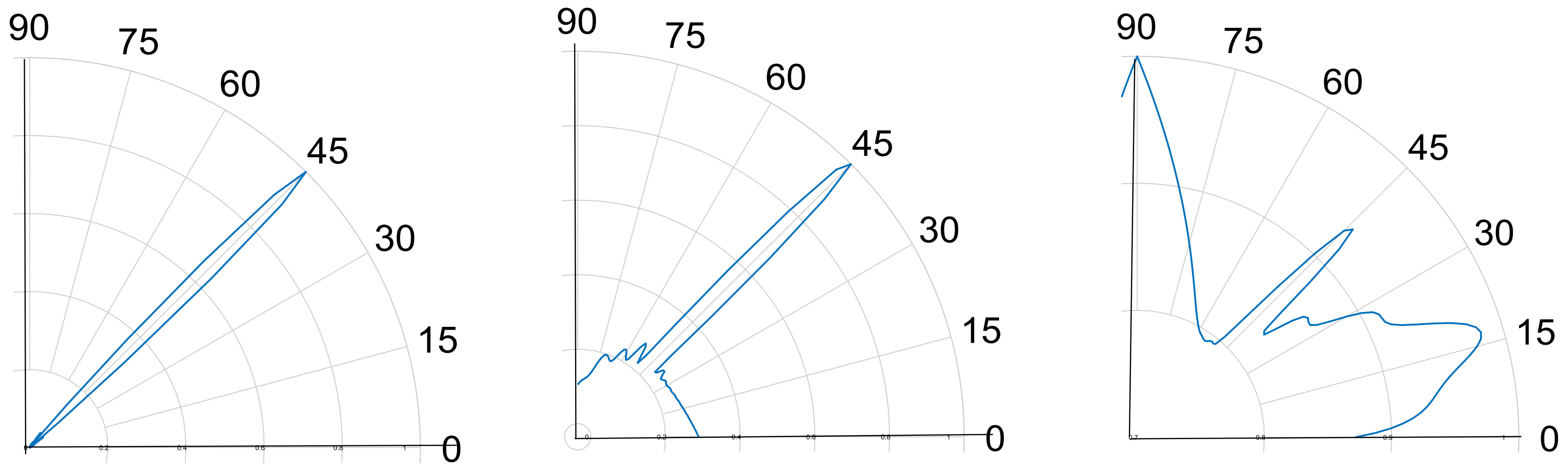


Fig 3. Simulated scattering coefficient (B-K model) at 300GHz

In the B-K model, the maximum scattering lobe is also steered to the direction of specular reflection. The higher the roughness σ_h, the wider the lobe.

The dependence of the scattered power on surface roughness σ_h

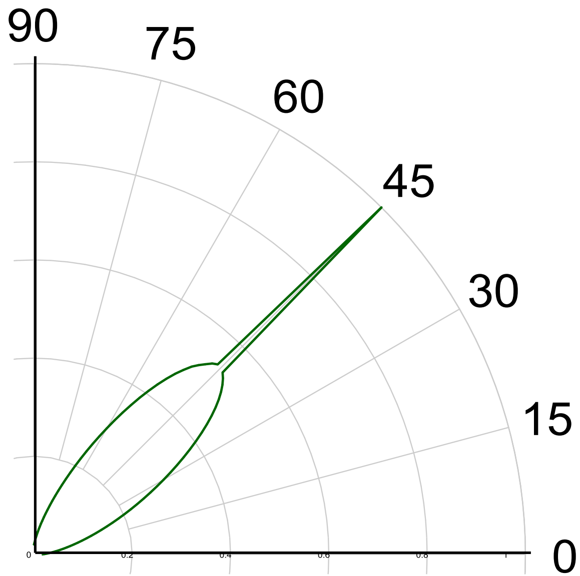
- When the surface is smooth, there is a single reflected wave in the specular direction.
- When the surface is slightly rough, the specular term is the dominating term.
- When the surface is very rough, the specular term has lost its privileged position.



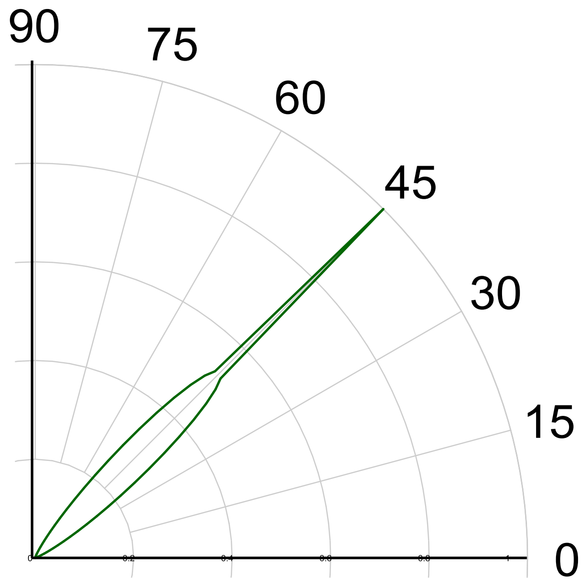
smooth slightly rough very rough

The dependence of the scattered power on the correlation distance l_{corr}.

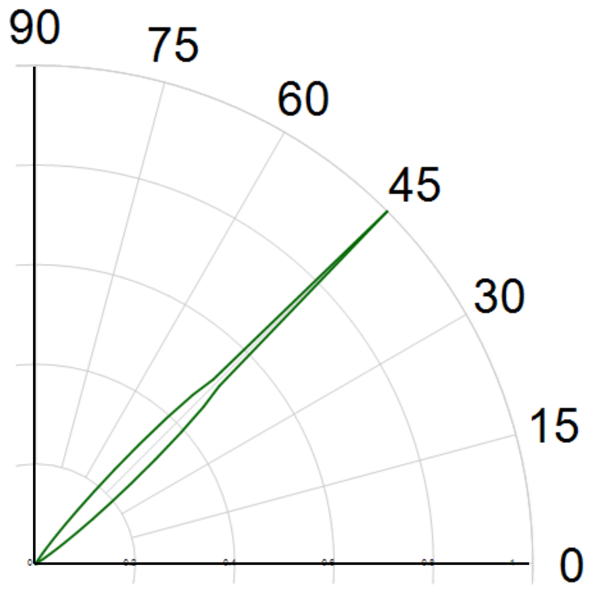
- Differentiating the scattering coefficient with respect to l_{corr}, and we find that for v_x = 0 (specular direction), the result is positive, which means as l_{corr} is increased, the specular direction will receive more power at the expense of other directions.



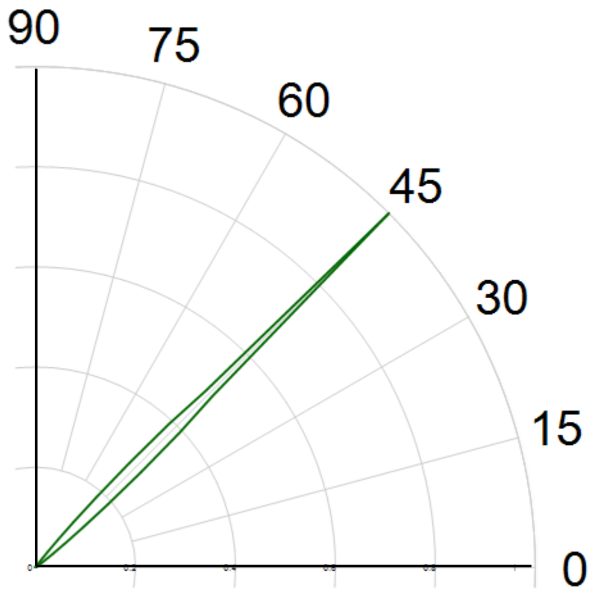
$l_{corr} = 2.3\ mm$



$l_{corr} = 5\ mm$



$l_{corr} = 8\ mm$



$l_{corr} = 12\ mm$

- In the simulation, we set l_{corr}=2.3, 5, 8, 12mm, l_x=l_y=10·l_{corr}, θ_1=45°, $f_c = 300GHz$. From the figures, it can be seen that when the correlation length l_{corr} is increased, the specular direction will receive more power.

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1. Comparison under a single surface element

- In the first step, we focus on a single surface element which has the size of $A = 23mm \times 23mm$. As the ER model only consider the influence of diffuse scattering, we eliminate the impact of specular direction in B-K model.

B-K model

$$\mathrm{E}\{\rho\rho^*\}_\infty = \left(\frac{\pi l_{corr}^2 F^2}{dS}\sum_{m=1}^{\infty}\frac{g^m}{m!\sqrt{m}}e^{-\frac{v_{xy}^2 l_{corr}^2}{4m}}\right)e^{-g},$$

$$\mathrm{E}\{\rho\rho^*\}_{finite} = \mathrm{E}\{\Gamma\Gamma^*\}\cdot\mathrm{E}\{\rho\rho^*\}_\infty,$$

$$\mathrm{E}\{R_{power}\} = \frac{4dS^2\cos^2(\Theta_1)}{\lambda^2 r_2^2}\mathrm{E}\{\rho\rho^*\}_{finite}.$$

ER model

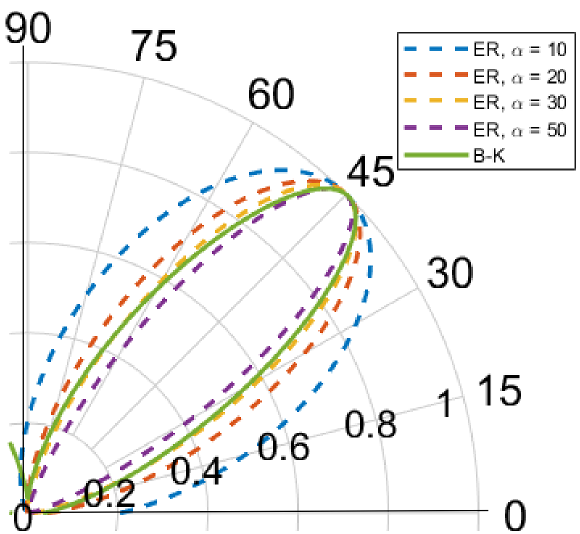
$$\left|E_S^{Dir}\right|^2 = \left(\frac{KS}{|\boldsymbol{r}_i||\boldsymbol{r}_s|}\right)^2 \frac{\mathrm{dScos}\,\theta_i}{F_\alpha}\left(\frac{1+\cos\Psi_R}{2}\right)^\alpha,$$

$$F_\alpha = \int_0^{\pi/2}\int_0^{2\pi}\left(\frac{1+\cos\Psi_R}{2}\right)^\alpha \sin\theta_s d\theta_s d\varphi_s,$$

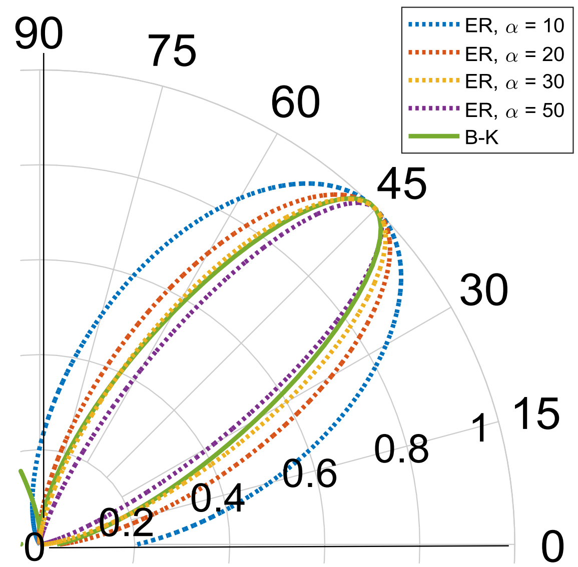
$$S = \sqrt{1-\rho^2}|R|,$$

$$\rho = \exp\{-\frac{1}{2}\left(\frac{4\pi\sigma_h\cos\theta_i}{\lambda}\right)\}.$$

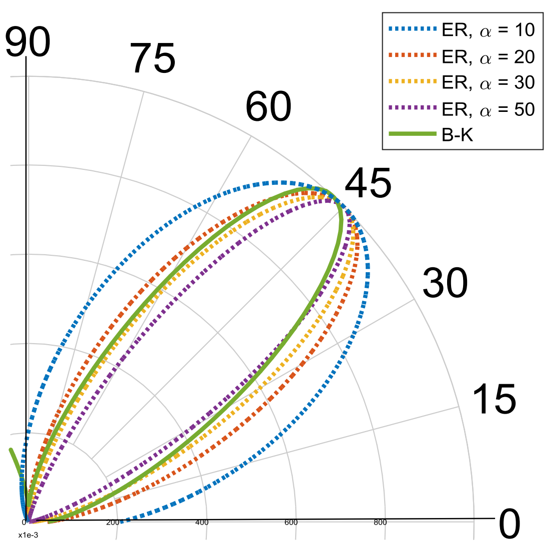
Compare the ER model with B-K model on different frequencies



$f_c = 300GHz$



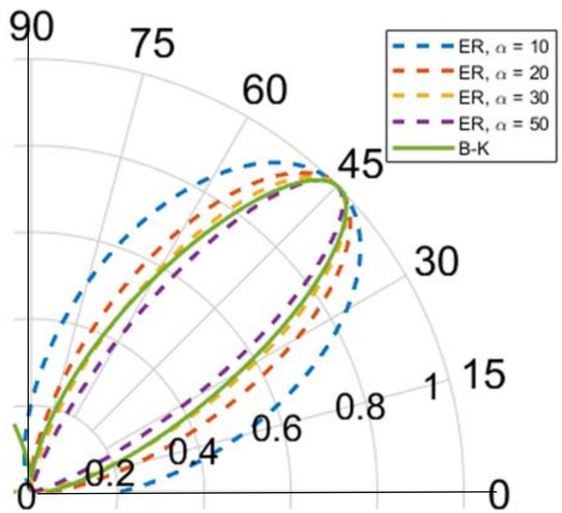
$f_c = 250GHz$



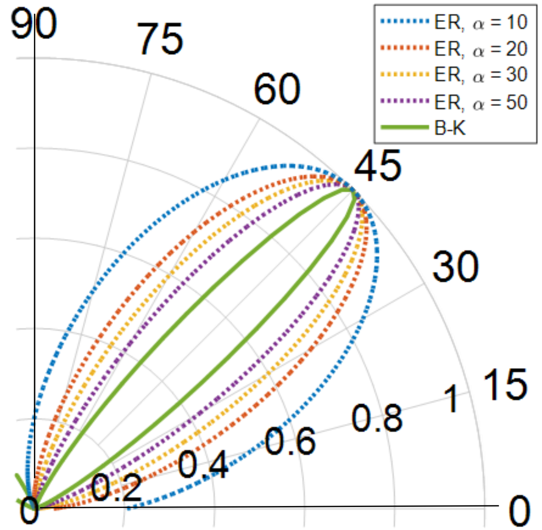
$f_c = 200GHz$

From the result, we can see that when $\alpha = 30$, the scattering pattern of ER model is mostly close to the B-K model.

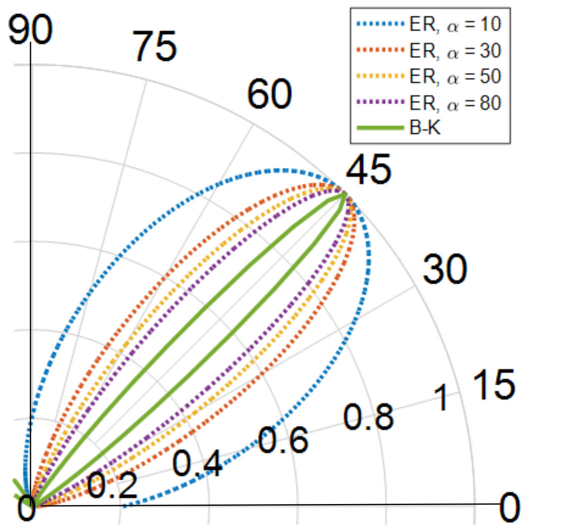
Compare the ER model with B-K model on different correlation distance



$l_{corr} = 2.3\ mm$



$l_{corr} = 5\ mm$

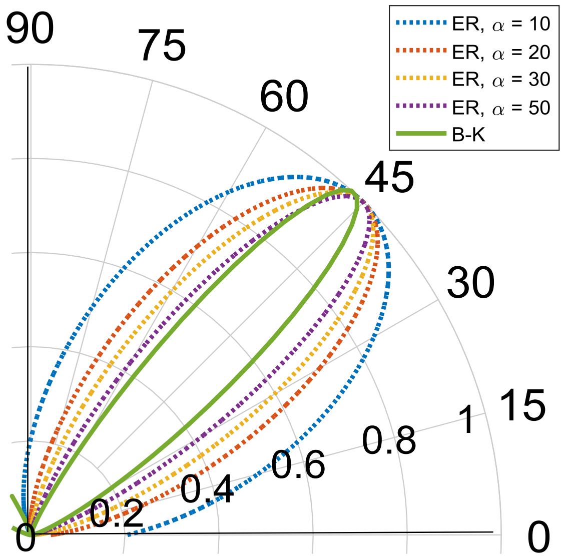


$l_{corr} = 8\ mm$

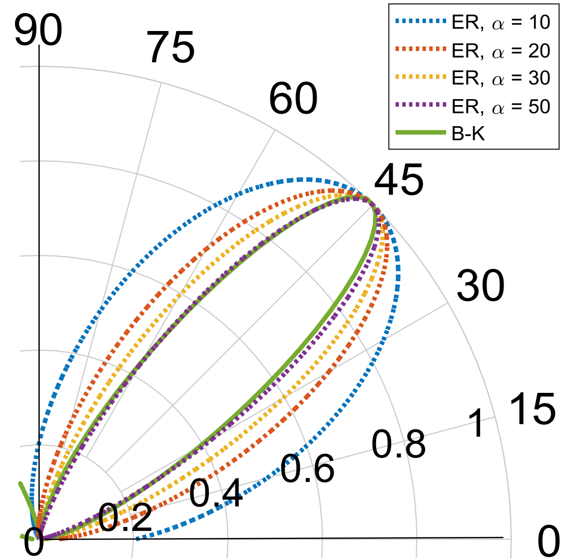
From the result, we can see that when the correlation distance increases, the scattering lobe of B-K model gets narrower.

Therefore, the value of α which makes the ER model mostly close to the B-K model needs to be lager.

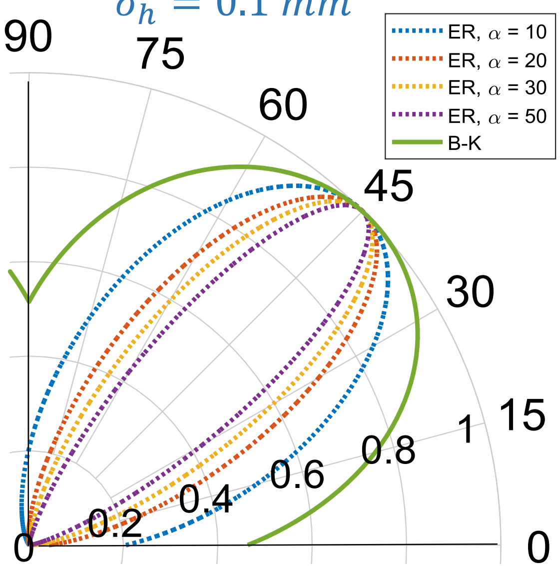
Compare the ER model with B-K model on different **roughness**



$\sigma_h = 0.1\ mm$



$\sigma_h = 0.2\ mm$



$\sigma_h = 0.5\ mm$

From the result, we can see that when the surface gets rougher, the value of $\boldsymbol{\alpha}$ which makes the ER model mostly close to the B-K model should be smaller.

2. Comparison under the rough surface

- Same as the ER model simulation, the procedure is as follows

1. Generate a Gaussian rough surface with correlation length

2. Calculate the scattered electric field from each surface element utilizing ER and B-K model

3. Sum up the contributions from each surface element to obtain the scattering pattern

Detailed simulation process

1 Generate a Gaussian rough surface with correlation length

Step 1.1: generate a Gaussian surface with zero mean and standard deviation σ_h. The rough surfaces with Gaussian statistics can be generated using a method outlined by Garcia and Stoll [4], where the uncorrelated distribution of surface points is convolved with a Gaussian filter to achieve correlation.

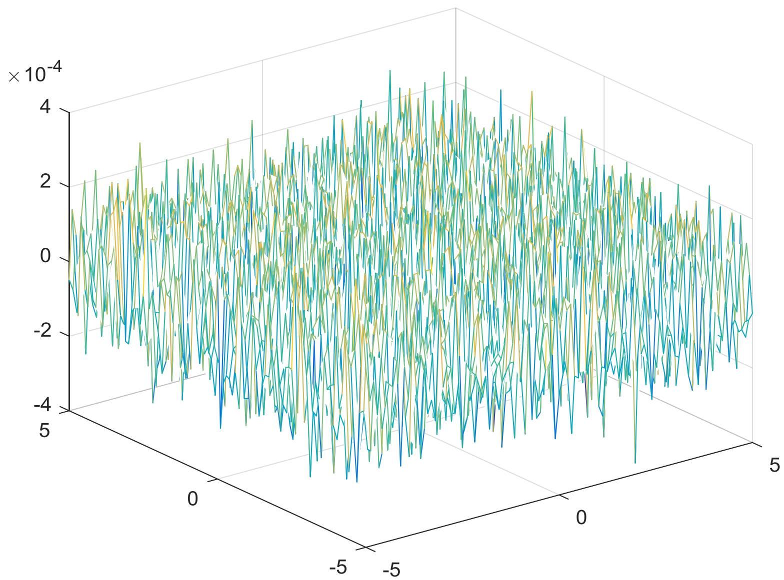


Fig 4. The rough surface with $\mu = 0$, $\sigma_h =$ 0.5mm, l_corr=10mm.

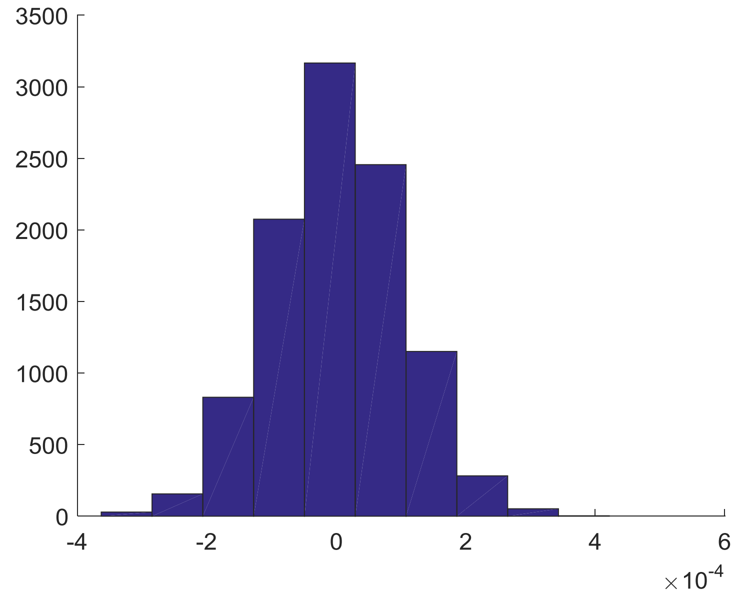
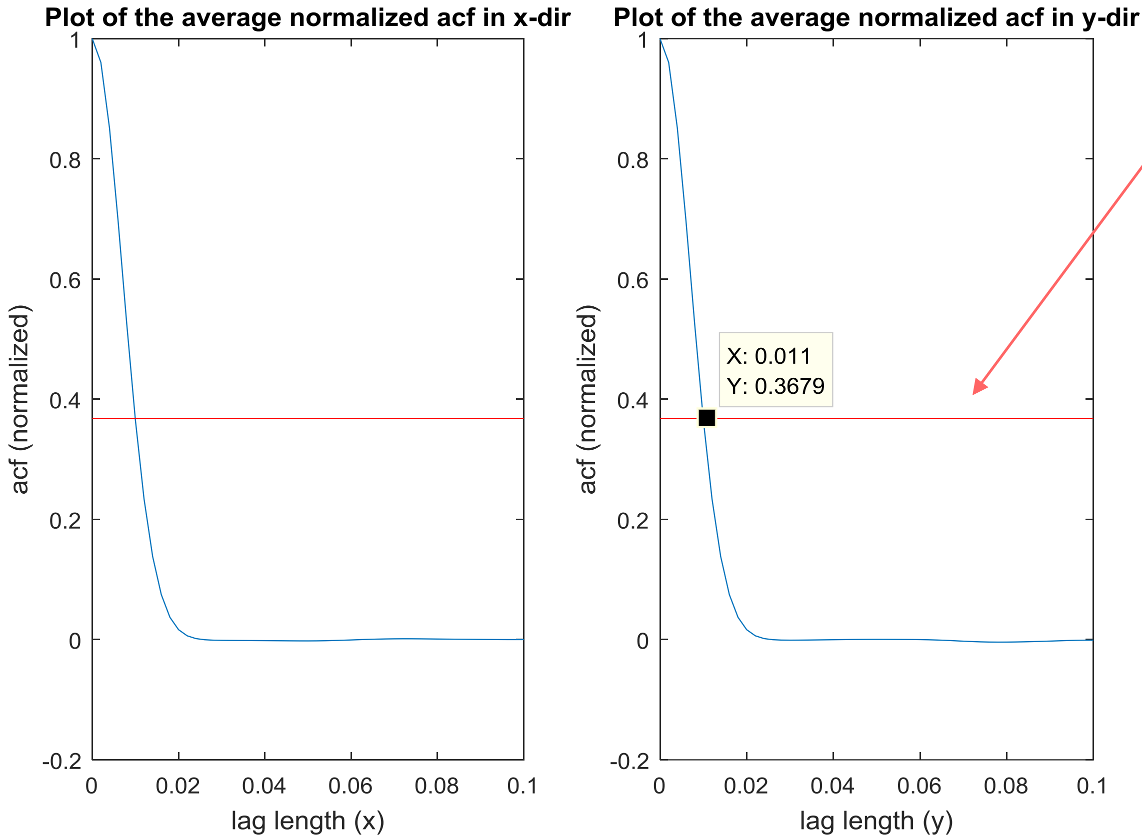


Fig 5. Histogram bar chart of the elements in the rough surface with $\mu = 0$, $\sigma_h = 0.5$mm.

[4] Garcia, N.; Stoll, E.: "Monte Carlo Calculation of Electromagnetic-Wave Scattering from Random Rough Surfaces", *Physical Review Letters*, Volume 52, Issue 20, pp. 1798-1801 (1984).



$$y = \frac{1}{e} \approx 0.3679$$

From the figure, the correlation length is around 10mm.

Fig 6. MATLAB plots of the normalized autocorrelation function

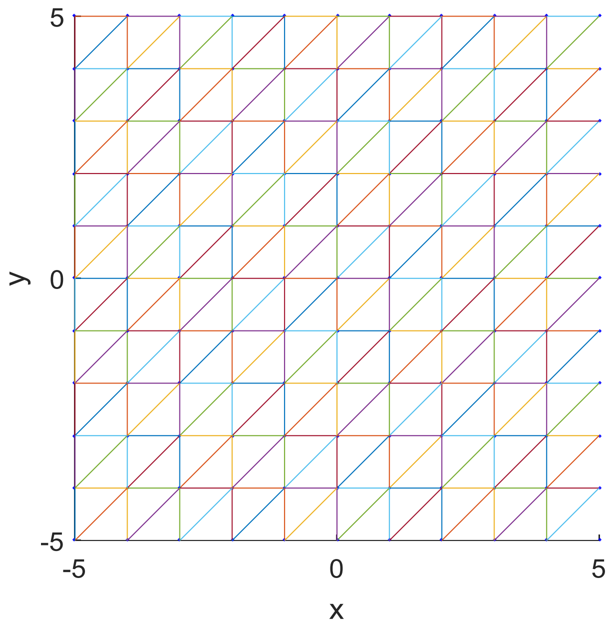


Fig 7. Top view of the rough surface

Step 1.2: In order to apply the surface data to the ER and B-K model, the surface is divided into multiple triangular surface elements.

Step 1.3: Find the gravity and normal vector of each surface element.

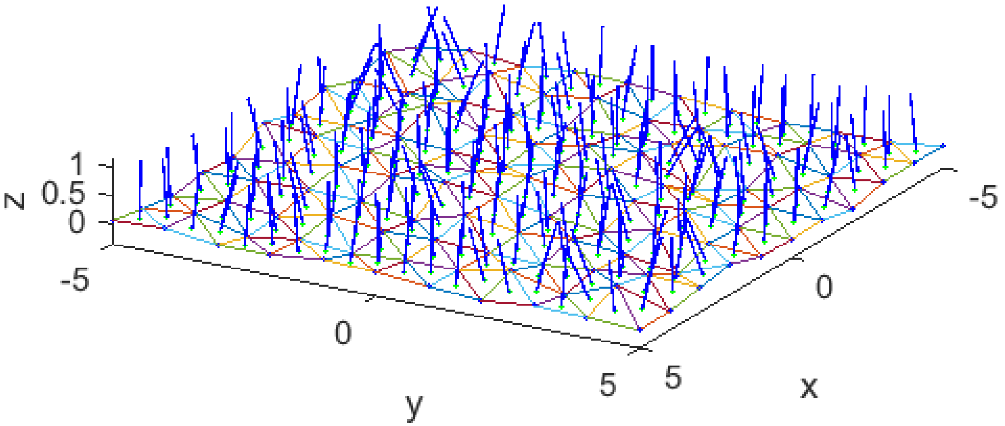
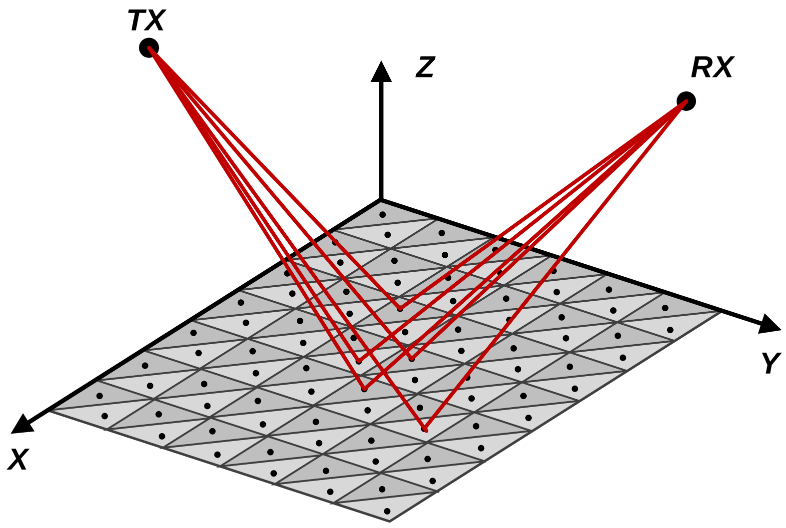


Fig 8. The rough surface with triangular elements and normal vector

2. Calculate the scattered electric field from rough surface utilizing ER and B-K model

Step 2.1: Set TX position. For RX with θ_{RX}, find the ER model parameters ($\boldsymbol{r}_i$, $\boldsymbol{r}_s$, θ_i, etc) for directive model from each surface element. Then calculate the scattered electric field from this triangular element.

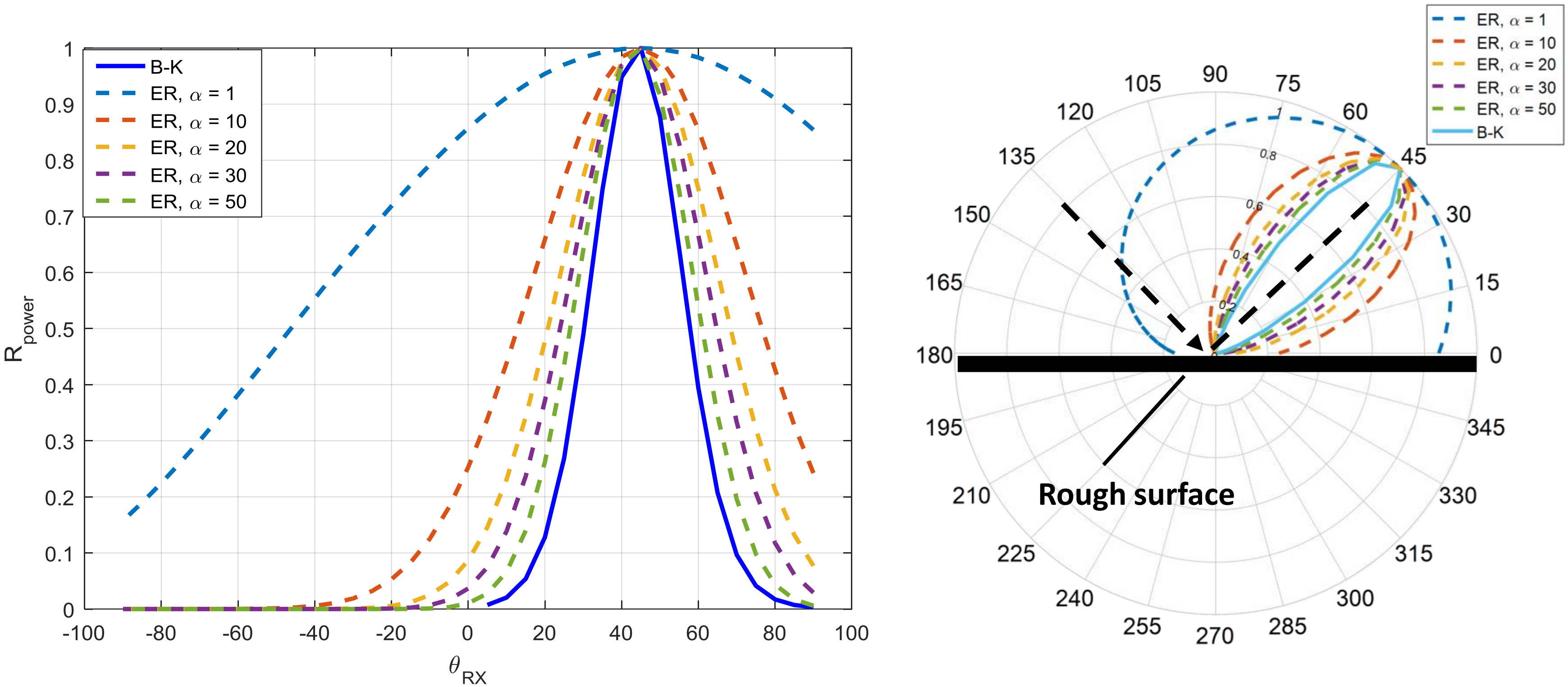
Step 2.2: Sum up the contributions of all the rays to obtain the total electric field.



Step 2.3: calculate the electric field for different RX position (θ_{RX}) to get the scattering pattern.

Fig 9. Surface elements of scattering object.

The simulation result



When $\alpha = 50$, the scattering pattern of ER model is mostly close to the B-K model.

Fig 10. Scattering pattern using ER and B-K model with $f_c = 100$GHz, σ_h=0.5mm, l_corr=10mm, $\alpha = 1, 10, 20, 30, 50$.

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Conclusion

- Review the B-K model.
- Simulate the B-K model, and analyze the impact of different parameters.
- Compare the simulation results between ER model and B-K model.

Future plan

- Further compare the two scattering models and analyze the simulation results.
- Prepare for the measurement in the anechoic chamber.

Thank you for your attention!