Quasi Monte Carlo Kalman filter for Nonlinear and Non-Gaussian State Space Models*

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Abstract

In this study, we present a new filtering approach for nonlinear and non-Gaussian state space models. This approach builds on the well-established Kalman filter, combined with approximations of least square linearization for nonlinear function and Gaussian mixture for non-Gaussian noises, and applies the quasi Monte Carlo method for numerical integration during computation. We compare our approach with other existing methods, such as Particle filter, using simulated data, and we show the proposed approach can outperform these methods in terms of speed and accuracy. This study also provides analysis on the stability of this new filtering approach. In addition, we propose two methods to estimate the unknown parameters in the model, and show the consistency of the proposed quasi-maximum likelihood estimator under general conditions. To illustrate the proposed approach, we discuss several numerical examples. We also introduce two applications of our approach. The first one is a popular stochastic volatility model, and we apply it to foreign exchange data between Sterling and Dollar. In the second application, we discuss a jump model, and show the jump size has a Gaussian mixture representation. We use the 3-month T-bill data to estimate the jump probability and investigate the jump sources based on macroeconomic events.

Keywords: Quasi Monte Carlo Kalman filter, nonlinear and non-Gaussian state space models, stability

JEL codes: C11, C32, C41

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