Quasi Monte Carlo Kalman filter for Nonlinear and Non-Gaussian State Space Models*

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Abstract

In this study, we present a new filtering approach for nonlinear and non-Gaussian state space models. This approach builds on the well-established Kalman filter, featuring a state-dependent least-square linearization of the nonlinear function and a Gaussian-mixture approximation to the error distribution, and it applies the quasi Monte Carlo method for numerical integration during the computation. We compare our approach with other existing methods using simulated data, and we find that the proposed approach can outperform these methods in terms of speed and accuracy. This study also provides analysis on the stability of this new filtering approach. In addition, we propose two methods to estimate the unknown parameters, and establish the consistency of the proposed quasi-maximum likelihood estimator under general conditions. To illustrate the proposed approach, we discuss several numerical examples. We also consider two empirical applications. The first is a stochastic volatility model for foreign exchange data between Sterling and Dollar. The second is a jump model for the 3-month T-bill rate data, where we show that the jump size has a Gaussian mixture representation. We estimate the jump probability and investigate the jump sources based on macroeconomic events.

Keywords: Quasi Monte Carlo Kalman filter, nonlinear and non-Gaussian state space models, stability

JEL codes: C11, C32, C41

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