Hermite Polynomial-based Valuation of American Options with General Jump-Diffusion Process*

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Abstract

We present a new approximation scheme for both of the price and exercise policy of American options. The scheme is based on Hermite polynomials expansion of the transition density of the underlying asset dynamics and the early exercise premium (EEP) representation of the American option price. The advantage of the proposed approach is threefold. First, our approach does not require the transition density and characteristic functions of the underlying asset dynamics to be attainable in closed form. Second, our approach is shown to be fast and accurate, while the prices and exercise policy could be jointly produced. Third, our approach has a wide range of application scopes, and can be easily extended to higher dimensional cases and jump-diffusion models. We show that the proposed approximations of the price and optimal exercise boundary will be convergent to the true ones. We also provide a numerical method based on a step function for the implementation of our proposed approach. Examples such as nonlinear mean-reverting model, double mean-reverting model, Merton's and Kou's jump-diffusion models are presented and discussed.

Keywords: Hermite polynomials, American option, early exercise premium, jump-diffusion process

JEL codes: C22, C41, G12, G13.

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