Physics 247 HW 10

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Problem 1

Let $u = 0.6c$ be the particle's velocity relative to the lab and $E_{rest} = 0.511\, MeV$ - the particle's rest energy.

Part a

$$\gamma = \frac{1}{\sqrt{1 - \frac{u}{c}^2}} = \frac{1}{\sqrt{1 - 0.6^2}} = \frac{1}{0.8} = 1.25.$$

Part b

$$p = \gamma m u = \gamma u \frac{E_{rest}}{c^2} = 1.25 \times 0.6 \times 0.511 \frac{MeV}{c} = 0.383\, \frac{MeV}{c}.$$

Part c

Using the invariant quantity:

$$E^2 = (pc)^2 + (mc^2)^2 = 0.383^2 + 0.511^2 = 0.40781\, MeV^2;$$
$$E = 0.6386\, MeV.$$

Part d

$$E_k = E - E_{rest} = 0.6386 - 0.511 = 0.1276\, MeV.$$

Problem 2

Let $m_0 = 1\, kg$ be the rest mass of the iron slugs, $(u_1, u_2, u_3) = (0.9c, 0.99c, 0.999c)$ - the initial velocities of the slugs, and $M = 45 \times 10^6\, kg$ - the mass of the battleship.

Part a

Let $E_{rest} = m_0c^2$ be the rest mass of the slugs, E - the relativistic energy of the slugs, and E_k - the kinetic energy of the slugs. Let β denote $\frac{u}{c}$. Using the invariant quantity:

$$E^2 = (pc)^2 + E_{rest}^2 = (\gamma m_0 uc)^2 + E_{rest}^2 = E_{rest}^2(\frac{\gamma^2 u^2}{c^2} + 1) = E_{rest}^2(\frac{\beta^2}{1-\beta^2} + 1);$$

$$E = E_{rest}\sqrt{\frac{\beta^2}{1-\beta^2} + 1};$$

$$E_k = E - E_{rest};$$

$$E_{rest} = 1 \times (3 \times 10^8)^2 = 9 \times 10^{16}\,J\ or\ 5.61798 \times 10^{26}\,GeV.$$

For $u_1 = 0.9c$ ($\beta^2 = 0.81$):

$$E = 5.61798\sqrt{\frac{0.81}{1-0.81} + 1} \times 10^{26} = 12.88853 \times 10^{26}\,GeV;$$

$$or\ 12.88853 \times 10^{26} \times 1.6022 \times 10^{-10} = 20.65 \times 10^{16}\,J.$$

$$E_k = 12.88853 \times 10^{26} - 5.61798 \times 10^{26} = 7.27055 \times 10^{26}\,GeV;$$

$$or\ 7.27055 \times 10^{26} \times 1.6022 \times 10^{-10} = 11.64888 \times 10^{16}\,J.$$

For $u_2 = 0.99c$ ($\beta^2 = 0.9801$):

$$E = 5.61798\sqrt{\frac{0.9801}{1-0.9801} + 1} \times 10^{26} = 39.8248 \times 10^{26}\,GeV;$$

$$or\ 39.8248 \times 10^{26} \times 1.6022 \times 10^{-10} = 63.807 \times 10^{16}\,J.$$

$$E_k = 39.8248 \times 10^{26} - 5.61798 \times 10^{26} = 34.20682 \times 10^{26}\,GeV;$$

$$or\ 34.20682 \times 10^{26} \times 1.6022 \times 10^{-10} = 54.806167 \times 10^{16}\,J.$$

For $u_3 = 0.999c$ ($\beta^2 = 0.998$):

$$E = 5.61798\sqrt{\frac{0.998}{1-0.998} + 1} \times 10^{26} = 125.62185 \times 10^{26}\,GeV;$$

$$or\ 125.62185 \times 10^{26} \times 1.6022 \times 10^{-10} = 201.27 \times 10^{16}\,J.$$

$$E_k = 125.62185 \times 10^{26} - 5.61798 \times 10^{26} = 120.0 \times 10^{26}\,GeV;$$

$$or\ 120.0 \times 10^{26} \times 1.6022 \times 10^{-10} = 192.27 \times 10^{16}\,J.$$

Part b

Let v be the final velocity of the battleship. Using the conservation of momentum:

$$p_{initial} = p_{final};$$
$$0 = \gamma m_0 u + Mv;$$
$$v = -\frac{\gamma m_0 u}{M}.$$

For $u_1 = 0.9c$ ($\gamma = 2.2942$):

$$v \approx -13.765\, m/s.$$

For $u_2 = 0.99c$ ($\gamma = 7.0889$):

$$v \approx -46.7867\, m/s.$$

For $u_3 = 0.999c$ ($\gamma = 22.3663$):

$$v \approx -148.9596\, m/s.$$

Part c

Let $m_S = 50\, kg$ be the mass of the Sith warrior and v - her resulting velocity. According to the conservation of momentum:

$$v = \frac{\gamma m_0 u}{m_S}.$$

For $u = 0.9c$ ($\gamma = 2.2942$):

$$v \approx 1.2388 \times 10^7\, m/s.$$

Therefore, if the warrior were to start trying to stop the slugs, she would start rapidly accelerating to $\approx 0.04c$, which, as one could imagine, would not be comfortable for a human. Consequently, we can make a conclusion that this would not be a good idea.

Problem 3

Let u be the desired speed of the particle. The following amount of kinetic energy will be required to accelerate the particle to u:

$$E_k = mc^2(\gamma - 1).$$

Part a

For $u = 0.5c$, $\gamma \approx 1.1547$:

$$E_k \approx 0.1547\, mc^2.$$

Part b

For $u = 0.9c$, $\gamma \approx 2.29416$:

$$E_k \approx 1.29416\, mc^2.$$

Part c

For $u = 0.99c$, $\gamma \approx 7.08881$:

$$E_k \approx 6.08881\, mc^2.$$

Problem 4

Let $u = 2.5 \times 10^5\; m/s$ be the Sun's velocity, and m_0 - its rest mass.

Part a

$$\frac{p_{relativistic}}{p_{Newtonian}} = \frac{\gamma m_0 u}{m_0 u} = \gamma = \frac{1}{\sqrt{1 - (\frac{2.5\times 10^5}{3\times 10^8})^2}} = 1.00000035.$$

Part b

$$\frac{KE_{relativistic}}{KE_{Newtonian}} = \frac{m_0 c^2(\gamma - 1)}{\frac{1}{2} m_0 u^2} = \frac{2c^2}{u^2}(\gamma - 1) = 1.008.$$

Problem 5

Let m_e, m_μ be the masses of the particles, and u and v - the velocities of the electron and the muon respectively in the lab frame. Let $c = 1$. Since the kinetic energy of the electron is $5m_e c^2$, we can find γ_u (the Lorentz factor for the electron's velocity):

$$5m_e = m_e \gamma_u - m_e \Rightarrow \gamma_u = 6.$$

We can find u from γ_u:

$$\gamma_u = \frac{1}{\sqrt{1 - u^2}};$$

$$u = \sqrt{1 - \frac{1}{\gamma_u^2}} \approx 0.986013.$$

Next, according to the problem statement:

$$KE_e = KE_\mu;$$

$$m_e(\gamma_u - 1) = m_\mu(\gamma_v - 1);$$

$$\gamma_v = \frac{m_e}{m_\mu}(\gamma_u - 1) + 1.$$

Similarly to u, we can find v from γ_v:

$$\gamma_v = \frac{1}{\sqrt{1 - v^2}};$$

$$u = \sqrt{1 - \frac{1}{\gamma_v^2}} \approx 0.216018422.$$

Let us transform u to the muon's frame of reference using the relativistic velocity addition formula:

$$u\prime = \frac{u - v}{1 - uv} = \frac{-0.986013 - 0.216018422}{1 - (0.216018422)(-0.986013)} \approx -0.99095995.$$

The x-component of the four-momentum vector of the electron in the muon's frame of reference would then be:

$$p_1\prime = \gamma_{u\prime} u\prime m_e \approx -3.77444 \, \frac{MeV}{c^2}.$$

The transformed p_0 can be found using momentum transformation:

$$p_0\prime = \gamma_v(p_0 - vp_1) = \gamma_v(\gamma_u m_e - v\gamma_u u m_e) = \gamma_v \gamma_u m_e (1 - uv);$$

$$p_0\prime \approx 3.80894 \, \frac{MeV}{c^2}.$$

Therefore, the four-momentum vector of the electron in the muon's rest frame can be expressed as the following:

$$P_e\prime = (3.80894, -3.77444, p_2, p_3) \, \frac{MeV}{c^2}.$$