Physics 247 HW 13

Name: Hleb Zhynko

Due: December 12, 2022

Problem 1

Let $\tau = I\alpha$ be the torque of the bar, α - the final instantaneous angular acceleration of the bar, and $I = \frac{1}{3}ML^2$ - the moment of inertia of the bar.

$$\tau = F \times r = Mg\sin 30^\circ \times r = Mg\sin 30^\circ \times \frac{L}{2};$$

$$\alpha = \frac{Mgr\sin 30^\circ}{\frac{1}{3}ML^2} = \frac{3}{2}\frac{1}{2}\frac{g}{L} = \frac{3}{4}\frac{g}{L}.$$

Problem 2

Let $H = L\cos 30^\circ$ be the change in height of the tip of the bar. Using conservation of energy:

$$MgH = \frac{1}{2}I\omega^2;$$

$$MgL\cos 30^\circ = \frac{1}{2}\frac{1}{3}ML^2\omega^2;$$

$$\omega^2 = \frac{1}{L}6g\frac{\sqrt{3}}{2};$$

$$\omega = \sqrt{\frac{3\sqrt{3}g}{L}};$$

$$v = \omega R = \omega L = \sqrt{3\sqrt{3}gL}.$$

Problem 3

Using conservation of energy for all three cases:

For the cube:

$$\frac{1}{2}Mv^2 = Mgh;$$

$$h = \frac{v^2}{2g}.$$

For the hoop and the disk:

$$\frac{1}{2} I \frac{v}{r}^2 + \frac{1}{2} M v^2 = Mgh;$$

$$\frac{v^2}{2} \left(\frac{I}{r^2} + M \right) = Mgh;$$

$$h = \frac{v^2}{2Mg} \left(\frac{I}{r^2} + M \right).$$

For the hoop:

$$\begin{cases} I = Mr^2, \\ h = \frac{v^2}{2Mg} \left(\frac{I}{r^2} + M \right); \end{cases}$$

$$h = \frac{v^2}{g}.$$

For the disk:

$$\begin{cases} I = \frac{1}{2} Mr^2, \\ h = \frac{v^2}{2Mg} \left(\frac{I}{r^2} + M \right); \end{cases}$$

$$h = \frac{3}{4} \frac{v^2}{g}.$$

Assuming the initial velocity v is the same across all three cases, the hoop will get to the highest point.

Problem 4

See Figure 1 for the diagram of the problem. Let τ_A be the torque at point A, and τ_{Cright} - the torque of the right half of the ladder at point C.

$$\sin\theta = \frac{1}{4}; \cos\theta = \frac{\sqrt{15}}{4};$$

$$\tau_A = -\frac{3}{4} mg + 2N_B = 0; \Rightarrow N_B = \frac{3}{8} mg;$$

$$\tau_{Cright} = -2F_T \cos\theta + 4N_B \sin\theta = 0; \Rightarrow N_B = 2\frac{\sqrt{15}}{4} F_T;$$

$$\frac{3}{8} mg = 2\frac{\sqrt{15}}{4} F_T;$$

$$F_T = \frac{3}{4} \frac{1}{\sqrt{15}} mg \approx 132.98\, N.$$

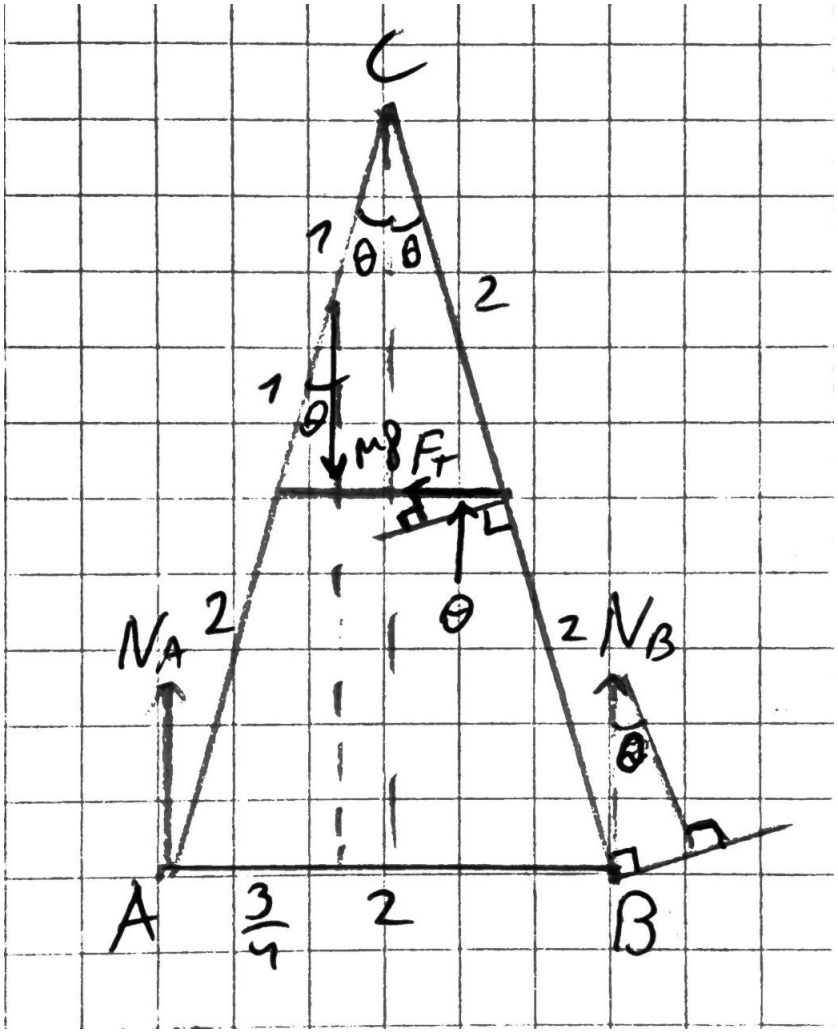

Figure 1: A diagram of the step ladder.