Physics 247 HW 10

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Due: November 18, 2022

Problem 1

Let $u = 0.6c$ be the particle's velocity relative to the lab and $E_{rest} = 0.511\, MeV$ - the particle's rest energy.

Part a

$$\gamma = \frac{1}{\sqrt{1 - \frac{u}{c}^2}} = \frac{1}{\sqrt{1 - 0.6^2}} = \frac{1}{0.8} = 1.25.$$

Part b

$$p = \gamma m u = \gamma u \frac{E_{rest}}{c^2} = 1.25 \times 0.6 \times 0.511 \frac{MeV}{c} = 0.383\, \frac{MeV}{c}.$$

Part c

Using the invariant quantity:

$$E^2 = (pc)^2 + (mc^2)^2 = 0.383^2 + 0.511^2 = 0.40781\, MeV^2;$$
$$E = 0.6386\, MeV.$$

Part d

$$E_k = E - E_{rest} = 0.6386 - 0.511 = 0.1276\, MeV.$$

Problem 2

Let $m_0 = 1\, kg$ be the rest mass of the iron slugs, $(u_1, u_2, u_3) = (0.9c, 0.99c, 0.999c)$ - the initial velocities of the slugs, and $M = 45 \times 10^6\, kg$ - the mass of the battleship.

Part a

Let $E_{rest} = m_0c^2$ be the rest mass of the slugs, E - the relativistic energy of the slugs, and E_k - the kinetic energy of the slugs. Let β denote $\frac{u}{c}$. Using the invariant quantity:

$$E^2 = (pc)^2 + E_{rest}^2 = (\gamma m_0 uc)^2 + E_{rest}^2 = E_{rest}^2(\frac{\gamma^2 u^2}{c^2} + 1) = E_{rest}^2(\frac{\beta^2}{1-\beta^2} + 1);$$

$$E = E_{rest}\sqrt{\frac{\beta^2}{1-\beta^2} + 1};$$

$$E_k = E - E_{rest};$$

$$E_{rest} = 1 \times (3 \times 10^8)^2 = 9 \times 10^{16}\, J \; or \; 5.61798 \times 10^{26}\, GeV.$$

For $u_1 = 0.9c$ ($\beta^2 = 0.81$):

$$E = 5.61798\sqrt{\frac{0.81}{1-0.81} + 1} \times 10^{26} = 12.88853 \times 10^{26}\, GeV;$$

$$or \; 12.88853 \times 10^{26} \times 1.6022 \times 10^{-10} = 20.65 \times 10^{16}\, J.$$

$$E_k = 12.88853 \times 10^{26} - 5.61798 \times 10^{26} = 7.27055 \times 10^{26}\, GeV;$$

$$or \; 7.27055 \times 10^{26} \times 1.6022 \times 10^{-10} = 11.64888 \times 10^{16}\, J.$$

For $u_2 = 0.99c$ ($\beta^2 = 0.9801$):

$$E = 5.61798\sqrt{\frac{0.9801}{1-0.9801} + 1} \times 10^{26} = 39.8248 \times 10^{26}\, GeV;$$

$$or \; 39.8248 \times 10^{26} \times 1.6022 \times 10^{-10} = 63.807 \times 10^{16}\, J.$$

$$E_k = 39.8248 \times 10^{26} - 5.61798 \times 10^{26} = 34.20682 \times 10^{26}\, GeV;$$

$$or \; 34.20682 \times 10^{26} \times 1.6022 \times 10^{-10} = 54.806167 \times 10^{16}\, J.$$

For $u_3 = 0.999c$ ($\beta^2 = 0.998$):

$$E = 5.61798\sqrt{\frac{0.998}{1-0.998} + 1} \times 10^{26} = 125.62185 \times 10^{26}\, GeV;$$

$$or \; 125.62185 \times 10^{26} \times 1.6022 \times 10^{-10} = 201.27 \times 10^{16}\, J.$$

$$E_k = 125.62185 \times 10^{26} - 5.61798 \times 10^{26} = 120.0 \times 10^{26}\, GeV;$$

$$or \; 120.0 \times 10^{26} \times 1.6022 \times 10^{-10} = 192.27 \times 10^{16}\, J.$$

Part b

Let v be the final velocity of the battleship. Using the conservation of momentum:

$$p_{initial} = p_{final};$$
$$0 = \gamma m_0 u + Mv;$$
$$v = -\frac{\gamma m_0 u}{M}.$$

For $u_1 = 0.9c$ ($\gamma = 2.2942$):

$$v \approx -13.765\, m/s$$

For $u_2 = 0.99c$ ($\gamma = 7.0889$):

$$v \approx -46.7867\, m/s$$

For $u_3 = 0.999c$ ($\gamma = 22.3663$):

$$v \approx -148.9596\, m/s$$

Part c

Let $m_S = 50\, kg$ be the mass of the Sith warrior and v - her resulting velocity. According to the conservation of momentum:

$$v = \frac{\gamma m_0 u}{m_S}$$

For $u = 0.9c$ ($\gamma = 2.2942$):

$$v \approx 1.2388 \times 10^7\, m/s$$

Therefore, if the warrior were to start trying to stop the slugs, her resulting velocity would be $\approx 0.04c$, which, as one could imagine, would not be comfortable for a human. Consequently, we can make a conclusion that this would not be a good idea.