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CS 405 / Yang
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Assignment 1

1. Written Question

$$n = \frac{V1}{|V1|}, \quad \text{where } |V1| = \sqrt{V1_x^2 + V1_y^2 + V1_z^2}$$

$$n_x = \frac{V1_x}{|V1|}$$

$$n_y = \frac{V1_y}{|V1|}$$

$$n_z = \frac{V1_z}{|V1|}$$

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u is a unit vector orthogonal to both $V1$ and $V2$. Therefore

$$u = \frac{V1 \times V2}{|V1 \times V2|}$$

$$u_x = \frac{(V1_y V2_z - V2_y V1_z)}{|V1 \times V2|}$$

$$u_y = \frac{(V1_x V2_z - V2_x V1_z)}{|V1 \times V2|}$$

$$u_z = \frac{(V1_x V2_y - V2_x V1_y)}{|V1 \times V2|}$$

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v is a unit vector orthogonal to both n and u. Therefore

$$v = \frac{n \times u}{|n \times u|}$$

$$v_x = \frac{n_y u_z - u_y n_z}{|n \times u|}$$

$$v_y = \frac{u_x u_z - u_x n_z}{|n \times u|}$$

$$v_z = \frac{n_x u_y - u_x n_y}{|n \times u|}$$

2. Programming Question

Refer to the files Calc_uvn.cpp
Vector3D.h
Vector3D.cpp

3. Written and Programming Question

a) Verify that $T * T^{-1} = I$

$$T * T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VPP_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & VRP_x \\ 0 & 1 & 0 & VRP_y \\ 0 & 0 & 1 & VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & VRP_x - VPP_x \\ 0 & 1 & 0 & VRP_y - VRP_y \\ 0 & 0 & 1 & VRP_z - VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) An orthogonal matrix is one in which the vector defined by each row is a unit vector orthogonal to all the other rows.

b. (cont'd)

Each component of $R^* R^T$ is equal to the dot product of two rows of R.

The components on the diagonal of $R^* R^T$ will be equal to the dot product of two identical unit vectors, given by $\cos 0° = 1$.

The components not on the diagonal will be equal to the dot product of two orthogonal unit vectors, given by $\cos 90° = 0$.

Therefore

$$R^* R^{-1} = R^* R^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I.$$