The upper 2×2 submatrix is a composite rotation and scale matrix, whereas t_x and t_y are composite translations. Calculating $M \cdot P$ as a vector multiplied by a 3×3 matrix takes nine multiplies and six adds. The fixed structure of the last row of Eq. (5.35), however, simplifies the actual operations to

$$x' = x \cdot r_{11} + y \cdot r_{12} + t_x, \qquad (5.36)$$
$$y' = x \cdot r_{21} + y \cdot r_{22} + t_y,$$

reducing the process to four multiplies and four adds—a significant speedup, especially since the operation can be applied to hundreds or even thousands of points per picture. Thus, although 3×3 matrices are convenient and useful for composing 2D transformations, we can use the final matrix most efficiently in a program by exploiting its special structure. Some hardware matrix multipliers have parallel adders and multipliers, thereby diminishing or removing this concern.

5.7 MATRIX REPRESENTATION OF 3D TRANSFORMATIONS

Just as 2D transformations can be represented by 3×3 matrices using homogeneous coordinates, 3D transformations can be represented by 4×4 matrices, provided that we use homogeneous-coordinate representations of points in 3-space as well. Thus, instead of representing a point as (x, y, z), we represent it as (x, y, z, W), where two of these quadruples represent the same point if one is a nonzero multiple of the other; the quadruple $(0, 0, 0, 0)$ is not allowed. As in 2D, a standard representation of a point (x, y, z, W) with $W \neq 0$ is given by $(x/W, y/W, z/W, 1)$. Transforming the point to this form is called **homogenizing**, as before. Also, points whose W coordinate is zero are called points at infinity. There is a geometrical interpretation as well. Each point in 3-space is being represented by a line through the origin in 4-space, and the homogenized representations of these points form a 3D subspace of 4-space that is defined by the single equation $W = 1$.

The 3D coordinate system used in this text is **right-handed**, as shown in Fig. 5.16. By convention, positive rotations in a right-handed system are such that, when looking from a positive axis toward the origin, a 90° *counterclockwise* rotation will transform one positive axis into the other. This table follows from this convention:

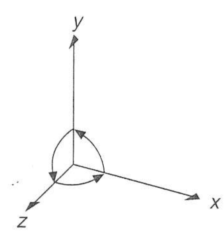

Figure 5.16
The right-handed coordinate system

Axis of rotation is	Direction of positive rotation
x	y to z
y	z to x
z	x to y

These positive directions are also depicted in Fig. 5.16. Be warned that not all graphics texts follow this convention.

We use a right-handed system here because it is the standard mathematical convention, even though it is convenient in 3D graphics to think of a left-handed