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The distribution of reflectances within the visual environment

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Abstract

The statistics of natural images have often been used to account for various properties of animal visual systems. However, for most visual tasks, the images themselves are not important; it is the physical properties of the surfaces which generated them that guide behaviour. Here, we present statistical characterisations of the surface reflectances encountered within four different visual environments (woodland, beach, urban and interior), sampled using a systematic, survey-based method. Of the distributions fitted to the data, the beta distribution provides the best description per number of free parameters. Such distributions may be used as priors in Bayesian models of lightness constancy, or to generate ecologically valid reflectance distributions for simulated environments. The implications of this for models of reflectance extraction within visual systems are discussed.

Keywords: Reflectance distributions; Bayesian models; Brightness constancy; Natural signals

1. Introduction

Natural visual signals are not random. As a result of the physical properties of the world we inhabit, the signals which animal visual systems have evolved to identify and process have strong statistical structure. Animals which have evolved to operate within such environments may be expected to exploit these regularities within natural visual signals in order to achieve accurate yet efficient representations of the world.

Over the past 30 years, a number of relatively simple statistical descriptions of natural visual signals have been used to explain various properties of animal visual systems. Laughlin (1981) demonstrated the optimality of the contrast response function of the fly eye in encoding the luminance contrast values encountered within the natural environment. Response functions for colour (Atick, Li, & Redlich, 1992) and spatio-temporal contrast (van Hateren, 1993) in human vision have also been accounted for using characterisations of the relevant aspects of natural images. The spatial statistics of natural images, such as their autocorrelation functions and $1/f$ property, are long-established (Kretzmer, 1952; Burton & Moorhead, 1987; Field, 1987; Ruderman & Bialek, 1994), and have been used to argue for the optimality of various visual receptive field properties in maximising information transmission from natural images. (Srinivasan, Laughlin, & Dubs, 1982; Field, 1987).

Aspects of images other than their spatial structure have also been explored. Chromatic characterisations of natural images have been used to account for the number of colour channels (Chiao, Cronin, & Osorio, 2000); the chromatic tuning of cone photopigments (Vorobyev & Osorio, 1998), and the nature of the colour-opponent system (Ruderman, Cronin, & Chiao, 1998; Tailor, Finkel, & Buchsbaum, 2000) within human vision.

Bayesian models of motion perception (Weiss, Simoncelli, & Adelson, 2002; Stocker & Simoncelli, 2005) and contour detection (Geisler, Perry, Super, & Gallogly, 2001) have been successfully developed using prior distributions consistent with real-world measurements of optical flow and edge co-occurrence, respectively. Similarly, Yang and Purves (2003) used a prior derived from measurements of the distance between an observer and objects within natural environments to account for the observed discrepancies between the perceived and actual distances of objects.

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Common to many of the studies outlined above is the assumption that the statistics of natural *images* describe the characteristics of the visual environment which animal visual systems have evolved to exploit. However, for most of the visual tasks which an animal may carry out, natural images themselves are not the signal *per se*. Rather, it is the physical properties of the surfaces and objects which generated them that are useful in guiding behaviour. Images are the result of environmental characteristics such as the illumination of the scene, the reflectance of surfaces within the scene, and the geometrical configuration of these surfaces, and are only useful insofar as they inform us about these properties. Barrow and Tennenbaum (1978) proposed representing the intrinsic characteristics of a scene as "intrinsic images", within which the value of a given point represents the value of the relevant characteristic at that point in the visual scene. Each intrinsic image will therefore contain all the information provided by a given characteristic. However, many studies of natural image statistics fail to acknowledge the composite nature of images and instead regard them as a single entity which may, in fact, be of little behavioural relevance.

Reflectance is the intrinsic physical property of a surface which determines how incident light is reflected. In its simplest form, reflectance is characterised as albedo, the proportion of incident light reflected, which provides a measure of the overall 'lightness' of a surface. This assumes that the surface is a lambertian reflector, and therefore cannot account for features such as specularities and surface texture. The most complete description of the reflectance properties of a surface is given by its Bidirectional Reflectance Distribution Function (BRDF), which specifies the proportion of light incident on a surface from each possible illumination direction which is reflected in each possible viewing direction (Dana, van Ginneken, Nayar, & Koenderink, 1999; Marschner, Westin, Lafortune, & Torrance, 2000; Matusik, Pfister, Brand, & McMillan, 2003). Alternatively, the reflectance properties of a surface may be characterised according to their effects on the spatial statistics of the reflected luminance signal (Dror, Adelson, & Willsky, 2001).

Although these characterisations describe the reflectance properties of real-world surfaces, they cannot tell us how likely an animal is to encounter a given reflectance within the visual environment. This information is provided by a probabilistic description of the distribution of reflectances. Such descriptions of environmental characteristics are important for three main reasons:

1. Aside from any specific use, there is intrinsic value in compiling a "Natural History" of the visual world we inhabit in terms of its image, colour and motion statistics.

2. Knowing how a certain characteristic of the environment, and hence its associated visual signal, varies, allows us to identify and extract that signal from the mixture of visual information and noise received from the environment. Traditionally, studies of noise within vision have focused on internal sources, e.g., photoreceptor noise, which is characterised as being white and of relatively low magnitude, with a Signal-to-Noise ratio (S/N) of approximately 10 for the mean receptor output (Osorio & Bossomaier, 1992; Vorobyev, Brandt, Peitsch, Laughlin, & Menzel, 2001). Vision, however, does not begin with a receptor, but with an environmental signal. When viewing a surface, the reflectance signal is not directly measurable. Instead, the luminance signal that we receive from the surface is the product of its reflectance and the incident illumination. Despite possessing strong structure (Dror, Willsky, & Adelson, 2004) and obvious behavioural utility, the illumination component of the signal gives us no information regarding reflectances in the scene. Therefore, when attempting to extract the intrinsic reflectance image, the luminance information received from the environment may be regarded as being composed of signal (variation in reflectance) plus noise (variation in illumination).

Physics constrains the reflectance signal to vary between approximately 0.04 and 1, i.e., over just one order of magnitude. In contrast, illumination may vary by up to three orders of magnitude between sunrise and sunset due to the position of the sun in the sky (Endler, 1993), and up to two orders of magnitude within a single visual scene, due to the effects of shadow. This suggests that, for animals dealing with visual signals within the real world, variation due to low-level internal noise within the visual system may be irrelevant when faced with the task of separating out the contributions of reflectance and illumination to the variation observed within the visual signal. To gain an understanding of the relative contributions of signal and noise, accurate probabilistic characterisations of environmental reflectance are essential.

3. Lastly, in recent years, probabilistic models of perception have become increasingly popular. Perhaps the most well known of these are the Bayesian models, which represent prior experience in the form of a probabilistic characterisation of the environment (the 'prior'). Simulations of the evolution of perceptual systems have also been carried out in which agents are evolved, through optimisation, to perform tasks within a probabilistically defined environment (e.g., Schlessinger et al., 2005).

The performance and ecological validity of both Bayesian and evolutionary optimisation models is fundamentally limited by the accuracy of their models of the environment. Often, the form of characterisation employed is decided upon for reasons of simplicity or computational convenience rather than its ability to faithfully represent the real-world distribution of the relevant variable. For example: In Schlessinger et al. (2005) environmental reflectances are arbitrarily assumed to be *uniformly* distributed between 0 and 1.

Gaussian distributions are commonly chosen to represent priors in Bayesian models of vision (e.g., Brainard & Freeman, 1997). Gaussianity, however, is a property associated with the probability distributions of random variables and, as stated above, "visual signals are not random". Gaussian distributions also assume that a variable is free to vary between $+\infty$ and $-\infty$, which clearly cannot be the case with variables such as reflectance which are constrained to lie between 0 and 1.

In order to produce accurate probabilistic models of environmental reflectance, three separate but related questions must be considered: (i) What is the distribution of reflectances within the visual world? In order to determine how reflectance varies within the real world, a range of visual environments must be systematically sampled. (ii) What is the most appropriate parametric form for characterising the distribution of reflectance? Possession of good closed-form approximations of the probability distribution of reflectances within different real-world visual environments is essential in the construction of ecologically valid analytic models of reflectance extraction. (iii) How statistically homogenous are the reflectance distributions for different environments? If the reflectance distributions within different visual environments are sufficiently homogenous, then a single characterisation may be used to describe the reflectance distributions within all environments. If, however, the distributions are quite different, then it may be more appropriate to describe each one using a specific characterisation.

In this paper we attempt to describe, probabilistically, the behaviourally important reflectance signal within a range of different visual environments. Here, a surface's reflectance is characterised in terms of its albedo. While albedo provides a much less detailed description of the reflectance characteristics of a surface than the BRDF, it may be measured quickly and simply within the field. Hence, by using a systematic, survey-based method we hope to build up a basic, but nonetheless very useful, picture of the distribution of surface lightnesses within each environment sampled.

Different parametric characterisations of the measured distributions of reflectances are compared, and the best of these (i.e., the most accurate and concise) are presented.

Mathematical techniques which assume Gaussian-distributed data are more familiar and widely used than those involving other parametric distributions. Hence, as a 'second-best' to characterising data using the most appropriate distribution, data may be transformed in order to increase its Gaussianity, thereby permitting the use of Gaussian techniques. To this end, a number of transforms are assessed in terms of their effectiveness in increasing the Gaussianity of the distributions of natural reflectance data.

2. Methods

2.1. Data collection

The reflectance of a given surface was determined using the following protocol: A photographic standard grey card of 18% reflectance (Rc = 0.18) was placed over the surface and the luminance of the card (Lc1) was measured. The card was then removed and the luminance of the surface itself (Ls) was measured. Finally, the card was replaced, and a second measurement of its luminance was made (Lc2). All luminances were measured using a Minolta LS-100 handheld luminance meter. The reflectance of the surface (Rs) was calculated using Eq. (1), where Lcm is the mean of Lc1 and Lc2.

$$\mathrm{Rs} = (\mathrm{Ls}/\mathrm{Lcm}) * \mathrm{Rc} \tag{1}$$

The reflectances of surfaces were measured within four different classes of visual environment: (i) Deciduous Woodland (ii) Beach (iii) Urban and (iv) Domestic Interior.

To ensure that the reflectance values obtained were as representative of the distribution of reflectances within the environment as possible, a 'linear transect' method of sampling from an environment, common in biological studies, was adopted. A straight line (the transect) is marked on the ground in the area to be sampled, and the relevant measurement of the environment (in this case, reflectance) is made at regular intervals along the length of the line (see Barnett, 2003; chapter 6).

In the deciduous woodland and beach environments the starting point of each transect was selected by blindly throwing a marker into the area to be sampled. The orientation of each transect was determined by randomly selecting a number between 1 and 360, corresponding to a bearing (in degrees) on a magnetic compass. The limitations of such methods in achieving randomness are fully acknowledged. In these two environments, two reflectance measurements were made at each 1 m interval along each transect; one from the surface at a point 0.5 m to the left, and the other 0.5 m to the right of the transect. The Beach dataset ($N = 50$) was obtained using a single 25 m-long linear transect across Brighton Beach, UK. The Woodland dataset ($N = 245$) comprises reflectances sampled from four linear transects laid down in different areas of deciduous woodland surrounding Bristol, UK.

Due to the spatial limitations imposed by the physical structure of domestic interiors, long linear transects could not be used to sample this environment. Instead, a grid of 1 m × 1 m squares was mapped onto the floor and walls of each room sampled, and the reflectance of the underlying surface was measured at each intersection point within the grid. The domestic interior dataset ($N = 176$) comprises reflectance samples taken from a living room ($N = 71$), kitchen ($N = 42$) and bedroom ($N = 63$).

Similar spatial constraints were in operation when sampling reflectances from an urban environment. For example: It was not practical to measure out a linear transect across public roads or pavements. In this environment, a 'spatio-temporal transect' was employed: An experimenter set out to walk across a built-up area of the city of Bristol, UK, and at 2 min intervals during the walk, took two reflectance measurements from the environment, one from a point 0.5 m to his left, and the other from a point 0.5 m to his right. The walk lasted 50 min, hence, for the Urban dataset, $N = 50$.

In all four environments, if a large structure such as a wall or tree occupied the point in space from which reflectance was to be sampled, then the reflectance of that structure's surface at a height of 1.5 m was taken in its place.

2.2. Data analysis

Three different probability distributions were fitted to each of the four environmental reflectance datasets: (i) A Gaussian distribution, chosen because of its common use as a 'default' distribution when the true distribution of a variable is unknown. (ii) A Beta distribution, chosen due to its versatility and the fact that, in common with reflectance, it is constrained to lie between 0 and 1. (iii) A mixture model composed of two Gaussian distributions, also chosen because of its versatility.

Estimates for the parameters of the best-fitting Beta and Gaussian distributions (α and β, and μ and σ, respectively) were determined using Maximum Likelihood Estimation. Estimates for the parameters of the best-fitting Gaussian mixture models (μ_1, σ_1, f_1, μ_2, σ_2, and f_2, where f_1 and f_2 are the mixing coefficients) were determined using an Expectation-Maximisation algorithm (Netlab Neural Network Toolbox v3.2, Bishop and Nabney, 2001). The parameters of each mixture model were deemed to have achieved maximum likelihood when the difference in log likelihood between two iterations of the algorithm was less than 0.0001. The maximum number of iterations per fitting was set at fifty. However, in every case, fewer iterations were required to achieve maximum likelihood.

To allow comparison of the goodness-of-fit of the three distributions fitted to each of the environmental datasets, the log likelihoods of the distributions were calculated. The resulting values were then compared using log-likelihood ratio tests (Lehmann, 1986).

To test for normality (and if better normality could be achieved using a data transform), a D'Agostino–Pearson test for normality was used (D'Agostino, 1986). The three transformations applied to the data were as follows (where $T(x)$ is the transformed data, and x the untransformed reflectance):

(i) Log transform.

$$T(x) = \ln x \tag{2}$$

(ii) Arcsine transform.

$$T(x) = \arcsin x^{0.5} \tag{3}$$

(iii) Arctangent transform.

$$T(x) = \arctan x^{0.5} \tag{4}$$

After noting the positive skew in the raw natural reflectance data (see Fig. 1) it was predicted that representing the data in log space would have the effect of producing a distribution with a more Gaussian profile by expanding the range at lower values while compressing those at higher values. Arcsine and arctangent transforms were selected as they act to increase the Gaussianity of proportional data (e.g., reflectance) in two main ways: (i) By converting proportional values, constrained to lie between 0 and 1, to scalar values which, in common with the Gaussian distribution, may vary between $+\infty$ and $-\infty$. (ii) By increasing the numerical spacing between values at the high and low ends of the proportion scale. This acts to reduce any skew, positive or negative, present in the original data. These three transforms are also the "standard" transforms used within exploratory data analysis (Tukey, 1977).

3. Results

Fig. 1 shows the reflectance data collected within the interior, beach, urban and woodland environments, along with plots of the best-fitting Gaussian, Mixture of Gaussian, and Beta distributions for these datasets. The parame-

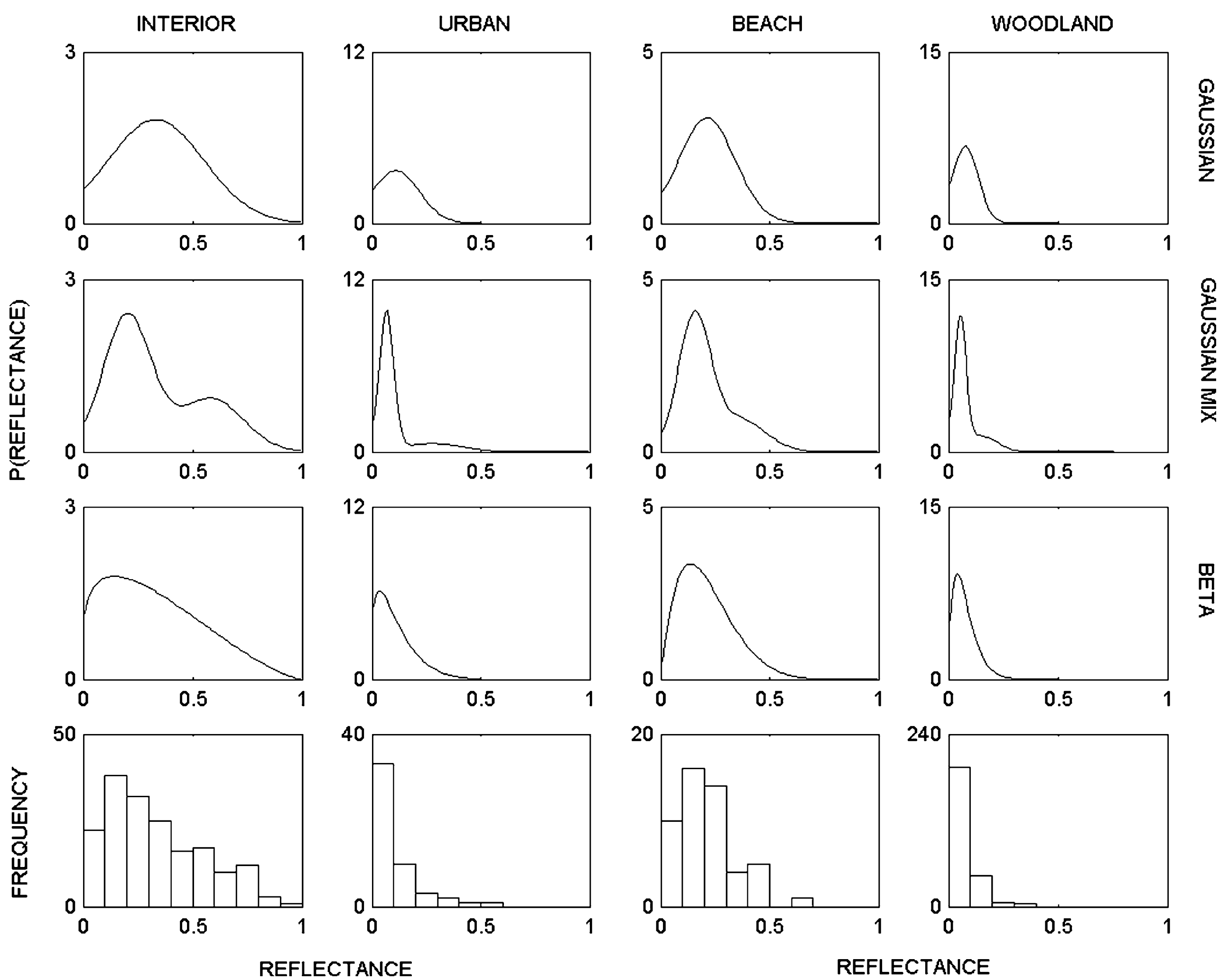

Fig. 1. Frequency distributions of surface reflectances within interior, urban beach and woodland visual environments (bottom row), along with the best-fitting Gaussian, mixture of Gaussian, and beta distributions for each environmental dataset.

Table 1
Lower order moments of the reflectance data sampled from each of the four classes of environment, with parameter estimates for distributions fitted to this data

Environment	Lower-order moments				Parameters of fitted distributions							
					Gaussian Mixture						Beta	
	μ	σ	Kurtosis	Skew	μ_1	σ_1	f_1	μ_2	σ_2	f_2	α	β
Woodland	0.08	0.06	8.77	2.03	0.05	0.03	0.75	0.15	0.07	0.25	1.91	22.60
Beach	0.21	0.13	3.51	0.92	0.15	0.07	0.70	0.36	0.12	0.30	2.04	7.57
Urban	0.11	0.11	8.34	2.32	0.07	0.03	0.80	0.28	0.14	0.20	1.35	10.72
Interior	0.33	0.22	2.42	0.61	0.21	0.11	0.62	0.61	0.13	0.38	1.29	2.30

Beta parameters were estimated using MLE. Gaussian Mixture parameters were estimated using an Expectation-Maximisation algorithm.

ters of these distributions are given in Table 1, along with the lower order moments of the data (mean, variance, kurtosis and skew). Within each of the four environments the mean reflectance is relatively low, with a maximum value of 0.33 within the interior environment. It is evident that both the median and the variance of reflectances differ significantly across environments (Kruskal–Wallis, $H = 219.18$, $p < 0.0001$, and Levene's test, $F = 94.73$, $df1 = 3$, $df2 = 517$, $p < 0.0001$, respectively), with the greatest variance and highest median reflectance being found within the interior environment, and the least variance and lowest median reflectance within the woodland environment. In all four environments, the distribution of reflectances is positively skewed, i.e., there is a bias towards lower reflectances.

The log-likelihood values in Table 2 reveal how well the three types of distribution fitted describe each environmental dataset. Values closer to zero indicate a better fit of a distribution to the data which it is to model. Comparison of these values using Log-Likelihood Ratio Tests shows that, under all four environmental conditions; (i) The best-fitting Mixture of Gaussian distribution achieves a significantly better fit to the data than the best-fitting single Gaussian distribution, ($df = 3$ $p < 0.005$). (ii) There is no significant difference between the fit of the best-fitting Beta and Mixture of Gaussian distributions to the data ($df = 3$ $p > 0.05$). This suggests that beta distributions are able to describe the distribution of natural reflectances equally as well as a Gaussian mixture model.

The effects of the log, arcsine and arctangent transforms on the Gaussianity of the reflectance datasets are given in Table 3. It is clear from the assessment of the raw data that the reflectances of surfaces are far from Gaussian distributed in any of the four environments sampled. The arcsine and arctangent transforms have very similar effects on the data; both producing normally distributed datasets when performed on the interior and beach datasets (those with the highest Gaussianity prior to transformation), but not on the urban and woodland data. The log transform was found to be the most successful transform of the three, producing Gaussian-distributed datasets from all but the interior data.

Table 2
Log likelihoods of distributions fitted to reflectance data sampled from each of the four classes of environment

Environment	Distribution fitted		
	Gaussian	Gaussian mix	Beta
Woodland	−1001.87[a]	−942.26	−936.50
Beach	−164.04[a]	−159.36	−158.24
Urban	−154.90[a]	−127.83	−133.80
Interior	−893.05[a]	−870.00	−874.05

[a] Indicates significant difference in log likelihood from the log likelihood of the Gaussian Mixture fitted to the same data at the 95% (or higher) confidence level.

4. Discussion

In this study, it was found that characteristic features of surface reflectance as measured within the real world are; (i) a low mean value and (ii) a positively skewed distribution. Hence, the reflectance of a surface encountered within the real world is likely to be low. Surfaces perceived as mid-grey, (e.g., a photographic standard grey), do not have a reflectance of 0.5, rather they are closer to 0.18 reflectance. This may be regarded as the result of adaptation to the reflectance information encoded within real-world visual environments.

The distributions of surface reflectances within a range of real-world environments are not Gaussian; rather they are better described by either beta or mixture of Gaussian distributions. Of the two, beta distributions are able to provide the most compact description of environmental reflectances; using only two free parameters rather than the five required to define a mixture of two Gaussians. Beta distributions are also particularly suitable for modelling reflectance as both the distribution and the variable are constrained to lie over the same interval; $[0, 1]$. While it is found that betas are able to follow the distributions of a range of environmental reflectance datasets with no significant statistical deviations (and hence act as good descriptors of the data), this does not necessarily mean that reverse is true, and the data follows the beta distribution due to any fundamental, underlying regularities within the visual world. Hence the beta distribution is given no special theoretical status in this context.

The beta characterisations of surface reflectance distributions presented here describe how the behaviourally use-

Table 3
Gaussianity of raw and transformed reflectance data, as assessed using the D'Agostino–Pearson test of normality

Environment	Data type											
	Raw			ln			Arcsine			Arctangent		
	χ^2	p	Gauss?	χ^2	p	Gauss?	χ^2	p	Gauss?	χ^2	p	Gauss?
Woodland	119.1	0.000	X	3.659	0.161	√	37.25	0.000	X	15.68	0.000	X
Beach	8.268	0.016	X	3.585	0.167	√	1.833	0.400	√	0.728	0.695	√
Urban	39.77	0.000	X	1.356	0.507	√	23.96	0.000	X	13.64	0.001	X
Interior	12.38	0.002	X	71.98	0.000	X	4.857	0.088	√	5.348	0.069	√

√ = Gaussian distribution X = Non-Gaussian distribution, at 95% significance level, df = 2.

ful reflectance signal is likely to vary within each class of environment. Hence, these descriptions allow us to identify the reflectance component of the environmental luminance signal. From a Bayesian viewpoint, they represent an animal's prior experience of the reflectance signal, and may therefore form the basis of environmental reflectance priors within models of reflectance extraction. For those wishing to carry out simulations using ecologically valid representations of surface reflectance, reflectance values may be obtained by random sampling from the relevant beta characterisation. A simple method for generating random beta-distributed variates from gamma variates, which are themselves simple to generate (Marsaglia & Tsang, 2000), is given by Devroye (1986,p432).

It was found that although reflectance distributions are far from normal, their Gaussianity can be greatly increased by applying a simple transformation. This may be useful given the analytic tractability of Gaussian approximations and the ubiquity of Gaussian techniques. In particular, the distribution of the log of reflectance is approximately normal for all the environments except the interior. Since image generation can be thought of as the multiplication of the illuminant by the reflectance of a surface (or the addition of their logs), this result may be of particular use. It is not known whether the distribution of local (log) illumination is approximately normal within real-world environments. It is acknowledged that applying histogram matching techniques to the data on a case-by-case basis would produce a set of optimal non-linear transforms for normalising our reflectance datasets. However, it is our intention here to identify simple, general transforms, familiar to researchers within a broad range of disciplines, which will encourage the use of ecologically valid reflectance datasets within Gaussian-assuming mathematical models.

The four different environments vary considerably in their reflectance variance. The relative contributions of reflectance and illumination to the luminance signal will therefore also differ between environments; being lower within environments with low reflectance variance (e.g., woodland), and higher within environments with high reflectance variance (e.g., interior). Hence, the accuracy and consistency with which a given animal is able to separate out these two components of the luminance signal may, in part, be determined by the reflectance characteristics of the environment.

The heterogeneity of environmental reflectance has implications for animals attempting to carry out visual tasks within a range of environments. Within Bayesian models of vision, heterogeneous environments are often dealt with using a system of competitive priors (Bülthoff & Yuille, 1996). This allows the perceptual system to employ different priors within different visual environments in order to maximise the accuracy and consistency of its inferences. Alternatively, an animal's visual system may be optimised to the reflectance statistics of a single, or average, environment. Animals will benefit from the use of competitive priors, but if they have only a single "average" prior we would expect to see strong and systematic errors in the performance of that animal when carrying out visual tasks within environments with non-average reflectance statistics.

Over the past 10 years there has been considerable interest in how the statistics of natural images can inform us about vision. We believe that to fully exploit these statistics, their relationship to the world needs to be known: An animal's survival depends not on knowing about images, but about the physical world that generated them. We have made a small step towards characterising the statistics of this world, in particular the distribution of reflectances. We found that they are; (1) on average dark (only 8% for the woodland environment), (2) far from Gaussian but well captured as beta distributed, and lastly (3) the reflectance environments are very heterogeneous (with the statistics of the indoor environment being particularly extreme). We believe these simple regularities will prove useful in understanding how animals extract useful information from their visual world.

References

Atick, J. J., Li, Z. P., & Redlich, A. N. (1992). Understanding retinal color coding from first principles. *Neural Computation, 4*(4), 559–572.

Barnett, V. (2003). *Environmental Statistics: Methods and Applications*. New York: Wiley.

Barrow, H. G., & Tenenbaum, J. M. (1978). Recovering intrinsic scene characteristics from images. In A. Hanson & E. Riseman (Eds.), *Computer vision systems* (pp. 3–26). New York: Academic Press.

Bishop, C., & Nabney, I. (2001). Netlab neural network software. Available from: http://ncrg.aston.ac.uk/netlab/index.php.

Brainard, D. H., & Freeman, W. T. (1997). Bayesian color constancy. *Journal of the Optical Society of America, A14*, 1393–1411.

Bülthoff, H. H., & Yuille, A. L. (1996). A Bayesian framework for the integration of visual modules. In J. McClelland (Ed.), *Attention and performance XVI: Information integration in perception and communication* (pp. 49–70). Cambridge, MA: MIT Press.

Burton, G. J., & Moorhead, I. R. (1987). Color and spatial structure in natural scenes. *Applied Optics, 26*, 157–170.

Chiao, C.-C., Cronin, T. W., & Osorio, D. (2000). Color signals in natural scenes: characteristics of reflectance spectra and effects of natural illuminants. *Journal of the Optical Society of America A, 2*, 218–224.

D'Agostino, R. B. (1986). Tests for normal distribution. In R. B. D'Agostino & M. A. Stepenes (Eds.), *Goodness-of-fit techniques*. New York: Macel Decker.

Dana, K. J., van Ginneken, B., Nayar, S. K., & Koenderink, J. J. (1999). Reflectance and texture of real world surfaces. *ACM Transactions on Graphics, 1*(18), 1–3.

Devroye, L. (1986). *Non-uniform random variate generation*. New-York: Springer-Verlag.

Dror, R. O., Adelson, E. H. & Willsky, A. S. (2001). Recognition of surface reflectance properties from a single image under unknown real-world illumination. In *Proceedings of the IEEE workshop on identifying objects across variations in lighting*.

Dror, R. O., Willsky, A. S., & Adelson, E. H. (2004). Statistical characterization of real-world illumination. *Journal of Vision, 4*, 821–837.

Endler, J. A. (1993). The color of light in forests and its implications. *Ecological Monographs, 63*(1), 1–27.

Field, D. J. (1987). Relations between the statistics of natural images and the response properties of cortical cells. *Journal of the Optical Society of America A, 4*, 2379–2394.

Geisler, W. S., Perry, J. S., Super, B. J., & Gallogly, D. P. (2001). Edge co-occurrence in natural images predicts contour grouping performance. *Vision Research, 41*, 711–724.

van Hateren, J. H. (1993). Spatiotemporal contrast sensitivity of early vision. *Vision Research., 33*, 257–267.

Kretzmer, E. R. (1952). Statistics of television signals. *Bell Systems Technical Journal, 31*, 751–763.

Laughlin, S. B. (1981). A simple coding procedure enhances a neuron's information capacity. *Zeitschrift für Naturforschung., 36*, 910–912.

Lehmann, E. L. (1986). *Testing statistical hypotheses* (2nd ed.). New York: John Wiley and Sons.

Marsaglia, G., & Tsang, W. W. (2000). A simple method for generating gamma variables. *ACM Transactions on Mathematical Software (TOMS), 26*(3), 363–372.

Marschner, S., Westin, S., Lafortune, E., & Torrance, K. (2000). Image-based measurement of the bidirectional reflectance distribution function. *Applied Optics, 39*, 16.

Matusik, W., Pfister, H., Brand, M., & McMillan, L. (2003). A data-driven reflectance model. *ACM Transactions on Graphics (TOG), 22*(3), 759–769.

Osorio, D., & Bossomaier, T. R. J. (1992). Human cone-pigment spectral sensitivities and the reflectances of natural surfaces. *Biological Cybernetics, 67*, 217–222.

Ruderman, D. L., & Bialek, W. (1994). Statistics of natural images: scaling in the woods. *Physical Review Letters, 73*, 814–817.

Schlessinger, E. Bentley, P. J. & Lotto, R. B. (2005). Evolving visually guided agents in an ambiguous virtual world. In *GECCO 2005: Proceedings of the Genetic and Evolutionary Computation Conference. June 25–29*, Washington DC.

Srinivasan, M. V., Laughlin, S. B., & Dubs, A. (1982). Predictive coding: A fresh view of inhibition in the retina. *Proceedings of the Royal Society of London. Series B, Biological Sciences, 216*(1205), 427–459.

Stocker, A. A., & Simoncelli, E. P. (2005). Constraining a Bayesian model of human visual speed perception. In L. K. Saul, Y. Weiss, & L. Bottou (Eds.), *Advances in neural information processing systems*. MIT Press: Cambridge, MA.

Ruderman, D. L., Cronin, T. W., & Chiao, C. C. (1998). Statistics of cone responses to natural images: implications for visual coding. *Journal of the Optical Society of America A, 15*, 2036–2045.

Tailor, D. R., Finkel, L. H., & Buchsbaum, G. (2000). Color-opponent receptive fields derived from independent components analysis of natural images. *Vision Research, 40*, 2671–2676.

Tukey, J. W. (1977). *Exploratory data analysis: reading*. Reading, MA: Addison-Wesley.

Vorobyev, M., & Osorio, D. (1998). Receptor noise as a determinant of colour thresholds. *Proceedings of the Royal Society of London B, 265*, 351–358.

Vorobyev, M., Brandt, R., Peitsch, D., Laughlin, S. B., & Menzel, R. (2001). Colour thresholds and receptor noise: behaviour and physiology. *Vision Research, 61*, 639–653.

Weiss, Y., Simoncelli, E. P., & Adelson, E. H. (2002). Motion illusions as optimal percepts. *Nature Neuroscience, 5*, 598–604.

Yang, Z., & Purves, D. (2003). A statistical explanation of visual space. *Nature Neuroscience, 6*, 632–640.