三、电容器电容的计算

1. 平板电容器

两带电平板间的电场强度

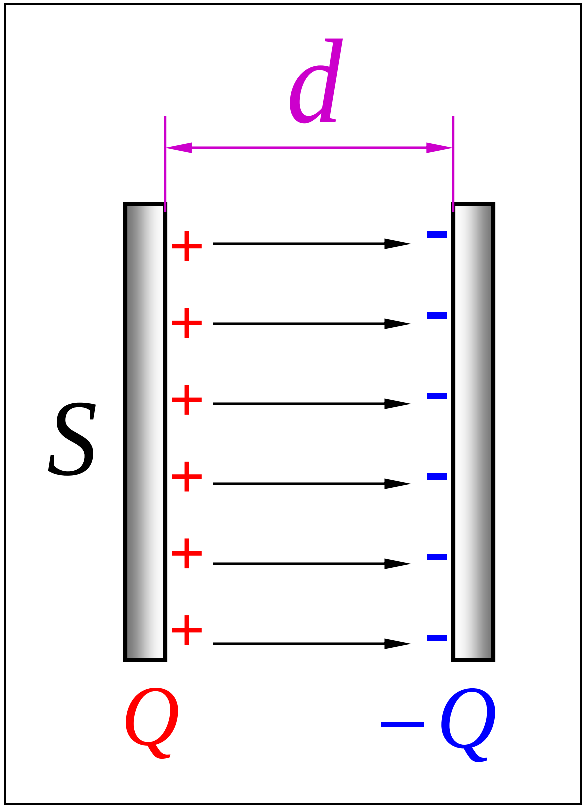
$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 S}$$

两带电平板间的电势差

$$U=Ed=\frac{Qd}{\varepsilon_0 S}$$

平板电容器电容

$$\boxed{C=\frac{Q}{U}=\frac{\varepsilon_0 S}{d}}$$



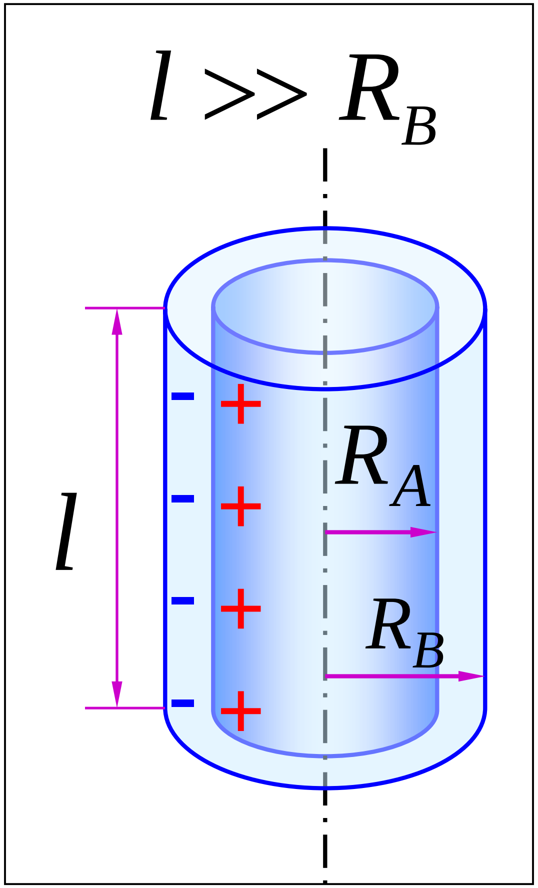
2. 圆柱形电容器

设两导体圆柱面单位长度上分别带电 $\pm\lambda$

$$E=\frac{\lambda}{2\pi\varepsilon_0 r},\quad (R_A<r<R_B)$$

$$\Delta U=\int_{R_A}^{R_B}\frac{\lambda \mathrm{d}r}{2\pi\varepsilon_0 r}=\frac{Q}{2\pi\varepsilon_0 l}\ln\frac{R_B}{R_A}$$

$$\boxed{C=\frac{Q}{\Delta U}=\left(2\pi\varepsilon_0 l\right)\Big/\ln\frac{R_B}{R_A}}$$

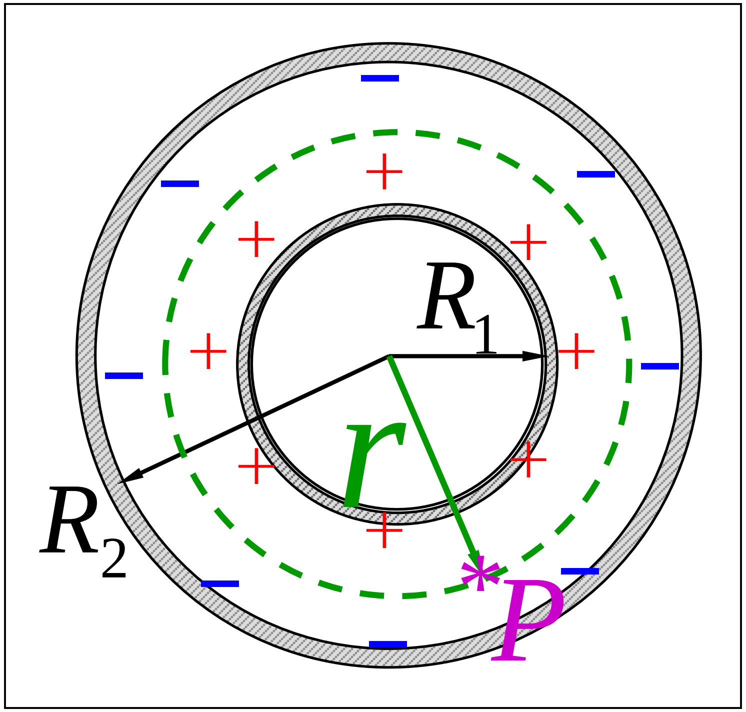


3. 球形电容器的电容

球形电容器是由半径分别为 R_1和 R_2的两同心金属球壳所组成.

解 设内球带正电（$+Q$），外球带负电（$-Q$）.

$$\vec{E}=\frac{Q}{4\pi\ \varepsilon_0 r^2}\vec{e}_{\mathrm{r}}\qquad (R_1<r<R_2)$$



$$U=\int_l \vec{E}\cdot\mathrm{d}\vec{l}=\frac{Q}{4\pi\ \varepsilon_0}\int_{R_1}^{R_2}\frac{\mathrm{d}r}{r^2}$$

$$=\frac{Q}{4\pi\ \varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

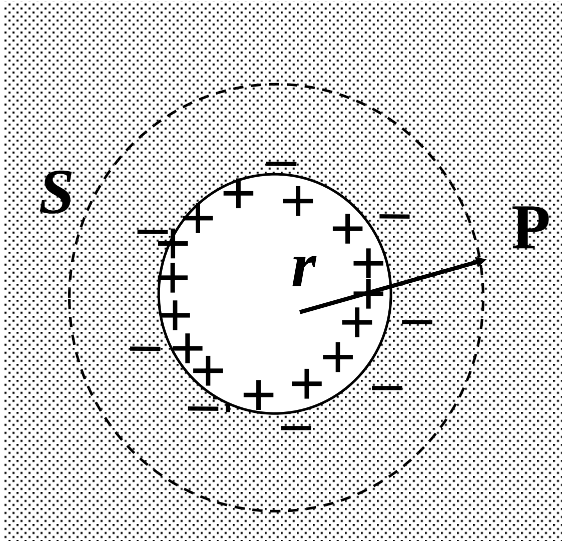
$$C=\frac{Q}{U}=4\pi\,\varepsilon_0\left(\frac{R_1R_2}{R_2-R_1}\right)$$

$$R_2\to\infty,\qquad C=4\pi\ \varepsilon_0 R_1$$

孤立导体球电容

例 1 一导体带电球壳，带电q，周围充满无限大均匀介质，相对电容率为ε_r，求球外一点P的场强、电势。

解: 由于导体和介质都满足球对称性，故自由电荷和极化电荷分布也满足球对称性，因而电场的**分布**也具有球对称性。如图在介质中作一半径为r的球面S，根据 D**的**高斯定理：



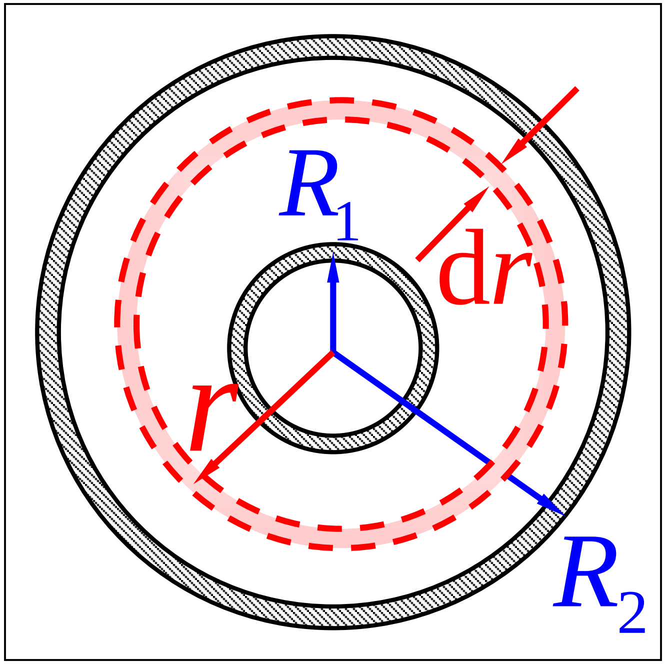
$$\oint_S \vec{D}\cdot \mathrm{d}\vec{S} = \sum q_0 = q$$

$$D4\pi r^2 = q, \quad D = \frac{q}{4\pi r^2}$$

$$\vec{D} = \varepsilon_0\varepsilon_r\vec{E}, \ \vec{E} = \frac{\vec{D}}{\varepsilon_0\varepsilon_r} = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\hat{e}_\mathrm{r}$$

$$U_P = \int_P^\infty \vec{E}\cdot \mathrm{d}\vec{l} = \int_r^\infty \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\mathrm{d}r = \frac{q}{4\pi\varepsilon_0\varepsilon_r r}$$

例2 如图所示,球形电容器的内、外半径分别为 R_1和 R_2，所带电荷为 $\pm Q$ 。问此电容器贮存的电场能量为多少?



解: $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{e}_r \quad (R_1 > r > R_2)$$

$$\mathrm{d}V = 4\pi r^2 \mathrm{d}r$$

$$w_\mathrm{e} = \frac{1}{2}\varepsilon_0 E^2 = \frac{Q^2}{32\pi^2\varepsilon_0 r^4}$$

$$\mathrm{d}W_\mathrm{e} = w_\mathrm{e}\mathrm{d}V = \frac{Q^2}{8\pi\varepsilon_0 r^2}\mathrm{d}r$$

$$W_e = \int dW_e = \frac{Q^2}{8\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2}$$

$$= \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0\frac{R_2R_1}{R_2-R_1}}$$

讨论： $$W_e = \frac{Q^2}{2\ C}$$

$$C = 4\pi\ \varepsilon\frac{R_2R_1}{R_2-R_1}$$

——球形电容器电容