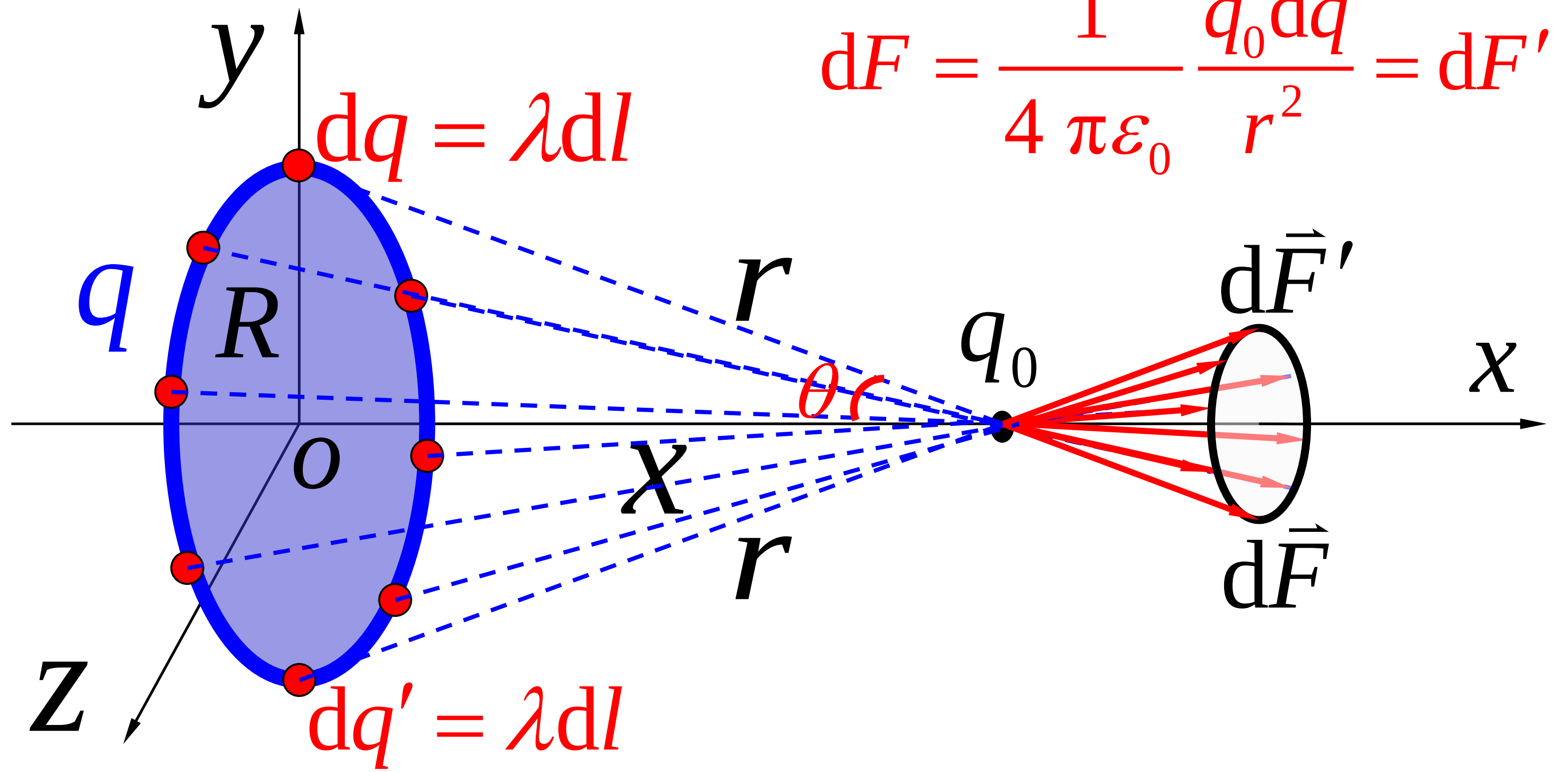
例1 正电荷q均匀分布在半径为 R 的圆环上。计算在环的轴线上任一点 P 处点电荷 q_0所受作用力。

解： $\lambda = \dfrac{q}{2\pi R}$ 　　　**取：** $\mathrm{d}q = \lambda \mathrm{d}l = \mathrm{d}q'$

$$\mathrm{d}F = \frac{1}{4\pi\varepsilon_0}\frac{q_0 \mathrm{d}q}{r^2} = \mathrm{d}F'$$



$$\mathrm{d}F=\frac{1}{4\pi\varepsilon_0}\frac{q_0\mathrm{d}q}{r^2}=\mathrm{d}F'$$

进行对称性分析：

建立 x 方向和与 x 方向垂直的 $\perp$ 方向。

$\mathrm{d}\vec{F}$和$\mathrm{d}\vec{F}'$关于 x 方向对称，可以把$\mathrm{d}\vec{F}$ 和 $\mathrm{d}\vec{F}'$向 x 方向和 $\perp$ 方向分解，其二者在 $\perp$ 方向等值反向相互抵消。

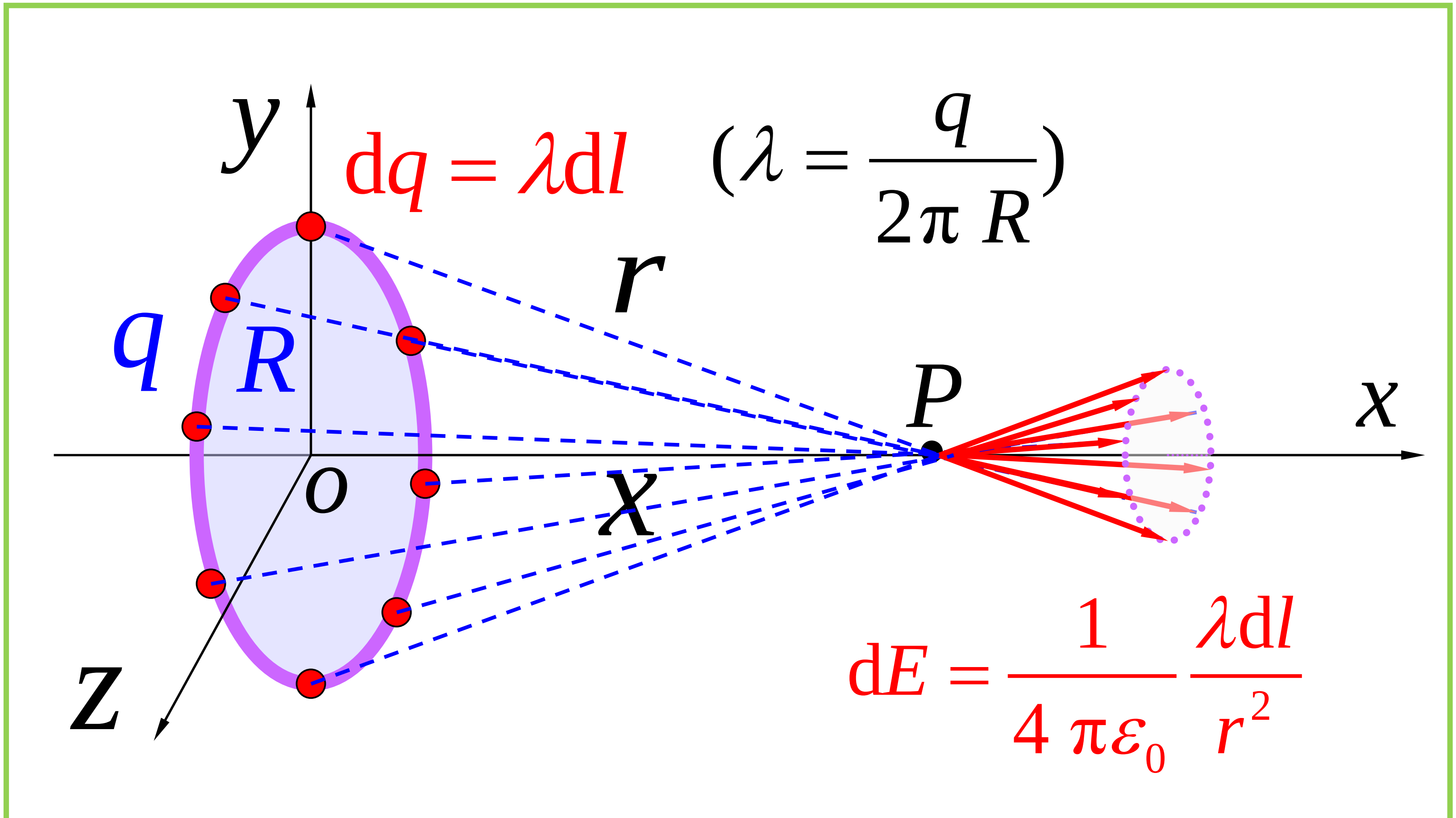
故由对称性有 $\vec{F}=\int \mathrm{d}F_x\hat{i}=F_x\hat{i}$

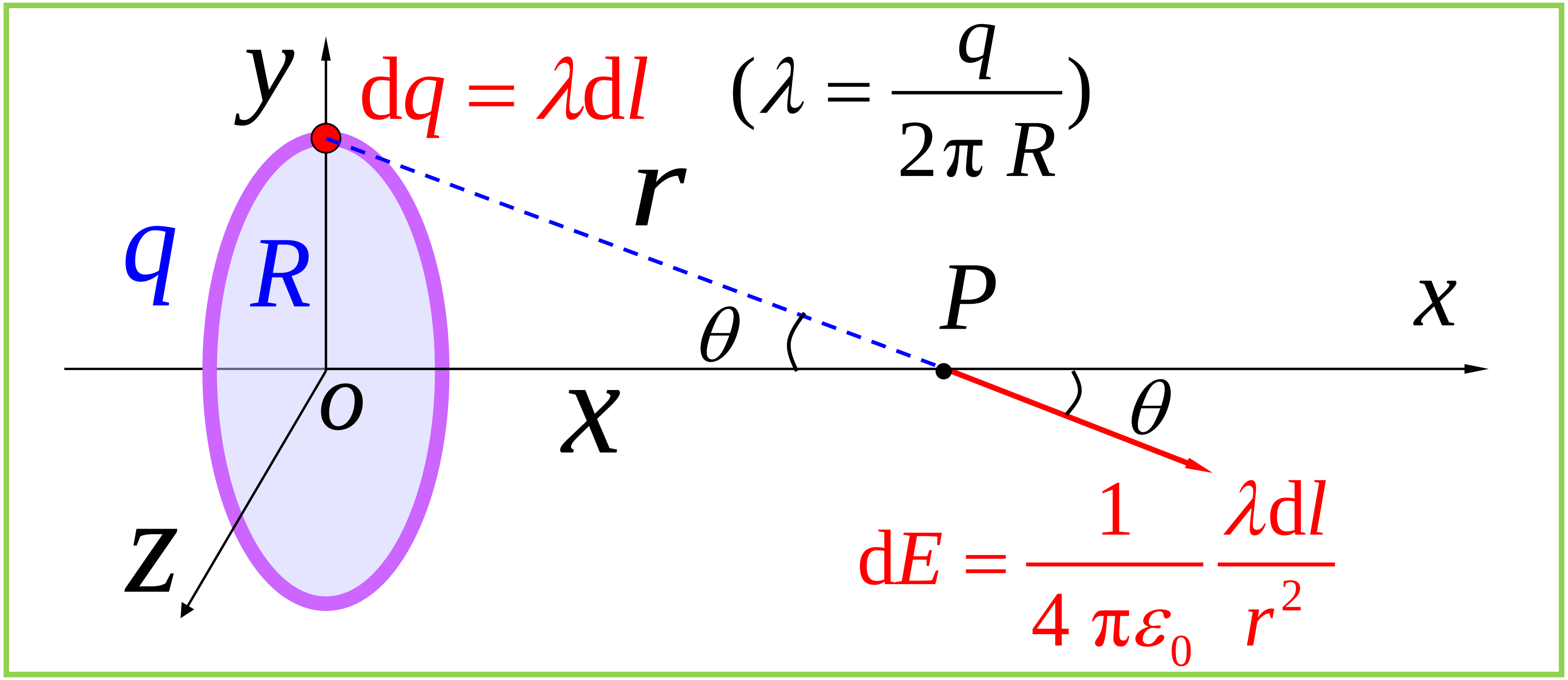
$$\mathrm{d}F_x=\mathrm{d}F\cos\theta=\frac{1}{4\pi\varepsilon_0}\frac{q_0\mathrm{d}q}{r^2}\cos\theta$$

$$F_x=\int_q \mathrm{d}F_x=\frac{q_0}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}\int_q \mathrm{d}q=\frac{q_0q}{4\pi\varepsilon_0}\frac{x}{\left(R^2+x^2\right)^{3/2}}$$

例2 正电荷 q 均匀分布在半径为R的圆环上.计算在环的轴线上任一点 P 的电场强度。

解: $\vec{E}=\int \mathrm{d}\vec{E}$　　　由对称性有 $\vec{E}=E_x\vec{i}$

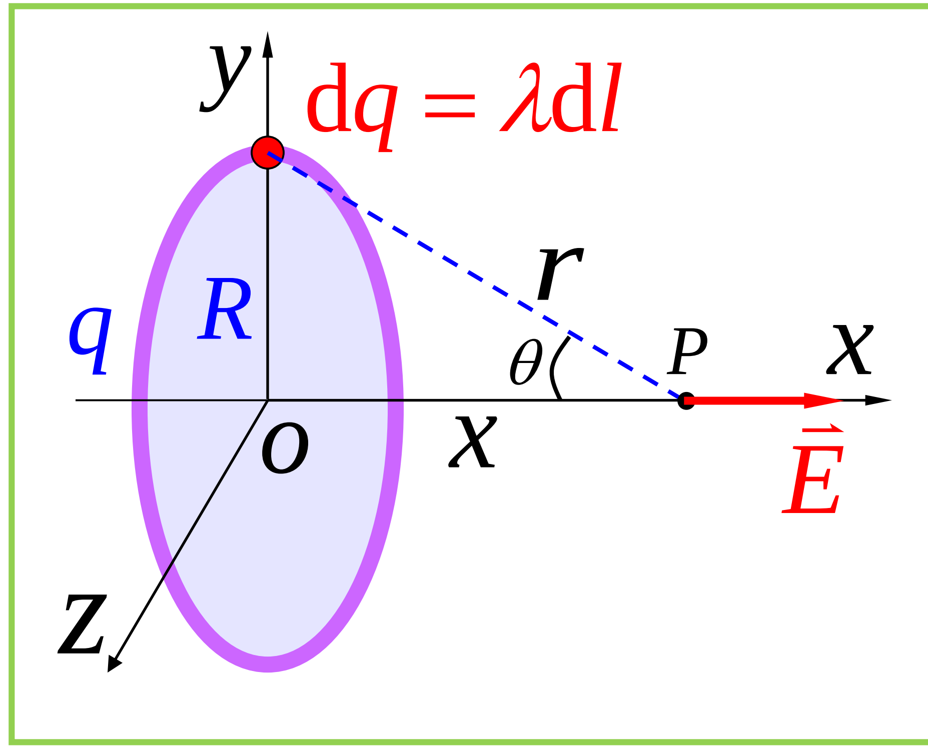




$$E = \int_l \mathrm{d}E_x = \int_l \mathrm{d}E\cos\theta = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi R}\frac{\lambda \mathrm{d}l}{r^2}\cdot\frac{x}{r}$$

$$= \frac{x\lambda}{4\pi\varepsilon_0 r^3}\int_0^{2\pi R}\mathrm{d}l = \frac{qx}{4\pi\varepsilon_0(x^2+R^2)^{3/2}}$$

$$E=\frac{qx}{4\pi\,\varepsilon_0(x^2+R^2)^{3/2}}$$



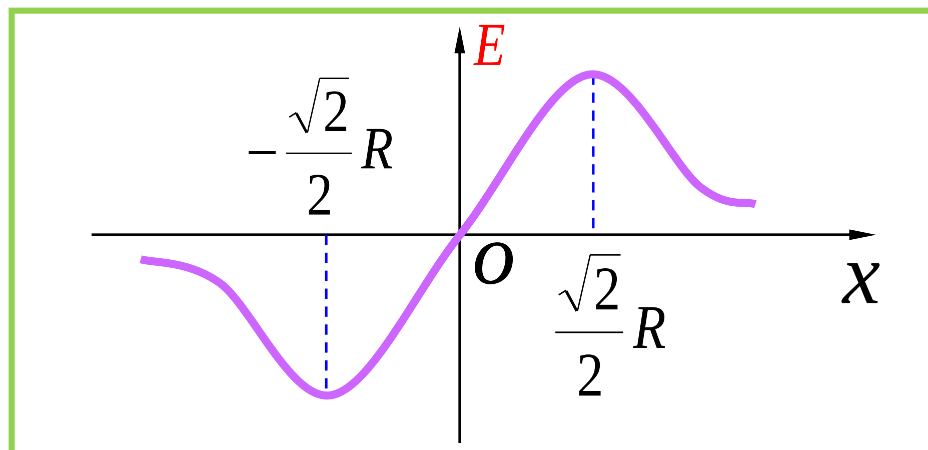
讨论：

(1) $x>>R$

$$E\approx\frac{q}{4\,\pi\varepsilon_0 r^2}$$

——**点电荷电场强度。**

(2) $x=0,\quad E_0=0$



(3) $\frac{\mathrm{d}E}{\mathrm{d}x}=0,\quad x=\pm\frac{\sqrt{2}}{2}R$

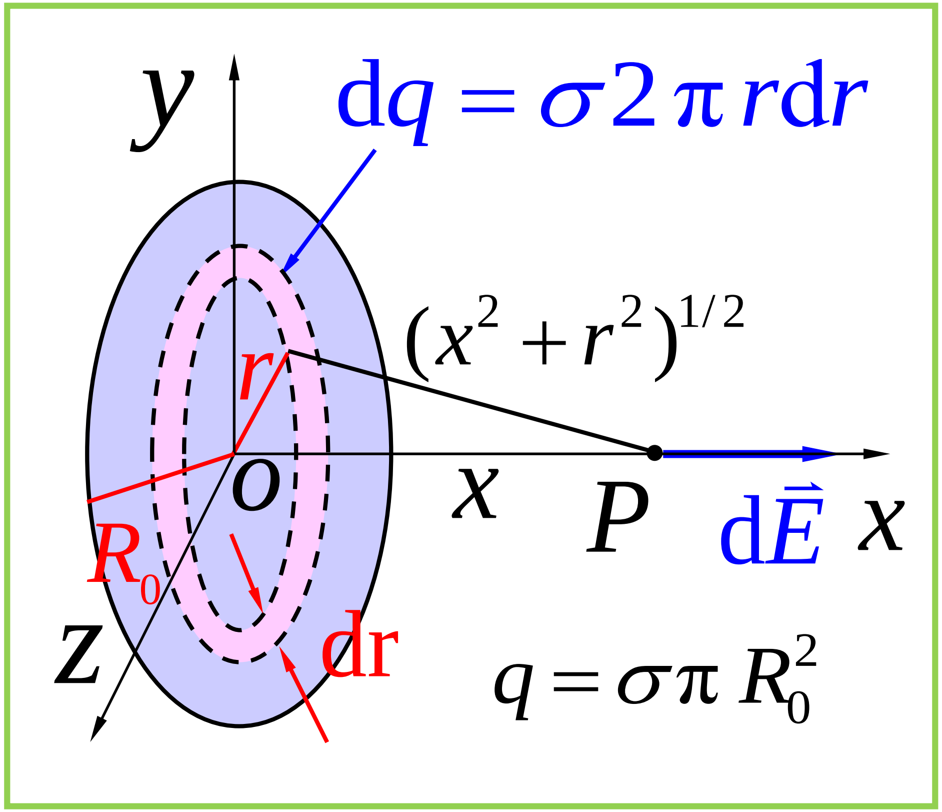
例2 有一半径为R，电荷均匀分布的薄圆盘，其电荷面密度为σ。求通过盘心且垂直盘面的轴线上任意一点处的电场强度。

解：

$$E=\frac{\boxed{q}\,x}{4\pi\varepsilon_0(x^2+r^2)^{3/2}}$$

$$\mathrm{d}E_x=\frac{\boxed{\mathrm{d}q}\cdot x}{4\pi\varepsilon_0(x^2+r^2)^{3/2}}$$

$$\mathrm{d}q=\sigma 2\pi r\mathrm{d}r$$



$$\mathrm{d}E_x=\frac{\sigma}{2\varepsilon_0}\frac{x r\mathrm{d}r}{(x^2+r^2)^{3/2}}=\frac{\sigma x}{2\varepsilon_0}\frac{r\mathrm{d}r}{(x^2+r^2)^{3/2}}$$

$$E=\int \mathrm{d}E_x=\frac{\sigma x}{2\varepsilon_0}\int_0^{R_0}\frac{r\mathrm{d}r}{(x^2+r^2)^{3/2}}$$

$$=\frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2}}-\frac{1}{\sqrt{x^2+R_0^2}}\right)$$

讨论：

(1) 若 $x << R_0$ $\qquad E\approx\frac{\sigma}{2\varepsilon_0}$ **无限大均匀带电平面外附近的电场强度**

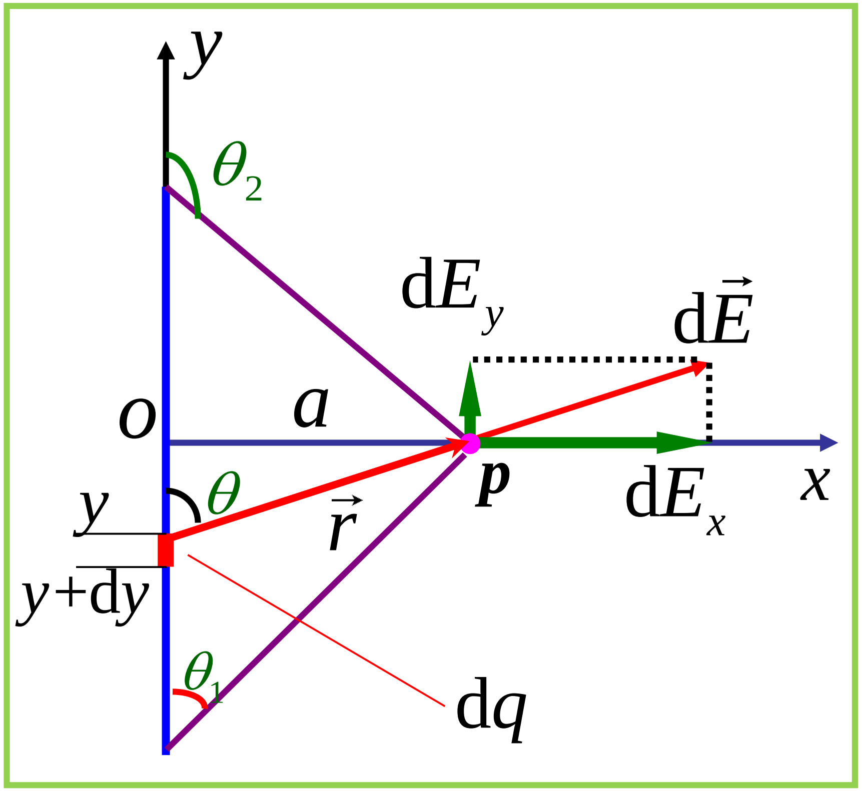
(2) 若 $x >> R_0$ **则** $\left(1+\frac{R_0^2}{x^2}\right)^{-\frac{1}{2}}=1-\frac{1}{2}\cdot\frac{R_0^2}{x^2}+\cdots$

$$E\approx\frac{q}{4\pi\varepsilon_0 x^2}$$

点电荷电场强度

例3 如图所示，求均匀带电直线周围电场分布。

解： 电荷的线密度为λ $\mathrm{d}q=\lambda \mathrm{d}y$



$$\mathrm{d}\vec{E}=\frac{\mathrm{d}q}{4\pi\varepsilon_0 r^2}\vec{r}_0=\frac{\lambda}{4\pi\varepsilon_0}\frac{\mathrm{d}y}{r^2}\vec{r}_0$$

$$\mathrm{d}E_x=\mathrm{d}E\sin\theta=\frac{\lambda}{4\pi\varepsilon_0}\frac{\mathrm{d}y}{r^2}\sin\theta$$

$$\mathrm{d}E_y=\mathrm{d}E\cos\theta=\frac{\lambda}{4\pi\varepsilon_0}\frac{\mathrm{d}y}{r^2}\cos\theta$$

$$r=\frac{a}{\sin\theta}\qquad y=-a\mathrm{ctg}\theta\qquad \therefore \mathrm{d}y=\frac{a\mathrm{d}\theta}{\sin^2\theta}$$

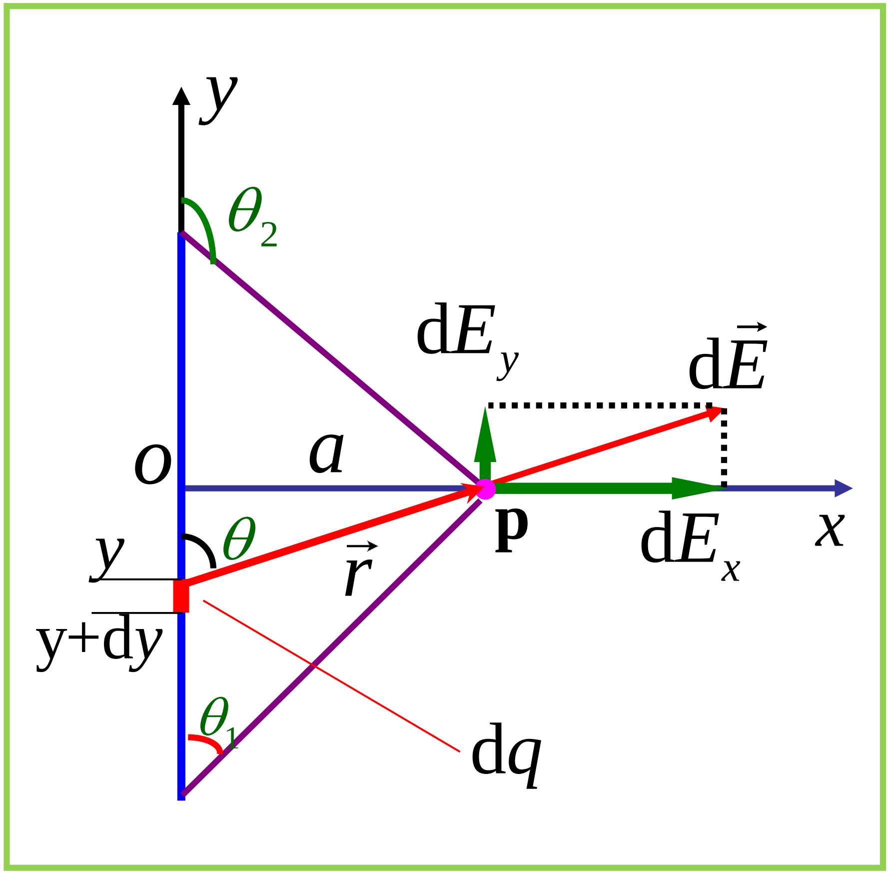
$$\mathrm{d}E_x=\frac{\lambda}{4\pi\varepsilon_0 a}\sin\theta\mathrm{d}\theta\qquad \mathrm{d}E_y=\frac{\lambda}{4\pi\varepsilon_0 a}\cos\theta\mathrm{d}\theta$$

$$E_x = \int \mathrm{d}E_x = \frac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2} \sin\theta \mathrm{d}\theta$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2)$$

$$E_y = \int \mathrm{d}E_y = \frac{\lambda}{4\pi\varepsilon_0 a}\int_{\theta_1}^{\theta_2} \cos\theta \mathrm{d}\theta$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$$



$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$$E_p = \sqrt{E_x^2 + E_y^2}$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0 a}(\cos\theta_1 - \cos\theta_2) \qquad E_y = \frac{\lambda}{4\pi\varepsilon_0 a}(\sin\theta_2 - \sin\theta_1)$$

讨论:

(1) 当 p 点落在带电直线的中垂线上时， $\theta_1 + \theta_2 = \pi$

$$E_y = 0$$

$$E_x = \frac{\lambda}{2\pi\varepsilon_0 a}\cos\theta_1$$

(2) 当带电直线为无限长时， $\theta_1 \to 0 \quad \theta_2 \to \pi$

$$E_y = 0$$

$$E_x = \frac{\lambda}{2\pi\varepsilon_0 a}$$

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 a}}$$