Numerical Methods, project A No. 25

1. Write a program finding *macheps* in the MATLAB environment on a lab computer or your computer.

2. Write a general program solving a system of n linear equations **Ax = b** using *the indicated method*. Using only elementary mathematical operations on numbers and vectors is allowed (command "A\b" cannot be used, except only for checking the results). Apply the program to solve the system of linear equations for given matrix **A** and vector **b**, for increasing numbers of equations n = 10,20,40,80,160,… until the solution time becomes prohibitive (or the method fails), for:

a) $a_{ij} = \begin{cases} 9 & for\ i = j \\ 1 & for\ i = j-1\ or\ i = j+1, \\ 0 & other\ cases \end{cases}$ $b_i = 1.4 + 0.6\,i,$ $i, j = 1,\ldots,n;$

b) $a_{ij} = 3/[4(i + j - 1)],$ $b_i = 1/i$, i – odd; $b_i = 0$, i – even, $i, j = 1,\ldots,n.$

For each case a) and b) calculate the solution error defined as the Euclidean norm of the vector of residuum **r = Ax–b,** where **x** is the solution, and plot this error versus n. For n = 10 print the solutions and the solutions' errors, make the residual correction and check if it improves the solutions.

The indicated method: Gaussian elimination with partial pivoting.

3. Write a general program for solving the system of n linear equations **Ax = b** using the Gauss-Seidel and Jacobi iterative algorithms. Apply it for the system:

$$8x_1 + 2x_2 - 3x_3 + x_4 = 7$$
$$2x_1 - 25x_2 + 5x_3 - 18x_4 = 12$$
$$x_1 + 3x_2 + 15x_3 - 8x_4 = 24$$
$$x_1 + x_2 - 2x_3 - 10x_4 = 28$$

and compare the results of iterations plotting norm of the solution error $\|\mathbf{Ax}_k - \mathbf{b}\|_2$ versus the iteration number k=1,2,3,… until the assumed accuracy $\|\mathbf{Ax}_k - \mathbf{b}\|_2 < 10^{-10}$ is achieved. Try to solve the equations from problem 2a) and 2b) for n=10 using a chosen iterative method.

4. Write a program of the QR method for finding eigenvalues of 5×5 matrices:
a) without shifts;
b) with shifts calculated on the basis of an eigenvalue of the 2×2 right-lower-corner submatrix.
Apply and compare both approaches for a chosen symmetric matrix 5×5 in terms of numbers of iterations needed to force all off-diagonal elements below the prescribed absolute value threshold 10^{-6}, print initial and final matrices. Elementary operations only permitted, commands "qr" or "eig" must not be used (except for checking the results).

All calculations should be done using MATLAB, the report should contain:
- brief description of the algorithms used and printout of programs (commented),
- presentation of results, as required above, with comments/conclusions.

The report is to be uploaded in the PDF format to the 'Reports' module on the 'Studia' server by Nov. 12, 2 p.m. Project A carries 15 points. Each day of delay results in deduction of 1 point.