Centralized and Decentralized Fault Diagnosis of Partially Observable Discrete-Event Systems

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The proofs of the following lemmas are omitted because they follow directly from the definitions of $\hat{\mathbf{G}}_I^{\sigma_{o0}}$ and $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$.

Lemma 1. *For any path* $p = q_{-1} \xrightarrow{\sigma_{o0}} (q_0, \ell_0) \xrightarrow{\sigma_{o1}} (q_1, \ell_1) \cdots (q_{k-1}, \ell_{k-1}) \xrightarrow{\sigma_{ok}} (q_k, \ell_k) \cdots (q_n, \ell_n) \xrightarrow{\sigma_{o(n+1)}} (q_k, \ell_k)$ *in* $\hat{\mathbf{G}}_I^{\sigma_{o0}}$ *ending with a cycle, The following hold:*

1) $\ell_j = \ell_r$ *for any* $j,\ r \in [k, n]$*;*
2) *There exists a path* $\bar{p} = \bar{q}_0 \xrightarrow{\sigma_1} \bar{q}_1 \xrightarrow{\sigma_2} \bar{q}_2 \cdots \bar{q}_{b-1} \xrightarrow{\sigma_b} \bar{q}_b \cdots \bar{q}_c \xrightarrow{\sigma_c} \bar{q}_b$ *in* $\mathbf{G}$ *such that* $\mathsf{P}(\sigma_1\sigma_2\cdots\sigma_b\cdots\sigma_c) = \sigma_{o0}\sigma_{o1}\cdots\sigma_{ok}\cdots\sigma_{o(n+1)}$ *and* $Q_p \subseteq Q_{\bar{p}}$*, where* Q_p *and* $Q_{\bar{p}}$ *are the set of states composed of paths* p *and* $\bar{p}$*, respectively.*

In a cycle, all the pairs have the same condition labels, which are either normal or F_i ($i \in [1, m]$), due to the assumption that all failure modes are permanent. We know that $\hat{\mathbf{G}}_I^{\sigma_{o0}}$ is defined on the basis of $\hat{\mathbf{G}}^{\sigma_{o0}}$ and $\hat{\mathbf{G}}^{\sigma_{o0}}$ derives from $\mathbf{G}$. Thus, there exists a transition path $\bar{p}$ in $\mathbf{G}$ corresponding to the path p with fault information in $\hat{\mathbf{G}}_I^{\sigma_{o0}}$.

Lemma 2. *For any path* $p = (q_{-1}, q_{-1}) \xrightarrow{\sigma_{o0}} (q^1_{o0}, q^2_{o0}) \xrightarrow{\sigma_{o1}} (q^1_{o1}, q^2_{o1}) \cdots (q^1_{o(k-1)}, q^2_{o(k-1)}) \xrightarrow{\sigma_{ok}} (q^1_{ok}, q^2_{ok}) \cdots (q^1_{on}, q^2_{on}) \xrightarrow{\sigma_{o(n+1)}} (q^1_{ok}, q^2_{ok})$ *with* $q^1_{ot} = (q^1_{\mathbf{n}t}, \ell^1_t) \in \hat{Q}_{\mathbf{n}}$ *and* $q^2_{ot} = (q^2_{\mathbf{f}t}, \ell^2_t) \in \hat{Q}_{\mathbf{f}}$ *(*$0 \le t \le n$*) in* $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$ *ending with a cycle, the following hold:*

1) *There exist a path* $p^1_{\mathbf{n}}$ *in* $\hat{\mathbf{G}}_{\mathbf{n}}^{\sigma_{o0}}$ *and a path* $p^2_{\mathbf{f}}$ *in* $\hat{\mathbf{G}}_{\mathbf{f}}^{\sigma_{o0}}$ *ending with cycles, namely,* $p^1_{\mathbf{n}} = q_{-1} \xrightarrow{\sigma_{o0}} q^1_{o0} \xrightarrow{\sigma_{o1}} q^1_{o1} \cdots q^1_{o(k-1)} \xrightarrow{\sigma_{ok}} q^1_{ok} \cdots q^1_{on} \xrightarrow{\sigma_{o(n+1)}} q^1_{ok}$ *and* $p^2_{\mathbf{f}} = q_{-1} \xrightarrow{\sigma_{o0}} q^2_{o0} \xrightarrow{\sigma_{o1}} q^2_{o1} \cdots q^2_{o(k-1)} \xrightarrow{\sigma_{ok}} q^2_{ok} \cdots q^2_{on} \xrightarrow{\sigma_{o(n+1)}} q^2_{ok}$*;*
2) $\ell^1_j = \ell^1_r = \{N\}$ *and* $\ell^2_j = \ell^2_r$ *for any* $j,\ r \in [k, n]$*.*

Based on Lemmas 1 and 2. the proof of Theorem 1 is shown as follows.

Proof. (only if): Suppose $\mathbf{G}$ is F_i-asynchronously diagnosable w.r.t. Σ_f and Σ_o, but there exists a cycle $cl = (q^1_{ok}, q^2_{ok}) \xrightarrow{\sigma_{o(k+1)}} (q^1_{o(k+1)}, q^2_{o(k+1)}) \cdots (q^1_{on}, q^2_{on}) \xrightarrow{\sigma_{o(n+1)}} (q^1_{ok}, q^2_{ok})$ with $q^1_{oj} = (q^1_{\mathbf{n}j}, \ell^1_j) \in \hat{Q}_{\mathbf{n}}$ and $q^2_{oj} = (q^2_{\mathbf{f}j}, \ell^2_j) \in \hat{Q}_{\mathbf{f}}$ ($0 \le k \le j \le n$) in $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$ such that $\ell^2_j = \{F_i\}$ ($i \in [1, m]$).

Since $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$ is reachable, there exists a path p in $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$ ending with the cycle cl, i.e., $p = (q_{-1}, q_{-1}) \xrightarrow{\sigma_{o0}} (q^1_{o0}, q^2_{o0}) \xrightarrow{\sigma_{o1}} (q^1_{o1}, q^2_{o1}) \cdots (q^1_{o(k-1)}, q^2_{o(k-1)}) \xrightarrow{\sigma_{ok}} (q^1_{ok}, q^2_{ok}) \cdots (q^1_{on}, q^2_{on}) \xrightarrow{\sigma_{o(n+1)}} (q^1_{ok}, q^2_{ok})$. Based on Lemma 2, we know that there exist one path $p^1_{\mathbf{n}}$ in $\hat{\mathbf{G}}_{\mathbf{n}}^{\sigma_{o0}}$ and one path $p^2_{\mathbf{f}}$ in $\hat{\mathbf{G}}_{\mathbf{f}}^{\sigma_{o0}}$ ending with cycles, namely, $p^1_{\mathbf{n}} = q_{-1} \xrightarrow{\sigma_{o0}} q^1_{o0} \xrightarrow{\sigma_{o1}} q^1_{o1} \cdots q^1_{o(k-1)} \xrightarrow{\sigma_{ok}} q^1_{ok} \cdots q^1_{on} \xrightarrow{\sigma_{o(n+1)}} q^1_{ok}$ and $p^2_{\mathbf{f}} = q_{-1} \xrightarrow{\sigma_{o0}} q^2_{o0} \xrightarrow{\sigma_{o1}} q^2_{o1} \cdots q^2_{o(k-1)} \xrightarrow{\sigma_{ok}} q^2_{ok} \cdots q^2_{on} \xrightarrow{\sigma_{o(n+1)}} q^2_{ok}$ with $\ell^1_j = \ell^1_r = \{N\}$ and $\ell^2_j = \ell^2_r = \{F_i\}$ for any $j,\ r \in [k, n]$. Further from Lemma 1, we know that for the path $p^2_{\mathbf{f}}$ there exists a path $\bar{p}^2 = \bar{q}^2_0 \xrightarrow{\sigma_1} \bar{q}_1 \xrightarrow{\sigma_2} \bar{q}^2_2 \cdots \bar{q}^2_{b-1} \xrightarrow{\sigma_b} \bar{q}^2_b \cdots \bar{q}_c \xrightarrow{\sigma_c} \bar{q}^2_b$ in $\mathbf{G}$ such that $\mathsf{P}(\sigma_1\sigma_2\cdots\sigma_b\cdots\sigma_c) = \sigma_{o0}\sigma_{o1}\cdots\sigma_{ok}\cdots\sigma_{o(n+1)}$ and $Q_{p^2_{\mathbf{f}}} \subseteq Q_{\bar{p}^2}$.

Let $\mathcal{Q}_e(q, n')$ denote the set of state estimations calculated after the occurrence of n' events from state q. Suppose the state estimation $x_{0e} \in \mathcal{Q}_e(\bar{q}^2_s, 0)$ ($s \in [1, b]$), where $\bar{q}^2_s$ is the first faulty state in the path $\bar{p}^2$, is given. When the system evolves along the path $\bar{p}^2$, there exists a state estimation $x_e^{n''} \in \mathcal{Q}_e(\bar{q}^2_s, n'')$ ($n'' = d + m_2 * k_2$, $d = b - s$, and $m_2 = c - b + 1$ is the length of $\bar{q}^2_b \cdots \bar{q}^2_c$) such that $q^1_k \in x_e^{n''}$ and $q^2_k \in x_e^{n''}$ for any nonnegative integer k_2. Then $\mathbf{D}(x_e^{n''}) = -1$. Since $\mathbf{G}$ is asynchronously diagnosable, there exists an integer N_i such that for any $x_e^{n'} \in \mathcal{Q}(q^2_s, n')$, $x_e^{n'} \subseteq Q_{F_i}$ holds for $n' \ge N_i$. We choose an integer k_2 such that $n'' \ge N_i$. Then we have that $x_e^{n''} \subseteq Q_{F_i}$, i.e., $\mathbf{D}(x_e^{n''}) = i$, which leads to a contraction. So the necessity holds.

(if): Suppose for every cycle $cl = (q^1_{ok}, q^2_{ok}) \xrightarrow{\sigma_{o(k+1)}} (q^1_{o(k+1)}, q^2_{o(k+1)}) \cdots (q^1_{on}, q^2_{on}) \xrightarrow{\sigma_{o(n+1)}} (q^1_{ok}, q^2_{ok})$ with $q^1_{oj} = (q^1_{\mathbf{n}j}, \ell^1_j) \in \hat{Q}_{\mathbf{n}}$ and $q^2_{oj} = (q^2_{\mathbf{f}j}, \ell^2_j) \in \hat{Q}_{\mathbf{f}}$ ($0 \le k \le j \le n$), we have $\ell^1_j = \ell^2_j = \{N\}$ ($j \in [k, n]$). From the second clause of Lemma 2, we can infer that for any $q^d = ((q^1, \ell^1), (q^2, \ell^2))$ in $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$, q^d is not contained in a loop if $\ell^1 \ne \ell^2$, which further implies that for any path $p = q^d_1 \xrightarrow{\sigma_1} q^d_2 \cdots \xrightarrow{\sigma_l} q^d_l$ in $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$ with $q^d_r = ((q^1_r, \ell^1_r), (q^2_r, \ell^2_r))$ ($r \in [1, l]$) if $\ell^1_r \ne \ell^2_r$, then the length of this path is finite, which is less than the number of states in $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$.

Suppose the state estimation $x_{0e} \in \mathcal{Q}_e(q_{F_i}, 0)$ ($i \in [1, m]$) is given when the system first reaches the fault state q_{F_i}. For any $x_e^{n'} \in \mathcal{Q}_e(q_{F_i}, n')$ with $n' > |\hat{Q}_d| \times (|\Sigma| - 1)$, we claim that $\mathbf{D}(x_e^{n'}) = i$. After n' transitions from state q_{F_i}, we have the path $p' = q_{F_i} \xrightarrow{\sigma_1} q^1_{F_i} \cdots \xrightarrow{\sigma_{n'}} q^{n'}_{F_i}$ with the observed event sequence $s_o = \mathsf{P}(\sigma_1\sigma_2\cdots\sigma_{n'})$ ($n'' = |s_o|$ is the length of s_o). From above, for any state $q^d \in \hat{Q}_d$ that can be reached from (q_{-1}, q_{-1}) in $\hat{\mathbf{G}}_{dc}^{\sigma_{o0}}$, we have that for any path starting from q^d, a state $\hat{q}^d = ((\hat{q}^1, \hat{\ell}^1), (\hat{q}^2, \hat{\ell}^2)) \in \hat{Q}_d$ with $\hat{\ell}^1 = \hat{\ell}^2$ can be reached within $|\hat{Q}_d| - 1$ transitions. This implies that for any $n'' > |\hat{Q}_d| - 1$ and $x_e^{n''} \in \mathcal{Q}_e(q_{F_i}, n'')$, we have that $\mathcal{D}(x_e^{n''}) = i$. From the assumption in Remark 2, each observed event can be followed by at most $|\Sigma| - 1$ unobserved events. It follows that for the above path p', $n' \le (n'' + 1) \times (|Q| - 1)$, i.e., $n'' \ge n'/(|\Sigma| - 1) - 1$. So if $n' > |\hat{Q}_d| \times (|\Sigma| - 1)$, then $n'' \ge n'/(|Q| - 1) - 1 > |\hat{Q}_d| - 1$, establishing our claim.

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Note that we have assumed implicitly that $|Q| > 1$; otherwise if $|Q| = 1$, then from the assumption of no path cycles, no transition labeled by a failure event exists, so that the system is trivially diagnosable. Based on Definition 7, we can conclude that $\mathbf{G}$ is diagnosable w.r.t, Σ_f and Σ_o. So the sufficiency also holds. $\square$