Asynchronous Fault Diagnosis of Discrete-Event Systems With Partially Observable Outputs

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APPENDIX

The proofs of the following lemmas are omitted because they follow directly from the definitions of $\hat{\mathcal{D}}_{oc}$ and $\hat{\mathcal{D}}_{dc}$.

Lemma 1. *For any path $p = \{(q_{-1}, l_0, q_0), (q_0, l_1, q_1), \ldots, (q_{k-1}, l_k, q_k), \ldots, (q_n, l_k, q_k)\}$ in $\hat{\mathcal{D}}_{oc}$ ending with a cycle, The following hold:*

1) *$\kappa(q_j) = \kappa(q_r)$ for any $j,\ r \in [k, n]$;*
2) *There exists a path $\bar{p} = \{(\bar{q}_0, \sigma_1, \bar{q}_1), (\bar{q}_1, \sigma_2, \bar{q}_2), \ldots, (\bar{q}_{b-1}, \sigma_b, \bar{q}_b), \ldots, (\bar{q}_c, \sigma_c, \bar{q}_b)\}$ in $\mathbf{G}$ such that $O(\lambda(\bar{q}_0\bar{q}_1 \cdots \bar{q}_c\bar{q}_b^1)) = l_0l_1 \cdots l_nl_k$ and $Q_p \subseteq Q_{\bar{p}}$, where Q_p and $Q_{\bar{p}}$ are the set of states composed of paths p and $\bar{p}$, respectively.*

In a cycle, all states correspond to the same condition, which is either normal or F_i $(i \in [1, m])$, owing to the assumption that all failure modes are permanent. We know that $\hat{\mathcal{D}}_{oc}$ is defined on the basis of $\hat{\mathcal{D}}_{ds}$ and $\hat{\mathcal{D}}_{ds}$ derives from $\mathbf{G}$. Thus, there exists a transition path $\bar{p}$ in $\mathbf{G}$ corresponding to the indication path p in $\hat{\mathcal{D}}_{oc}$.

Lemma 2. *For any path $p = \{((q_{-1}, q_{-1}),\ l_0,\ (q_0^1, q_0^2)), ((q_0^1, q_0^2),\ l_1,\ (q_1^1, q_1^2)), \ldots, ((q_{k-1}^1,\ q_{k-1}^2),\ l_k,\ (q_k^1, q_k^2)), \ldots, ((q_n^1, q_n^2), l_k, (q_k^1, q_k^2))\}$ $(k < n)$ in $\hat{\mathcal{D}}_{dc}$ ending with a cycle, the following hold:*

1) *There exist a path p_{noc}^1 in $\hat{\mathcal{D}}_{noc}$ and a path p_{foc}^2 in $\hat{\mathcal{D}}_{foc}$ ending with cycles, namely $p_{noc}^1 = \{(q_{-1}, l_0, q_0^1), (q_0^1, l_1, q_1^1), \ldots, (q_{k-1}^1,\ l_k,\ q_k^1), \ldots, (q_n^1,\ l_k,\ q_k^1)\}$ and $p_{foc}^2 = \{(q_{-1},\ l_0,\ q_0^2), (q_0^2,\ l_1,\ q_1^2), \ldots, (q_{k-1}^2,\ l_k,\ q_k^2), \ldots, (q_n^2,\ l_k,\ q_k^2)\}$;*
2) *$\kappa(q_j^1) = \kappa(q_r^1) = 0$ and $\kappa(q_j^2) = \kappa(q_r^2)$ for any $j,\ r \in [k, n]$.*

$\diamondsuit$

Based on Lemmas 1 and 2. the proof of Theorem 1 is shown as follows.

Proof. (only if): Suppose $\mathbf{G}$ is asynchronously diagnosable w.r.t. $l_0 \in \Lambda_o$, but there exists a cycle cl in $\hat{\mathcal{D}}_{dc}$, $cl = \{((q_k^1, q_k^2),\ l_{k+1},\ (q_{k+1}^1, q_{k+1}^2)), \ldots, ((q_n^1, q_n^2),\ l_k,\ ((q_k^1, q_k^2))\}$ $(k < n)$ such that $\kappa(q_j^2) = i \in [1, m]$ $(j \in [k, n])$.

Since $\hat{\mathcal{D}}_{dc}$ is reachable, there exists a path p in $\hat{\mathcal{D}}_{dc}$ ending with the cycle cl, i.e., $p = \{((q_{-1}, q_{-1}),\ l_0,\ (q_0^1, q_0^2)), ((q_0^1, q_0^2),\ l_1,\ (q_1^1, q_1^2)), \ldots, ((q_{k-1}^1,\ q_{k-1}^2),\ l_k,\ (q_k^1, q_k^2)), \ldots, ((q_n^1, q_n^2), l_k, (q_k^1, q_k^2))\}$. Based on Lemma 2, we know that there exist one path p_{noc}^1 in $\hat{\mathcal{D}}_{noc}$ and one path p_{foc}^2 in $\hat{\mathcal{D}}_{foc}$ ending with cycles, namely $p_{noc}^1 = \{(q_{-1}, l_0, q_0^1), (q_0^1, l_1, q_1^1), \ldots, (q_{k-1}^1, l_k, q_k^1), \ldots, (q_n^1, l_k, q_k^1)\}$ and $p_{foc}^2 = \{(q_{-1}, l_0, q_0^2), (q_0^2, l_1, q_1^2), \ldots, (q_{k-1}^2, l_k, q_k^2), \ldots, (q_n^2, l_k, q_k^2)\}$ and $\kappa(q_j^1) = \kappa(q_r^1) = 0$ and $\kappa(q_j^2) = \kappa(q_r^2) = i$ for any $j,\ r \in [k, n]$. Further from Lemma 1, we know that for the path p_{noc}^1 there exists a path $\bar{p}^2 = \{(\bar{q}_0^2, \sigma_1^2, \bar{q}_1^2), (\bar{q}_1^2, \sigma_2^2, \bar{q}_2^2), \ldots, (\bar{q}_{b-1}^2, \sigma_b^2, \bar{q}_b^2), \ldots, (\bar{q}_c^2, \sigma_b, \bar{q}_b^2)\}$ in $\mathbf{G}$ such that $O(\lambda(\bar{q}_0^2\bar{q}_1^2 \cdots \bar{q}_c^2\bar{q}_b^2)) = l_0l_1 \cdots l_nl_k$ and $Q_{p_{foc}^2} \subseteq Q_{\bar{p}^2}$.

Let $\mathcal{Q}_e(q, n)$ denote the set of state estimations calculated after the occurrence of n events from state q. Suppose the state estimation $x_{0e} \in \mathcal{Q}_e(\bar{q}_s^2, 0)$ $(s \in [1, b])$, where $\bar{q}_s^2$ $(\kappa(\bar{q}_s^2) = i)$ is the first fault state in the path $\bar{p}^2$, is given. When the system evolves along the path $\bar{p}^2$, there exists a state estimation $x_e^{n'} \in \mathcal{Q}_e(\bar{q}_s^2, n')$ $(n' = d + m_2 * k_2$, $d = b - s$, and $m_2 = c - b + 1$ is the length of $\bar{q}_b^1 \cdots \bar{q}_c^1)$ such that $q_k^1 \in x_e^{n'}$ and $q_k^2 \in x_e^{n'}$ for any nonnegative integer k_2. Then $\mathbf{D}(x_e^{n'}) = -1$. Since $\mathbf{G}$ is asynchronously diagnosable w.r.t. l_0, there exists an integer N_i such that for any $x_e^n \in \mathcal{Q}(q_s^2, n)$, $x_e^n \subseteq Q_{F_i}$ holds for $n \geq N_i$. We choose an integer k_2 such that $n' \geq N_i$. Then we have that $x_e^{n'} \subseteq Q_{F_i}$, i.e., $\mathbf{D}(x_e^{n'}) = i$, which leads to a contraction. So the necessity holds.

(if): Suppose for every cycle $cl = \{((q_k^1, q_k^2),\ l_{k+1},\ (q_{k+1}^1, q_{k+1}^2)), \ldots, ((q_n^1, q_n^2),\ l_k,\ ((q_k^1, q_k^2))\}$ $(k < n)$ in $\hat{\mathcal{D}}_{dc}$, we have $\kappa(q_j^1) = \kappa(q_j^2) = 0$ $(j \in [k, n])$. From the second clause of Lemma 2, we can infer that for any $q^d = (q^1, q^2)$ in $\hat{\mathcal{D}}_{dc}$, q^d is not contained in a loop if $\kappa(q^1) \neq \kappa(q^2)$, which further implies that for any state sequence $(q_1^d, q_2^d, \ldots, q_k^d)$ in $\hat{\mathcal{D}}_{dc}$ with $q_r^d = (q_r^1, q_r^2)$ $(r \in [1, k])$ if $\kappa(q_r^1) \neq \kappa(q_r^2)$, then the length of this state sequence is finite, which is less than the number of states in $\hat{\mathcal{D}}_{dc}$.

Suppose the state estimation $x_{0e} \in \mathcal{Q}_e(q_{F_i}, 0)$ $(i \in [1, m])$ is given when the system first reach the fault state q_{F_i} $(\kappa(q_{F_i}) = i)$. For any $x_e^n \in \mathcal{Q}_e(q_{F_i}, n)$ with $n > |\hat{Q}_d| \times (|Q| - 1)$, we claim that $\mathbf{D}(x_e^n) = i$. After n transitions from state q_{F_i}, we have the state sequence $s = q_{F_i}q_{F_i}^1 \cdots q_{F_i}^n$ with the observed output sequence $O(\lambda(s))$ $(n' = |O(\lambda(s))|)$. From above, for any state $q^d \in \hat{Q}_d$ that can be reached from (q_{-1}, q_{-1}) in $\hat{\mathcal{D}}_{dc}$, we have that for any state sequence starting from q^d, a state $\hat{q}^d = (\hat{q}^1, \hat{q}^2) \in \hat{Q}_d$ with $\kappa(\hat{q}^1) = \kappa(\hat{q}^2)$ can be reached within $|\hat{Q}_d| - 1$ transitions. This implies that for any $n' > |\hat{Q}_d| - 1$ and $x_e^{n'} \in \mathcal{Q}_e(q_{F_i}, n')$, we have that $\mathcal{D}(x_e^{n'}) = i$. From the assumption in Remark 1, each observed output can be followed by at most $|Q| - 1$ unobserved outputs. It follows that for the above state sequence s, $n \leq (n' + 1) \times (|Q| - 1)$,

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i.e., $n' \geq n/(|Q|-1)-1$. So if $n > |\hat{Q}_d| \times (|Q|-1)$, then $n' \geq n/(|Q|-1)-1 > |\hat{Q}_d|-1$, establishing our claim. Note that we have assumed implicitly that $|Q| > 1$; otherwise if $|Q| = 1$, then from the assumption of no path cycles, no transition labeled by a failure event exists, so that the system is trivially diagnosable. Based on Definition 9, we can conclude that $\mathbf{G}$ is diagnosable w.r,t, l_0. So the sufficiency also holds. $\square$