Asynchronous Fault Diagnosis of Discrete-Event Systems With Partially Observable Outputs

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The proofs of the following lemmas are omitted because they follow directly from the definitions of $\hat{\mathcal{D}}^{l_o^0}_{oc}$ and $\hat{\mathcal{D}}^{l_o^0}_{dc}$.

Lemma 1. *For any path $p = q_{-1} \xrightarrow{l_o^0} q_0 \xrightarrow{l_o^1} q_1 \cdots q_{k-1} \xrightarrow{l_o^k} q_k \cdots q_n \xrightarrow{l_o^{n+1}} q_k$ in $\hat{\mathcal{D}}^{l_o^0}_{oc}$ ending with a cycle, The following hold:*

1) *$\kappa(q_j) = \kappa(q_r)$ for any j, $r \in [k, n]$;*
2) *There exists a path $\bar{p} = \bar{q}_0 \xrightarrow{\sigma_1} \bar{q}_1 \xrightarrow{\sigma_2} \bar{q}_2 \cdots \bar{q}_{b-1} \xrightarrow{\sigma_b} \bar{q}_b \cdots \bar{q}_c \xrightarrow{\sigma_c} \bar{q}_b$ in $\mathbf{G}$ such that $O(\lambda(\bar{q}_0\bar{q}_1 \cdots \bar{q}_c\bar{q}_b)) = l_o^0 l_o^1 \cdots l_o^n l_o^k$ and $Q_p \subseteq Q_{\bar{p}}$, where Q_p and $Q_{\bar{p}}$ are the set of states composed of paths p and $\bar{p}$, respectively.*

In a cycle, all states correspond to the same condition, which is either normal or F_i $(i \in [1, m])$, owing to the assumption that all failure modes are permanent. We know that $\hat{\mathcal{D}}^{l_o^0}_{oc}$ is defined on the basis of $\hat{\mathcal{D}}^{l_o^0}_{dm}$ and $\hat{\mathcal{D}}^{l_o^0}_{dm}$ derives from $\mathbf{G}$. Thus, there exists a transition path $\bar{p}$ in $\mathbf{G}$ corresponding to the indication path p in $\hat{\mathcal{D}}^{l_o^0}_{oc}$.

Lemma 2. *For any path $p = (q_{-1}, q_{-1}) \xrightarrow{l_o^0} (q_0^1, q_0^2) \xrightarrow{l_o^1} (q_1^1, q_1^2) \cdots (q_{k-1}^1, q_{k-1}^2) \xrightarrow{l_o^k} (q_k^1, q_k^2) \cdots (q_n^1, q_n^2) \xrightarrow{l_o^{n+1}} (q_k^1, q_k^2)$ $(k < n)$ in $\hat{\mathcal{D}}^{l_o^0}_{dc}$ ending with a cycle, the following hold:*

1) *There exist a path p^1_{noc} in $\hat{\mathcal{D}}_{\mathsf{noc}}$ and a path p^2_{foc} in $\hat{\mathcal{D}}_{\mathsf{foc}}$ ending with cycles, namely, $p^1_{\mathsf{noc}} = q_{-1} \xrightarrow{l_o^0} q_0^1 \xrightarrow{l_o^1} q_1^1 \cdots q_{k-1}^1 \xrightarrow{l_o^k} q_k^1 \cdots q_n^1 \xrightarrow{l_o^{n+1}} q_k^1$ and $p^2_{\mathsf{foc}} = q_{-1} \xrightarrow{l_o^0} q_0^2 \xrightarrow{l_o^1} q_1^2 \cdots q_{k-1}^2 \xrightarrow{l_o^k} q_k^2 \cdots q_n^2 \xrightarrow{l_o^{n+1}} q_k^2$;*
2) *$\kappa(q_j^1) = \kappa(q_r^1) = N$ and $\kappa(q_j^2) = \kappa(q_r^2)$ for any j, $r \in [k, n]$.*

Based on Lemmas 1 and 2. the proof of Theorem 1 is shown as follows.

Proof. (only if): Suppose $\mathbf{G}$ is F_i-asynchronously diagnosable w.r.t. $l_o^0 \in \Lambda_o$, but there exists a cycle cl in $\hat{\mathcal{D}}^{l_o^0}_{dc}$, $cl = (q_k^1, q_k^2) \xrightarrow{l_o^{k+1}} (q_{k+1}^1, q_{k+1}^2) \cdots (q_n^1, q_n^2) \xrightarrow{l_o^{n+1}} (q_k^1, q_k^2)$ $(k < n)$ such that $\kappa(q_j^2) = F_i$ $(i \in [1, m], j \in [k, n])$.

Since $\hat{\mathcal{D}}^{l_o^0}_{dc}$ is reachable, there exists a path p in $\hat{\mathcal{D}}^{l_o^0}_{dc}$ ending with the cycle cl, i.e., $p = (q_{-1}, q_{-1}) \xrightarrow{l_o^0} (q_0^1, q_0^2) \xrightarrow{l_o^1} (q_1^1, q_1^2) \cdots (q_{k-1}^1, q_{k-1}^2) \xrightarrow{l_o^k} (q_k^1, q_k^2) \cdots (q_n^1, q_n^2) \xrightarrow{l_o^{n+1}} (q_k^1, q_k^2)$. Based on Lemma 2, we know that there exist one path p^1_{noc} in $\hat{\mathcal{D}}_{\mathsf{noc}}$ and one path p^2_{foc} in $\hat{\mathcal{D}}_{\mathsf{foc}}$ ending with cycles, namely, $p^1_{\mathsf{noc}} = q_{-1} \xrightarrow{l_o^0} q_0^1 \xrightarrow{l_o^1} q_1^1 \cdots q_{k-1}^1 \xrightarrow{l_o^k} q_k^1 \cdots q_n^1 \xrightarrow{l_o^{n+1}} q_k^1$ and $p^2_{\mathsf{foc}} = q_{-1} \xrightarrow{l_o^0} q_0^2 \xrightarrow{l_o^1} q_1^2 \cdots q_{k-1}^2 \xrightarrow{l_o^k} q_k^2 \cdots q_n^2 \xrightarrow{l_o^{n+1}} q_k^2$ with $\kappa(q_j^1) = \kappa(q_r^1) = 0$ and $\kappa(q_j^2) = \kappa(q_r^2) = i$ for any j, $r \in [k, n]$. Further from Lemma 1, we know that for the path p^1_{foc} there exists a path $\bar{p}^2 = \bar{q}_0^2 \xrightarrow{\sigma_1^2} \bar{q}_1^2 \xrightarrow{\sigma_2^2} \bar{q}_2^2 \cdots \bar{q}_{b-1}^2 \xrightarrow{\sigma_b^2} \bar{q}_b^2 \cdots \bar{q}_c^2 \xrightarrow{\sigma_c^2} \bar{q}_b^2$ in $\mathbf{G}$ such that $O(\lambda(\bar{q}_0^2\bar{q}_1^2 \cdots \bar{q}_c^2\bar{q}_b^2)) = l_o^0 l_o^1 \cdots l_o^n l_o^k$ and $Q_{p^2_{\mathsf{foc}}} \subseteq Q_{\bar{p}^2}$.

Let $\mathcal{Q}_e(q, n)$ denote the set of state estimations calculated after the occurrence of n events from state q. Suppose the state estimation $x_{0e} \in \mathcal{Q}_e(\bar{q}_s^2, 0)$ $(s \in [1, b])$, where $\bar{q}_s^2$ $(\kappa(\bar{q}_s^2) = F_i)$ is the first faulty state in the path $\bar{p}^2$, is given. When the system evolves along the path $\bar{p}^2$, there exists a state estimation $x_e^{n'} \in \mathcal{Q}_e(\bar{q}_s^2, n')$ $(n' = d + m_2 * k_2$, $d = b - s$, and $m_2 = c - b + 1$ is the length of $\bar{q}_b^1 \cdots \bar{q}_c^1)$ such that $q_k^1 \in x_e^{n'}$ and $q_k^2 \in x_e^{n'}$ for any nonnegative integer k_2. Then $\mathbf{D}(x_e^{n'}) = -1$. Since $\mathbf{G}$ is asynchronously diagnosable w.r.t. l_o^0, there exists an integer N_i such that for any $x_e^n \in \mathcal{Q}(q_s^2, n)$, $x_e^n \subseteq Q_{F_i}$ holds for $n \geq N_i$. We choose an integer k_2 such that $n' \geq N_i$. Then we have that $x_e^{n'} \subseteq Q_{F_i}$, i.e., $\mathbf{D}(x_e^{n'}) = i$, which leads to a contraction. So the necessity holds.

(if): Suppose for every cycle $cl = (q_k^1, q_k^2) \xrightarrow{l_o^{k+1}} (q_{k+1}^1, q_{k+1}^2) \cdots (q_n^1, q_n^2) \xrightarrow{l_o^{n+1}} (q_k^1, q_k^2)$ $(k < n)$ in $\hat{\mathcal{D}}^{l_o^0}_{dc}$, we have $\kappa(q_j^1) = \kappa(q_j^2) = N$ $(j \in [k, n])$. From the second clause of Lemma 2, we can infer that for any $q^d = (q^1, q^2)$ in $\hat{\mathcal{D}}^{l_o^0}_{dc}$, q^d is not contained in a loop if $\kappa(q^1) \neq \kappa(q^2)$, which further implies that for any state sequence $q_1^d q_2^d \cdots q_k^d$ in $\hat{\mathcal{D}}^{l_o^0}_{dc}$ with $q_r^d = (q_r^1, q_r^2)$ $(r \in [1, k])$ if $\kappa(q_r^1) \neq \kappa(q_r^2)$, then the length of this state sequence is finite, which is less than the number of states in $\hat{\mathcal{D}}^{l_o^0}_{dc}$.

Suppose the state estimation $x_{0e} \in \mathcal{Q}_e(q_{F_i}, 0)$ $(i \in [1, m])$ is given when the system first reaches the fault state q_{F_i} $(\kappa(q_{F_i}) = F_i)$. For any $x_e^n \in \mathcal{Q}_e(q_{F_i}, n)$ with $n > |\hat{Q}_d| \times (|Q| - 1)$, we claim that $\mathbf{D}(x_e^n) = i$. After n transitions from state q_{F_i}, we have the state sequence $s = q_{F_i} q_{F_i}^1 \cdots q_{F_i}^n$ with the observed output sequence $O(\lambda(s))$ $(n' = |O(\lambda(s))|)$. From above, for any state $q^d \in \hat{Q}_d$ that can be reached from (q_{-1}, q_{-1}) in $\hat{\mathcal{D}}^{l_o^0}_{dc}$, we have that for any state sequence starting from q^d, a state $\hat{q}^d = (\hat{q}^1, \hat{q}^2) \in \hat{Q}_d$ with $\kappa(\hat{q}^1) = \kappa(\hat{q}^2)$ can be reached within $|\hat{Q}_d| - 1$ transitions. This implies that for any $n' > |\hat{Q}_d| - 1$ and $x_e^{n'} \in \mathcal{Q}_e(q_{F_i}, n')$, we have that $\mathcal{D}(x_e^{n'}) = i$. From the assumption in Remark 2, each observed output can be followed by at most $|Q| - 1$ unobserved outputs. It follows that for the above state sequence s, $n \leq (n' + 1) \times (|Q| - 1)$,

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i.e., $n' \geq n/(|Q|-1)-1$. So if $n > |\hat{Q}_d| \times (|Q|-1)$, then $n' \geq n/(|Q|-1)-1 > |\hat{Q}_d|-1$, establishing our claim. Note that we have assumed implicitly that $|Q| > 1$; otherwise if $|Q| = 1$, then from the assumption of no path cycles, no transition labeled by a failure event exists, so that the system is trivially diagnosable. Based on Definition 9, we can conclude that $\mathbf{G}$ is diagnosable w.r,t, l_o^0. So the sufficiency also holds. $\square$