EV Charging Network Design with Transportation and Power Grid Constraints

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Abstract—**Connected electric vehicles (EVs) are a key component of future intelligent and green transportation systems, and the penetration of EVs depends on convenient and cost-effective charging services. In addition to being charged at home or on parking lots, a charging network is needed for EVs right off the road. This paper first focuses on the optimal charging network design for charging service providers, considering the time-varying and location-dependent demands from vehicles and constraints of power grids. To optimize the charging station locations and the number of chargers in each station, we first model the coverage area of each possible location to estimate the dynamic charging requirements of EVs. Then, we formulate the problem as profit maximization, which is a mixed-integer program. To make the problem tractable, we investigate the features of the problem and obtain a necessary condition to deploy a charging station and derive the upper and lower bounds of the number of chargers in each station. Given the analysis, we take two steps to transform and relax the problem to convex optimization. A fast-converging search algorithm is further proposed based on the profit of each possible location. Using real vehicle traces, simulation results show that the proposed algorithm can maximize the total profit when fewer charging stations and chargers are initially needed, which is more attractive for charging service providers.**

Index Terms—**Electric Vehicles, Charging Service, Charing Station, Power Grid.**

I. Introduction

Electric Vehicles (EVs) have made rapid development recently thanks to their prominent advantages in reducing greenhouse gas emissions and saving fuel consumption and maintenance costs. Also, governments worldwide have established various policies and economic incentives to speed up the rollout of EVs. Currently, the majority of EVs' charging is done at residences or limited public charging infrastructures, taking several hours to fully charge the batteries due to the low charging power and sometimes long waiting time. As the EV penetration continues to grow, the EV charging problem has been becoming more serious and it is difficult for the public charging facilities to satisfy all the charging requirements of EVs. Thus, commercial charging stations, especially fast charging stations, should be developed by charging service providers to offer convenient charging services.

EV charging infrastructure deployment in a city needs to consider several important factors, including the transportation and the charging requirements of EVs, EV owners' behaviors, the possible locations and their space limits in the city, and the stability requirements and charging load limits in the power grid. There is a rich set of literature aiming at addressing the EV charging infrastructure deployment problem, and the optimal charging station placement problems have been formulated. Several flow refueling location models have been proposed to satisfy the charging requirements of EVs as much as possible [1]–[3]. A few charging station deployment schemes under the limit of power grid have been proposed to minimize the total cost including power generation, power transmission loss and construction cost of charging stations [4]–[8]. A number of charging station placement strategies have been designed to maximize the social welfare under the constraints of transportation and power grid [9]–[13]. Several strategies of charging station deployment have been designed to minimize the charging station construction cost under the constraints of charging requirements and charging powers [14]–[16]. These works either focused on how to satisfy the charging requirements of all the EVs or to maximize the social warfare. Most of them were formulated from the perspective of a central urban planner rather than the charging service providers who intend to maximize their profits by building the charging network at the possible locations.

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Similar problems for gas station deployment have been widely studied over the past decades and numerous efficient gas station deployment schemes have been designed in [17]–[21]. However, the gas station deployment schemes cannot be directly applied to the charging station deployment given the following differences: i) The space limits for the charging station planner are much more important than that for the gas station planner since the charging station needs a large space to accommodate more chargers. ii) The construction cost of gas station mainly depends on the cost of building the gas station while that of charging station depends on building costs of both the charging station and plenty of chargers. iii) The service capacity of a gas station generally is large given the relatively short refueling time, while that of a charging station is limited due to a long charging duration and the constraint of the power grid. Consequently, how to determine the location of charging stations and the number of chargers in each charging station to reduce the long waiting time and blocking probability is an important open issue for charging service providers.

The main objective of charging service providers is to maximize the total profit of the charging network, considering the construction cost, the charging requirements of EVs given correlated vehicle traffic dynamics and the power constraint from the power grid. To address the issue, we first formulate a profit maximization problem, which is a mixed-integer program. Since it is difficult to solve directly, we investigate the features of the problem and obtain a necessary condition to deploy a charging station and derive the upper and lower bounds of the number of chargers in each station. Given the analysis, we take two steps to transform and relax the problem to a convex optimization problem. Then, we propose a heuristic Removing and Merging Possible Locations (RMPL) algorithm, based on the relationship between the total profit and the decision for each possible location to improve the total profit for the charging service provider. At last, the performance of the proposed scheme is investigated using simulations with real traffic data. The contributions of this paper can be summarized as follows:

- We formulate the design of charging network as a profit maximization problem considering the time-varying charging demand in each area, the construction cost and the limits of location space and charging power.
- We analyze the range of the maximal profit of each possible location and classify them into three categories, such that one of the integer variables can be removed. Also, we prove that the transformed problem is a convex optimization problem.
- We propose a Removing and Merging Possible Locations algorithm to improve the total profit of the charging network by excluding and merging some unprofitable and less profitable locations.
- Simulations based on the real traffic data are conducted to demonstrate the efficiency of the proposed scheme.

The rest of the paper is organized as follows. Section II presents the system model and problem formulation. A method has been proposed to classify the possible locations and exclude the unprofitable locations, such that the primal problem can be simplified, and then a heuristic algorithm has been designed to improve the total profit in Section III. Section IV demonstrates the operational performance analysis based on simulation results. Finally, Section V concludes our work.

II. System Model and Problem Formulation

Considering a charging service provider, who plans to build several charging stations in a city to provide charging services for EVs. Due to urban planning factors and space limits, the possible locations for operating charging stations are given, and the restrictions for each location, such as the upper bound of the number of chargers and the charging load constraint from the power grid, are defined. How to find the potential locations and how to negotiate with the power grid on the upper bound of charging load are out of the scope of this paper. The key problem is to identify, among the possible locations, where to build the charging stations, and how many chargers should be installed in each charging station.

In the following, we first model the expected profit of each possible location, and then introduce the charging requirement model of each charging station, which depends on the location and the transportation in its coverage area. Next, we introduce the operation model for charging stations, which depends on charging requirements and the limits of the possible locations and the power grid, and formulate the design of charging network as a profit maximization problem.

A. Profit Model for Possible Locations

Denote by I the total number of all the possible locations, and denote by i the i-th location. Let (S_i, N_i) denote the decision of the charging service provider, where $S_i \in \{0, 1\}$ represents whether or not to build a charging station at location i, and $N_i \in \{1, 2, \cdots, \tilde{N}_i\}$ represents the number of chargers to be installed at location i. N_i in location i cannot exceed its upper bound $\tilde{N}_i$ due to the space limits, etc.

Once the charging station has been built, its profit depends on the average construction cost and the revenue for providing charging services to EVs. Generally, the average construction cost of one possible location depends on the location, and the number of chargers, while the revenue depends on the number of EVs that it can serve.

To model the cost of each possible location, the charging service provider needs to estimate the average daily cost, including the property cost and fee, amortized construction fee, other initial construction costs, and construction cost of each charger at the possible location. Let $C_{1,i}$ denote the average daily cost of one charging station at possible location i and $C_{2,i}$ denote the average daily building and maintenance cost of each charger at possible location i, respectively. The total average daily cost of operating the charging station at possible location i, denoted by $\hat{C}_i^O$, is

$$\hat{C}_i^O = S_i(C_{1,i} + C_{2,i}N_i) \;\; \forall i. \tag{1}$$

Note that the total daily cost of possible location i depends on whether the charging station will be installed or not and how many chargers will be installed.

The expected revenue of one possible location depends on the total number of EVs being served in the charging station, their charging requirements, and the electricity price from the power grid. In this paper, we divide a day into T time slots and t denotes the t-th time slot.

In this paper, we assume that the EV arrivals to a charging station follow a Poisson distribution with the average charging requirement of $\bar{R}$ time slots per EV. Let C_3 denote the expected revenue for charging one EV in the charging station and $M_{i,t}$ denote the total number of EVs that are charged at charging station i in time slot t, respectively. The expected revenue of charging station i in time slot t, denoted by $C_{i,t}^I$, can be given by $C_{i,t}^I = C_3 M_{i,t}$, and the total expected revenue of charging station i during the day, denoted by $\hat{C}_i^I$, is

$$\hat{C}_i^I = \sum_t C_3 M_{i,t}. \tag{2}$$

Note that, the expected revenue of one charging station depends on the number of EVs that can be charged by the charging station successfully since $M_{i,t}$ not only depends on the charging requirements of EVs, but also depends on the service capacity of the charging station under the constraints of the location and the power grid.

The total expected profit from all the possible locations during one day, denoted by $\hat{C}$, can be given by

$$\hat{C} = \sum_i (\hat{C}_i^I - \hat{C}_i^O). \tag{3}$$

From the expression of $\hat{C}$, the total expected profit depends on the estimated daily cost and the revenue of each possible location. In order to maximize the total profit, the charging service provider could build the charging network based on the profit model of each possible location.

B. Charging Requirement Model for Charging Stations

Since the traffic is highly dynamic and the EV density changes in different places and time periods, we need to model the time-varying charging requirements for each possible location.

As EVs prefer to be served in a nearby location, the whole service area (e.g., a city) is divided into several zones centered by the possible locations as a Voronoi diagram [22]. Let Φ_i and $\lambda_{i,t}$ denote zone i and the expected number of EVs in Φ_i that need to be charged in time slot t, respectively. Assuming that the arrival rate of EVs requesting service at zone i follows a Poisson distribution with the average arrival rate of $\lambda_{i,t}$ in time slot t. In zone i, the probability for n EVs arriving in time slot t can be given by

$$P\{n\} = \frac{e^{\lambda_{i,t}}(\lambda_{i,t})^n}{n!}, \quad n = 0, 1, 2, \cdots.$$

Let R_i denote the expected charging requirement of one EV in Φ_i. The total amount of the expected charging requirements in zone i in time slot t is $R_i\lambda_{i,t}$. Here, we assume that EV arrivals at different zones are independent.

C. System Model for Charging Stations

In this section, we introduce the charging ability of each charging station and then define the transfer probabilities among the charging stations based on the queue length and the traffic among the possible locations.

Due to the space limits of each possible location, the total number of chargers that can be installed at possible location i, N_i, should satisfy

$$N_i \leq \tilde{N}_i. \tag{4}$$

Furthermore, for the safety and stability consideration, an upper bound on each charging station's charging load in a given time slot is issued by the power grid based on the power supply capability of the distribution network. Let $\bar{P}_{i,t}$ denote the upper bound on the charging load from charging station i in time slot t. To fulfill this upper bound, the charging station may only allow a maximum number of the chargers to work during one time slot, or equivalently slowing down all chargers. Let E denote the power rating of each charger. Thus, the available chargers at charging station i in time slot t, $\hat{N}_{i,t}$, is

$$\hat{N}_{i,t} = \min\{N_i, \lfloor \frac{\bar{P}_{i,t}}{E} \rfloor\}, \tag{5}$$

where $\lfloor\cdot\rfloor$ denotes the floor function. It can be found that the total number of available chargers in each charging station depends on not only the total number of chargers that are installed in the charging station, but also the upper bound on the charging load from the power grid in time slot t.

In time slot t, given the power rating of each charger, the expected charging time for each EV at charging station i is R_i/E, and the expected service rate for charging station i is $\hat{N}_{i,t}E/R_i$. Since the total number of the available chargers $\hat{N}_{i,t}$ is limited, the EVs will wait in the queue when the queue is not full. Otherwise, it may either move to a neighbor zone, or leave the charging system directly without being served (being blocked). Let $Q_{i,t}$ denote the queue length at charging station i in time slot t and $\bar{Q}_{i,t}$ denote the corresponding upper bound of the queue length, respectively. An EV will leave zone i in time slot t when $Q_{i,t} > \bar{Q}_{i,t}$.

Let $O(i)$ denote the set of the zones that are charging station i's neighbors. Let $p_{ii,t}$ denote the probability for EVs in Φ_i that will be charged in charging station i in time slot t and $p_{ij,t}$ denote the probability for EVs in Φ_i that will move to a possible neighbor zone j, $j \in O(i)$, and entering the charging station there in time slot t, respectively. Let $p_{il,t}$ denote the probability for EVs in Φ_i that an EV leaves the charging system directly (being blocked) in time slot t. The values of $p_{ii,t}$, $p_{ij,t}$, and $p_{il,t}$ satisfy the following constraints:

$$\begin{cases} p_{ii,t} = 1, & \text{if } Q_{i,t} \leq \bar{Q}_{i,t}; \\ 0 \leq p_{ii,t} < 1, & \text{Otherwise}, \end{cases} \tag{6}$$

$$\begin{cases} p_{ij,t} = 0 \ \& \ p_{il,t} = 0, & \text{if } Q_{i,t} \leq \bar{Q}_{i,t}; \\ 0 < p_{ij,t} \leq \alpha_{ij,t} \ \& \ 0 < p_{il,t} \leq \alpha_{il,t}, & \text{Otherwise}, \end{cases} \tag{7}$$

where $\alpha_{ij,t}$ and $\alpha_{il,t}$ denote the upper bound of the transfer probability that an EV moves from charging station i to charging station j and that leaves the charging system directly, respectively. The transfer probability $\alpha_{ij,t}$ satisfies the following properties:

- The value of $\alpha_{ij,t}$ decreases with the increase of travel distance d_{ij};
- The value of $\alpha_{ij,t}$ depends on the distribution of EVs' travel directions in time slot t.

Furthermore, the transfer probabilities of charging station i's neighbors, $\{\alpha_{ij,t}, j \in O(j)\}$, satisfy the following constraint:

$$\sum_{j \in O(i)} \alpha_{ij,t} + \alpha_{il,t} = 1. \tag{8}$$

In this paper, we define the probabilities $p_{ij,t}$ and $p_{il,t}$ as

$$p_{ij,t} = P_{i,t}^B \alpha_{ij,t} \text{ and } p_{il,t} = P_{i,t}^B \alpha_{il,t}, \tag{9}$$

where $P_{i,t}^B$ denotes the blocking probability $P\{Q_{i,t} \geq \bar{Q}_{i,t}\}$ for charging station i in time slot t. Probability $p_{ii,t}$ is given by

$$p_{ii,t} = 1 - P_{i,t}^B. \tag{10}$$

For the relationships among $p_{ii,t}$, $p_{ij,t}$ and $p_{il,t}$, we have

$$p_{ii,t} + \sum_{j \in O(i)} p_{ij,t} + p_{il,t} = 1, \quad \forall i, t. \tag{11}$$

According to the definition of $p_{ij,t}$ in (9), the expected number of EVs in Φ_j moving to charging station i in time slot t is

$$\lambda_{j,t} p_{ji,t} = \lambda_{j,t} P_{j,t}^B \alpha_{ji,t}. \tag{12}$$

Thus, the total expected arrival rate at charging station i can be given by $\lambda_{i,t} + \sum_{i \in O(j)} \lambda_{j,t} p_{ji,t}$. If the EVs in Φ_j are blocked at charging station i, they will leave the charging station system (being blocked).

In order to obtain the blocking probability $P_{i,t}^B$, we formulate the charging processes of each charging station as a first-in-first-out M/M/c/N model in queuing theory[1]. Let μ denote the mean service rate of one charger, c denote the total number of the available chargers in charging station i in time slot t, λ denote the arrival rate of EVs that need charging services at charging station i, and ρ denote the utilization factor of charging station i in time slot t, respectively. Thus, we have $\mu = E/R_{i,t}$, $c = \hat{N}_{i,t}$, $\lambda = \lambda_{i,t} + \sum_{j \in O(i)} \lambda_{j,t} p_{ji,t}$, $\rho = \lambda/(c\mu)$, and $N = \bar{Q}_{i,t}$ for charging station i. According to the M/M/c/N model, the steady state distributions of this queue are given by:

$$P_n = \begin{cases} \frac{1}{n!}(\frac{\lambda}{\mu})^n P_0, & 1 \le n \le c \\ \frac{1}{c^{n-c}c!}(\frac{\lambda}{\mu})^n P_0, & c \le n \le N, \end{cases} \tag{13}$$

where

$$P_0 = \begin{cases} \left[\sum_{n=0}^{c-1} \frac{1}{n!}(\frac{\lambda}{\mu})^n + \frac{1}{c!}(\frac{\lambda}{\mu})^c \frac{1-\rho^{N-c+1}}{1-\rho} \right]^{-1}, & \text{if } \rho \neq 1, \\ \left[\sum_{n=0}^{c-1} \frac{1}{n!}(\frac{\lambda}{\mu})^n + \frac{1}{c!}(\frac{\lambda}{\mu})^c (N-c+1) \right]^{-1}, & \text{if } \rho = 1, \end{cases}$$

and the blocking probability $P_{i,t}^B$ can be given by

$$P_{i,t}^B = \frac{1}{c^{N-c}c!}(\frac{\lambda}{\mu})^N P_0. \tag{14}$$

It means that the probability for EVs in Φ_i to leave charging station i in time slot t is $P_{i,t}^B$. Similarly, when EVs in Φ_j are blocked by charging station j, $j \in O(i)$, they may move to charging station i with probability $\alpha_{ji,t}$. According to the definition of $p_{ij,t}$ in (9), the expected number of EVs in Φ_j moving to charging station i in time slot t is

$$\lambda_{j,t} p_{ji,t} = \lambda_{j,t} P_{j,t}^B \alpha_{ji,t}. \tag{15}$$

Thus, the total expected arrival rate at charging station i can be given by $\lambda_{i,t} + \sum_{i \in O(j)} \lambda_{j,t} p_{ji,t}$. It can be found that the blocking probabilities for two neighbor charging stations may affect each other's arrival rate and blocking probabilities. It is very complicated to calculate the blocking probabilities of the charging stations considering the transfer probability between neighbor stations since they depend on the topology of charging networks and the transfer processes among all stations are coupled. Thus, we first find the lower and upper bounds of the blocking probability, $\underline{P}_{i,t}^B$ and $\overline{P}_{i,t}^B$. Since $P_{i,t}^B$ is bounded by the two bounds, we can use a simplified model to estimate the blocking probabilities by

$$P_{i,t}^B = \underline{P}_{i,t}^B + a_t(\overline{P}_{i,t}^B - \underline{P}_{i,t}^B) \tag{16}$$

where $\underline{P}_{i,t}^B$ denotes the blocking probability for charging station i when the arrival rate is $\lambda_{i,t}$, $\overline{P}_{i,t}^B$ denotes the blocking probabilities when the arrival rate is $\lambda_{i,t} + \sum_{j \in O(i)} \lambda_{j,t} \alpha_{ji,t}$, and a_t is a weight depending on the network congestion level, respectively. In this paper, we set $a_t = \sum_{j \in O(i)} [\lambda_j - N_j]^+ / \sum_{j \in O(i)} \lambda_j$.

[1] As the queueing model is not the main focus of this work, we adopt the simple M/M/c/N model, where the service time of EV charging here is modeled as an exponential R.V. If the service time fits other R.V., we can extend the work by applying the M/G/c/N model to obtain the blocking probability. From simulation results, we note that using other service time distributions such as Gaussian distribution does not have an obvious impact on the charging station deployment.

Since the EVs from another charging station j, $j \in O(i)$, may be blocked by charging station i, only part of these transferring EVs will be charged at charging station i. The total number of EVs that can be charged at charging station i in time slot t is $(\lambda_{i,t} + \sum_{i \in O(j)} \lambda_{j,t} p_{ji,t}) P_{ii,t}$, where $P_{ii,t} = 1 - P_{i,t}^B$. In this paper, we assume that the EVs from the other charging stations will not affect λ_i and have the similar blocking probability as the EVs in Φ_i. Furthermore, the EVs from another neighbor zone j, $j \in O(i)$, and being blocked at charging station i will leave the charging system. Thus, the expected number of EVs charged at charging station i in time slot t, denoted by $M_{i,t}$, is

$$M_{i,t} = \lambda_{i,t} p_{ii,t} + \sum_{i \in O(j)} \lambda_{j,t} p_{ji,t} p_{ii,t}, \quad \forall i, t. \tag{17}$$

The total number of EVs in Φ_i that leave the charging system without being charged, denoted by $L_{i,t}$, is given by

$$L_{i,t} = \lambda_{i,t}(p_{il,t} + \sum_{j \in O(i)} \alpha_{ij,t} P_{j,t}^B). \tag{18}$$

Note that, since the detour time of EVs is small comparing with each time slot, its impact on the system is omitted.

D. Problem Formulation

In this paper, we aim at maximizing the total profit of the charging service provider by designing an optimal charging network. Based on the above system model, we formulate the design of the charging network as the following profit maximization problem:

$$\textbf{P0}: \max_{S_i, N_i} \hat{C} \tag{19}$$

$$S.t.: \ M_{i,t} = \lambda_{i,t} p_{ii,t} + \sum_{i \in O(j)} \lambda_{j,t} p_{ji,t} p_{ii,t}, \quad \forall i, t, \tag{20}$$

$$\hat{N}_{i,t} = \min\{N_i, \lfloor \frac{\bar{P}_{i,t}}{E} \rfloor\}, \quad \forall i, t, \tag{21}$$

$$N_i \le \tilde{N}_i, \quad \forall i, \tag{22}$$

$$S_i = \{0, 1\}, \quad \forall i. \tag{23}$$

The objective function is to maximize the total profit for the charging service provider. The first constraint determines the number of EVs that will be charged at each charging station. The second equation defines the limit of the number of available chargers in each charging station. The third and the fourth constraints show the limits of the charging stations.

From the problem formulation, the decision variables, e.g., $\{S_i, N_i, i \in I\}$, are integers. Furthermore, the decision variables $\{N_i, i \in I\}$ are coupled by the objective function and the first constraint. Thus, the optimization problem is a mixed-integer programing problem, which is difficult to solve by the existing tools. Next, we analyze the possible profits for each possible location and exclude the unprofitable locations, such that the profit maximization problem can be simplified.

III. Problem Transformation and Solution

In this section, we first propose a method to classify the possible locations into three categories according to their range of the estimated profit. Since the charging service provider only builds the charging network at the possible locations in two categories, the problem can be transformed into another optimization problem. Then, we design a heuristic algorithm to solve the problem by removing the unprofitable locations and merging the possible locations to reduce the total cost.

A. Classification of Charging Stations

Due to the imbalance and the time-varying charging requirements of EVs in different areas, it is difficult for some possible locations to generate profit due to low charging requirements in their coverage area and/or high construction cost. Thus, we need to analyze the possible profit of each charging station and identify the unprofitable locations.

Given a possible location i, let $\overline{C}_i$ and $\underline{C}_i$ denote the upper bound and the lower bound of its maximal expected profit during one day, respectively. In order to obtain the values of $\overline{C}_i$ and $\underline{C}_i$, we make the following two assumptions:

Assumption I: Assume that all the possible neighbor zones of i, $\{j, j \in O(i)\}$, are closed (no charging station is built), and the EVs in $\Phi(j)$ move to charging station i for charging service with a probability $\alpha_{ji,t}$. Hence, the maximal number of EVs that could be charged at charging station i can be obtained. The upper bound of the maximal expected profit of charging station i can be obtained by solving Problem **A1**:

$$\mathbf{A1}: \overline{C}_i = \max_{N_i} \hat{C}_i \tag{24}$$

$$M_{i,t} = (\lambda_{i,t} + \sum_{i \in O(j)} \lambda_{j,t}\alpha_{ji,t})(1 - \overline{P}^B_{i,t}), \tag{25}$$

$$\hat{N}_{i,t} = \min\{N_i, \lfloor \frac{\bar{P}_{i,t}}{E} \rfloor\}, \tag{26}$$

$$N_i \leq \tilde{N}_i. \tag{27}$$

Assumption II: Assume that all the possible neighbor zones of i, $\{j, j \in O(i)\}$, have been installed with sufficient chargers, and no EVs in the neighbor zones will move to charging station i for charging service. Hence, the minimal number of EVs that can be charged at charging station i can be obtained. The lower bound of the maximal expected profit of charging station i can be obtained by solving Problem **A2**:

$$\mathbf{A2}: \underline{C}_i = \max_{N_i} \hat{C}_i \tag{28}$$

$$M_{i,t} = \lambda_{i,t}(1 - \underline{P}^B_{i,t}), \tag{29}$$

$$\hat{N}_{i,t} = \min\{N_i, \lfloor \frac{\bar{P}_{i,t}}{E} \rfloor\}, \tag{30}$$

$$N_i \leq \tilde{N}_i. \tag{31}$$

According to the results in the existing work [23], [24], blocking probability $P^B_{i,t}$ is decreasing and convex with respect to the number of servers N_i when the other parameters are given. It is easy to prove that both Problems **A1** and **A2** are convex optimization problems. Thus, optimal solutions to Problems **A1** and **A2** can be obtained in an efficient way [25]. Note that $\overline{C}_i \geq \underline{C}_i$ always holds for each charging station. Based on the values of $\overline{C}_i$ and $\underline{C}_i$, we can classify all the possible locations into three categories:

- **C1** Unprofitable location: $i \in$ **C1** if $\overline{C}_i < 0$;
- **C2** Possible profitable location: $i \in$ **C2** if $\overline{C}_i \geq 0$ and $\underline{C}_i \leq 0$;
- **C3** Profitable location: $i \in$ **C3** if $\underline{C}_i > 0$.

Theorem 1: For the optimal solution, the charging service provider only builds charging stations at the possible locations in **C2** and **C3** categories.

Proof: From the total profit given by (3), it can be found that the total profit of the charging network depends on the profit of each charging station. The maximal profit $\overline{C}_i$ for charging station i can be obtained by solving Problem **A1** since (25) shows the maximal number of EVs that can be served by charging station i. If $\overline{C}_i$ is negative, it means that charging station i cannot generate any profit even if all the possible EVs move to charging station i for charging services. Hence, in order to maximize the total profit, the possible locations in **C1** category cannot be installed. For the possible locations in **C2** and **C3** categories, they may generate profit if an adequate number of chargers are installed, such that building charging stations at the possible locations in **C2** and **C3** categories may increase the total profit. Thus, for the optimal solution, the charging service provider builds charging stations at the possible locations in **C2** and **C3** categories only. ■

Let $\overline{N}_i$ and $\underline{N}_i$ denote the optimal number of chargers at possible location i by solving Problems **A1** and **A2**, respectively. We have the following theorem for the optimal number of chargers, denoted by N_i^*, as

Theorem 2: For the optimal Problem **P0**, the optimal number of chargers N_i^* satisfies $\underline{N}_i \leq N_i^* \leq \overline{N}_i$.

Proof: It can be proved that the total revenue $C_3 M_{i,t}$ is an increasing and concave function of $N_{i,t}$. Thus, there are two possible conditions for $\overline{N}_i$: 1) $\overline{N}_i$ equals $\lfloor \frac{\bar{P}_{i,t}}{E} \rfloor$ or $\tilde{N}_i$; and 2) the derivative of $C_3 M_{i,t}$ with respect to N_i when $N_i = \overline{N}_i$ is larger than $C_{2,i}$ and that when $N_i = \overline{N}_i + 1$ is smaller than $C_{2,i}$. For condition 1), it means that the number of chargers reaches the upper bound of chargers, and thus the optimal solution N_i^* cannot exceed $\overline{N}_i$. For condition 2), it means that the total profit will decrease when $N_i \geq \overline{N}_i$ since $\partial C_3 M_{i,t} / \partial N_i < C_2$. Since $C_3 M_{i,t}$ is an increasing function of the arrival rate, which cannot exceed $\lambda_{i,t} + \sum_{i \in O(j)} \lambda_{j,t}\alpha_{ji,t}$, $N_i^* \leq \overline{N}_i$ should always hold. Similarly, it can be proved that $N_i^* \geq \underline{N}_i$ should always hold. The proof has been done. ■

According to the classification of the possible locations, the charging service provider only builds charging stations at the possible locations in **C2** and **C3** categories. Furthermore, the number of chargers in each possible location should satisfy Theorem 2. To solve this problem, we first assume that all the possible locations in **C2** and **C3** categories have been built with charging stations and we need to decide how many chargers should be installed. Thus, the problem can be transformed as the following problem:

$$\mathbf{P1}: \max_{N_i, i \in \{\mathbf{C2} \& \mathbf{C3}\}} \hat{C} \tag{32}$$

$$S.t.: M_{i,t} = \lambda_{i,t} p_{ii,t} + \sum_{i \in O(j)} \lambda_{j,t} p_{ji,t} p_{ii,t}, \quad \forall i, t, \tag{33}$$

$$\hat{N}_{i,t} = \min\{N_i, \lfloor \frac{\bar{P}_{i,t}}{E} \rfloor\}, \quad \forall i, t, \tag{34}$$

$$\underline{N}_i \leq N_i \leq \overline{N}_i, \quad \forall i. \tag{35}$$

In Problem **P1**, the objective is to maximize the total profit of all the charging stations at the possible locations in **C2** and **C3** categories, in which the value of $M_{i,t}$ is given by the first constraint. It can be found that the second and the third constraints are linear. However, since the variables are integers, the transformed problem also is an integer programming problem.

To solve this problem, we relax the number of chargers N_i for each charging station as a continuous variable, whose range is $[\underline{N}_i \overline{N}_i]$. Then, the following theorem can be obtained by analyzing the relationship among $\hat{C}_i$, N_i, and N_j:

Theorem 3: The total profit $\hat{C}_i$ is a concave function if $\{N_i, \forall i \in \mathbf{C2}\&\mathbf{C3}\}$ are continuous variables.

Proof: From the expression of $\hat{C}_i$, it includes the total daily cost C_i^O, which is a linear function of N_i, and the total revenue C_i^I, which is a linear function of $M_{i,t}$. All the constraints are linear, and thus the transformed problem is a convex optimization problem if $M_{i,t}$ is a convex function of the variables $\{N_i, N_j, j \in O(i)\}$. According to the existing works [23], [24], the blocking probability $P_{i,t}^B$ is convex in the number of servers N_i when the other parameters are given. Hence, $p_{ii,t}$, which equals $1 - P_{i,t}^B$, is concave with respect to N_i, and $p_{ji,t}$, which equals $P_{i,t}^B \alpha_{ji,t}$, is convex with respect to N_j. To prove one function is concave with respect to several variables, Hessian matrix of the second partial derivatives of the function can be employed since negative Hessian matrix denotes a concave function in these variables [25]. The Hessian matrix for $M_{i,t}$ with respect to $\{N_i, N_j, j \in O(i)\}$ is

$$\begin{bmatrix} \frac{\partial M_{i,t}^2}{\partial^2 N_i} & \frac{\partial M_{i,t}^2}{\partial N_i \partial N_j} & \cdots & \frac{\partial M_{i,t}^2}{\partial N_i \partial N_m} \\ \frac{\partial M_{i,t}^2}{\partial N_j \partial N_i} & \frac{\partial M_{i,t}^2}{\partial^2 N_j} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial M_{i,t}^2}{\partial N_m \partial N_i} & 0 & \cdots & \frac{\partial M_{i,t}^2}{\partial^2 N_m} \end{bmatrix}$$

Since $\frac{\partial M_{i,t}^2}{\partial^2 N_j} > 0$, $j \in O(i)$ and all the other elements in the matrix are smaller than 0, the Hessian matrix H is negative. Thus, the total profit $\hat{C}_i$ for charging station i is a concave function of the number of chargers $\{N_i, N_j, j \in O(i)\}$. ■

Since the transformed objective is a concave function of the number of the chargers in each charging station, there exists only one unique optimal solution [25]. Generally, a convex optimization problem can be solved by some existing tools, such as CVX, and Fmincon in Matlab. However, it is difficult for the existing tools to deal with the blocking probabilities due to the factorial function and the exponentiation function of variables. Furthermore, the solution is obtained based on the assumption that all the possible locations in **C2** and **C3** categories have been installed. According to the definition of the possible locations in **C2** category, it only can generate profit when their neighbor locations are not installed or have been installed but with a high blocking probability. Actually, some of the locations in **C2** category may not generate any profit or only generate a small profit. Even for the possible locations in **C3** category, due to the high construction cost, some of them may generate a low profit. Thus, merging some possible locations may increase the total profit for the charging service provider by reducing the high construction cost. Generally speaking, there may exist some margins to improve the total profit for the charging service provider.

Thus, we propose a heuristic algorithm, named Removing and Merging Possible Locations (RMPL), to exclude the unprofitable locations in **C1** and **C2** categories and merge the possible locations in **C2** and **C3** categories if beneficial, which can be summarized as Algorithm 1.

Algorithm 1 Removing and Merging Possible Locations

Initialization: Input $\lambda_{i,t}$, $\alpha_{ij,t}$, $\bar{P}_{i,t}$, E, $\bar{Q}_{i,t}$, $C_{1,i}$, $C_{2,i}$, C_3, $\underline{N}_i$ and $\overline{N}_i$ for each possible location;
I. Removing Possible Locations in C1
  Set $\underline{N}_i = 0$ and $\underline{P}_{i,t}^B = 1$ if $i \in \mathbf{C1}$;
  For each possible location i, $i \in \mathbf{C2}\&\mathbf{C3}$
    1) Set $\lambda'_{i,t} = \lambda_{i,t} + \sum_{j \in \mathbf{C1}} \lambda_{j,t} \alpha_{ji,t}$;
    2) Update $\underline{N}_i$, $\underline{P}_{i,t}^B$, and $\underline{C}_i$ by solving Problem **A2**;
  End for and go to **II**.
II. Removing Possible Locations in C2
  1) Set $\lambda''_{i,t} = \lambda'_{i,t} + \sum_{j \in \mathbf{C2}\&\mathbf{C3}} \lambda_{j,t} \underline{P}_{j,t}^B \alpha_{ji,t}$;
  2) Update $\underline{N}_i$, $\underline{P}_{i,t}^B$, and $\underline{C}_i$ by solving Problem **A2**;
  3) Sort $\{\underline{C}_i, \forall i \in \mathbf{C2}\&\mathbf{C3}\}$ from smallest to largest;
  4) If the smallest value is negative, remove the corresponding possible location i from **C2** and set $\underline{N}_i = 0$ and $\underline{P}_{i,t}^B = 1$; Otherwise, **end** and go to **III**.
III. Merging Possible Locations in C2&C3
  For each possible location i, $i \in \mathbf{C2}\&\mathbf{C3}$
    1) Remove location i from **C2**&**C3** and set $\underline{P}_{i,t}^B = 1$;
    2) Calculate the updated profits $\{\underline{C}_j, \forall j \in \mathbf{C2}\&\mathbf{C3}\}$ according to step **II** 1)-2);
    3) If total profit is increased, remove the possible location i and sets $\underline{N}_i = 0$ and $\underline{P}_{i,t}^B = 1$; Otherwise, return it to **C2**&**C3** and recover $\underline{P}_{i,t}^B$;
  End for.
Return $\{i, N_i = \underline{N}_i, i \in \mathbf{C2}\&\mathbf{C3}\}$.

The proposed RMPL algorithm can be divided into three steps: **I)** Removing Possible Locations in **C1**, **II)** Removing Possible Locations in **C2**, and **III)** Merging Possible Locations in **C2**&**C3**. Taking the profit of each possible location as the reference, some unprofitable possible locations are removed in steps **I)** and **II)** since they cannot generate any profit to the charging network. Then, we attempt to remove each possible location to reduce the high construction cost, which can increase the total profit of the charging network until removing any possible location will decrease the total profit. Such that, the maximal profit of the charging network can be reached.

In the proposed RMPL algorithm, the computation complexities for steps **I)** and **II)** are $O(\log N)$ and that for step **III)** is $O(IM \log N)$ where N is the maximal number of chargers at each possible location, I is the number of locations in **C2** and **C3** categories, and M is the maximal number of the neighbor charging stations of one location, respectively. Thus, the computation complexity of the proposed RMPL algorithm is $O(IM \log N)$.

IV. Case Study and Numerical Simulations

A. Case Study

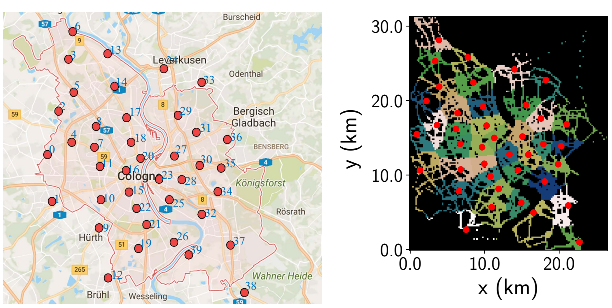
In this paper, we use the traffic data in Cologne in Germany as the case study [26]. The synthetic trace of the car traffic covers a region of 400 square kilometers for a period of 24 hours in a typical working day, and comprises more than

TABLE I
THE AVERAGE CONSTRUCTION COST AND LIMITS OF EACH POSSIBLE LOCATION

Locations	0-14, 33-39	15-18	19, 26-28	20-25	29-32
$C_{1,i}$	\$150	\$260	\$220	\$300	\$180
$C_{2,i}$	\$35/pile	\$30/pile	\$30/pile	\$40/pile	\$30/pile
$\bar{N}_i$	50	30	30	30	50
$\bar{P}_i$	5000kW	4000kW	3500kW	3000kW	5000kW

700,000 individual car trips. From the data, we should obtain not only the real-time traffic density information, but also the statistics of the travel direction, the traffic volume in a certain area, etc. The road densities are time-varying and the numbers of EVs at different locations are different. We assume that the charging requirements during different time slots are different and the percentage of EVs requiring charging services are 5% during [6am, 10am), 8% during [10am, 2pm), and 10% during [2pm, 9pm], respectively. The charging requirement of one EV follows a Gaussian distribution (R, δ^2), where $R = 40kWh$ and $\delta = 4kWh$.

Generally, EVs always select the nearest charging station for charging services, so the service area of each possible location can be obtained using the Voronoi diagram. The possible locations and their service areas are shown in Fig. 1. Based on the construction cost estimation in [27], we assume that the range of the average daily cost for building one charging station is [\$150- \$300] depending on the locations and that for building and maintenance one charger at the possible locations is [\$30- \$35]. Different locations may have different space limits and charging load limits from the power grid. The details about the costs and the limits can be found in Table I. The charging power for each charger is $E = 120kW$ and the expected revenue for charging one EV is $C_3 = \$5$, respectively.

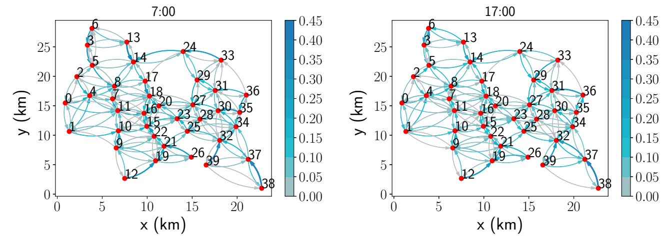


(a) Possible locations (b) Service areas

Fig. 1. The possible locations and their service areas in the city.

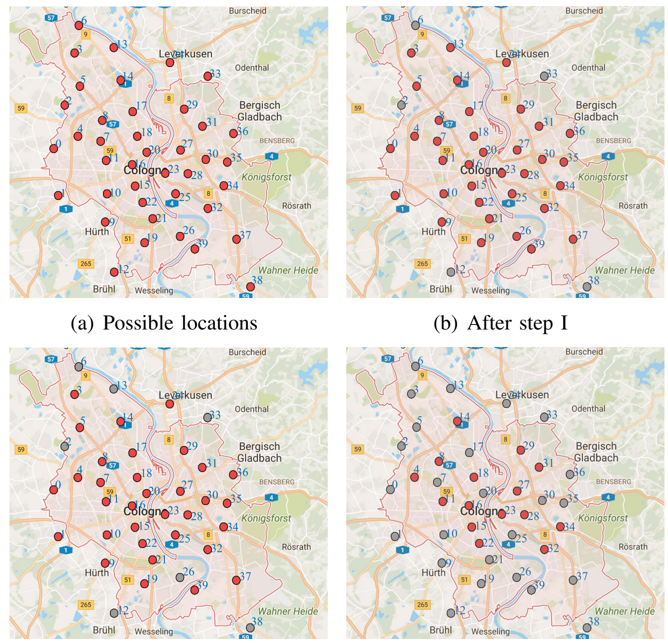
An EV may move to its neighbor charging stations when the queue in the nearest charging station is too long. Considering the EV owners' behaviors, we model the transfer probabilities based on the statistics of the traffic between two possible locations. The time-varying transfer probabilities among two neighbor charging stations at 7am and 5pm are shown in Fig. 2. The maximal queue length for each charging station is $\bar{Q}_{i,t} = 10$ and the probability for one EV leaving the charging system directly $\alpha_{il,t}$ is randomly selected in $[0, 0.3]$.

The decision for each step in the RMPL algorithm can be found in Fig. 3. At step I, the unprofitable locations, which have low charging requirements in their service areas and an insufficient number of EVs that may transfer from their neighbor locations, will be removed, e.g., locations No. 6, 38.



(a) Transfer at 7am (b) Transfer at 5pm

Fig. 2. The transfer probabilities between two neighbor locations at 7am and 5pm during one day, where the arrows denote the transfer directions and both colors and widths denote the value of blocking probability.



(a) Possible locations (b) After step I

(c) After step II (d) After step III

Fig. 3. The decision of each step in the RMPL algorithm (gray denotes the unselected location and red denotes the selected location).

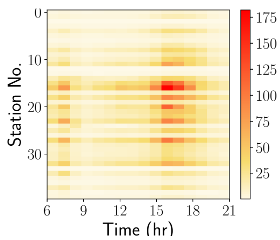
TABLE II
THE FINAL DECISION OF THE CHARGING SERVICE PROVIDER

Location No.	4	8	11	14	15	16	18	22
N_i	8	11	15	11	26	30	24	11
Location No.	23	27	28	29	31	32	34	Others
N_i	23	25	9	8	10	19	9	0

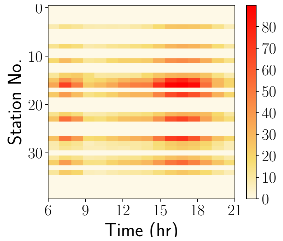
Note that $N_i \neq 0$ implies $S_i = 1$ and $N_i = 0$ implies $S_i = 0$.

At step II, the possible locations, which have low charging requirements in their service area and few EVs may come due to low blocking probabilities of their neighbor stations, will be removed since they cannot generate any profit, e.g., locations No. 13, 26. By now, all the unprofitable locations have been removed. At step III, the possible locations, which have low charging requirements, high construction cost and at least one neighbor charging station with redundant charging capacity, will be removed if the total profit of the charging network can be increased by doing so. It can be found that all the charging stations are installed at the possible locations with high charging requirements. The final decision of the charging service provider is shown in Table II.

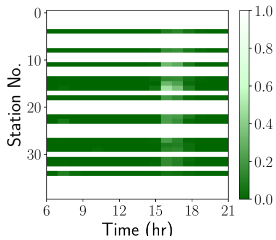
The tendency of EV's charging requirements, the distribution of the served EVs and the blocking probabilities in each charging station are shown in Fig. 4. If no charging station is installed at one possible location, its blocking probabilities



(a) Arrive rate



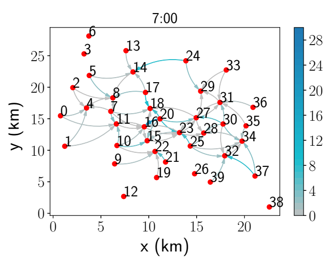
(b) Number of served EVs



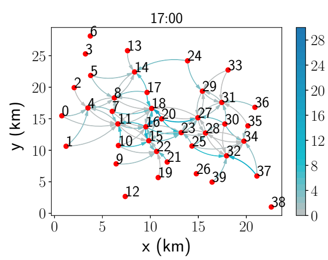
(c) Blocking probabilities

Fig. 4. The distributions of arrival rates, the number of served EVs and the blocking probabilities for possible locations, respectively.

during the day are set to 1. From Fig. 4(a), it can be seen that the charging requirements at different time slots are different and there are several rush hours. Figs. 4(b) and 4(c) show that the distribution of the served EVs follows the similar distribution of the arrival rates since most of EVs are served in their service areas and EVs from other possible locations are few. Fig. 4(c) shows that the charging stations have high blocking probabilities during the rush hours and very low blocking probabilities during non-rush hours.



(a) Transfer during 7am

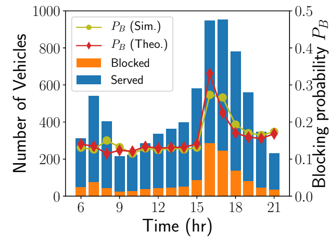


(b) Transfer during 5pm

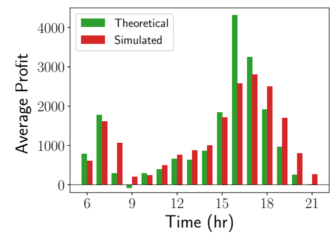
Fig. 5. The number of EVs transferred from their zones to their neighbor charging stations at 7am and 5pm, respectively.

Due to different traffic densities and travel directions, the transfers of the charging requirements from one possible location to another possible location may be different. In this paper, part of the transfers of the charging requirements between two possible neighbor locations are shown in Fig. 5. It can be found that the transfers, which depend on the arrival rate and the service ability of each charging station, are time-varying. The transfers during the rush hours are much more complex and higher than those in non-rush hours.

The distributions of served EVs and unserved EVs, as well as the average blocking probabilities in the entire city, are shown in Fig. 6(a). There exists a small gap between the theoretical and simulation blocking probabilities. That is because the transfer delays for the transferred EVs in simulation change the arrival rates of charging stations at the following hour while theoretical values do not take the transfer delay into consideration. Generally, the blocking probabilities are higher than 10%, even in non-rush hours, since some possible



(a) Blocking probabilities



(b) Total profit

Fig. 6. The average blocking probabilities and the average profit during each time slot.

TABLE III
THE DECISION AND PERFORMANCE FOR EACH STEP OF RMPL.

Schemes	Number of stations	Number of chargers	Profit ($)
RMPL	15	240	$20,000$
Benchmark	35	289	$19,000$
Ratio	43%	83%	105%

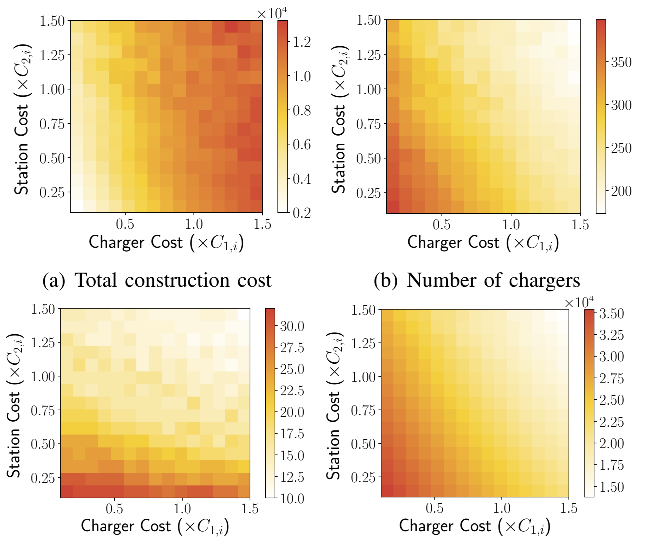
locations have no charging stations and part of EVs will leave the charging system directly due to the long travel distance to another neighbor station. The total profit can be found in Fig. 6(b), in which the daily construction costs are equally divided into each hour. It can be found that the total profits during different hours are time-varying and there also exists a small gap between the theoretical profit and the simulation profit. That is because the transfer delay for EVs moving from their zones to another charging station may smooth the blocking probabilities in rush hours, e.g., [16:00, 17:00], such that more EVs can be served by the charging system. Note that, even though we employ the M/M/c/N model to analyze the blocking probability and use the service times follow the Gaussian distribution to conduct the simulation in real-time trace, similar results with theoretical analysis can be obtained.

In order to compare the performance of the proposed RMPL algorithm, we take the following scheme as the benchmark: based on the arrival rate of each zone, the charging service provider builds charging stations at all the profitable locations with the optimal number of chargers by solving Problem **A2**. The results of the Benchmark and the proposed RMPL are shown in Table III. It can be found that RMPL can reduce the number of charging stations by 57% and chargers by 17% while increasing the total profit of the charging network by 5%, respectively. This gain is because the proposed RMPL takes the transfer between the neighbor charging stations and the total construction cost into consideration.

B. Numerical Simulation

In order to explore the effects of the construction costs on the design of the charging network, more numerical simulations are conducted by adjusting the construction cost of each charging station and construction cost of each charger. We set the range of the construction cost of charging stations and chargers as $[0.1C_{1,i}, 1.5C_{1,i}]$ and $[0.1C_{2,i}, 1.5C_{2,i}]$, respectively. The numerical results for the number of charging stations, the total number of chargers, the total construction cost, and the total profit of the charging network can be found in Fig. 7.

From the simulation results, it can be found that the construction cost of each charging station affects the number of charging stations greatly while the construction cost of



(a) Total construction cost

(b) Number of chargers

(c) Number of stations

(d) Total profit

Fig. 7. The effects of the construction costs of charging stations and chargers on the decision of the charging service provider.

the charger decreases the number of chargers sharply. Both of them affect the total profit of charging stations since they dictate the construction cost of the charging network. Furthermore, the construction cost of each charging pile will increase the total construction cost greatly due to its huge number. Especially, when the construction cost of charging stations or that of chargers increases, the total construction cost of the charging network will increase while the number of charging stations and chargers, as well as the total profit, will decrease. That is because the charging service provider needs to make a decision by comparing the construction cost and the revenue for providing charging services. Some possible locations, at which reducing the construction cost can increase the total profit, will be removed, such that the total profit for the charging service provider can be maximized.

V. Conclusion

In this paper, we addressed the charging station deployment problem for charging service providers, who intend to build charging networks at some possible locations, to generate profit by providing charging services for EVs. Taking the limits of the locations and the power grid into consideration, we formulated a profit maximization problem, which is a mixed-integer program and difficult to solve directly. By analyzing the range of the maximal profit of each possible location, we classify all the possible locations into three categories and exclude some possible locations since they cannot generate any profit, such that the primal problem can be transformed into another integer programming problem, which has been proved to have a unique optimal solution. Then, we design a Removing and Merging Possible Locations algorithm to remove the unprofitable locations and merge some possible locations by reducing the construction cost, such that the total profit for the charging service providers can be increased. A case study using the real vehicle traces and numerical experiments are conducted to verify the performance of RMPL. The simulation results showed that RMPL can reduce the number of charging stations and chargers and increase the profit for the charging service providers at the same time.

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