portfolio_2

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Factor Analysis and Independent Component Analysis

Task 1

For this task we are going to use the communities and crime dataset:

```
library(mogavs)
data(crimeData)
```

Using the factor model on this dataset is a reasonable assumption as the number of samples will be larger than the number of factors and there is no perfect multicolinearity between any of the variables (ie. the covariance matrix will be full rank). Let's first analyse the difference between the number of constraints and the number of free parameters in the factor model for different values of k (the number of factors):

```
p <- ncol(crimeData) -1
for(k in 1:p){
  print( paste( k, ": ", ((p-k)**2 /2 ) - ((p+k)/2) )  )
}
```

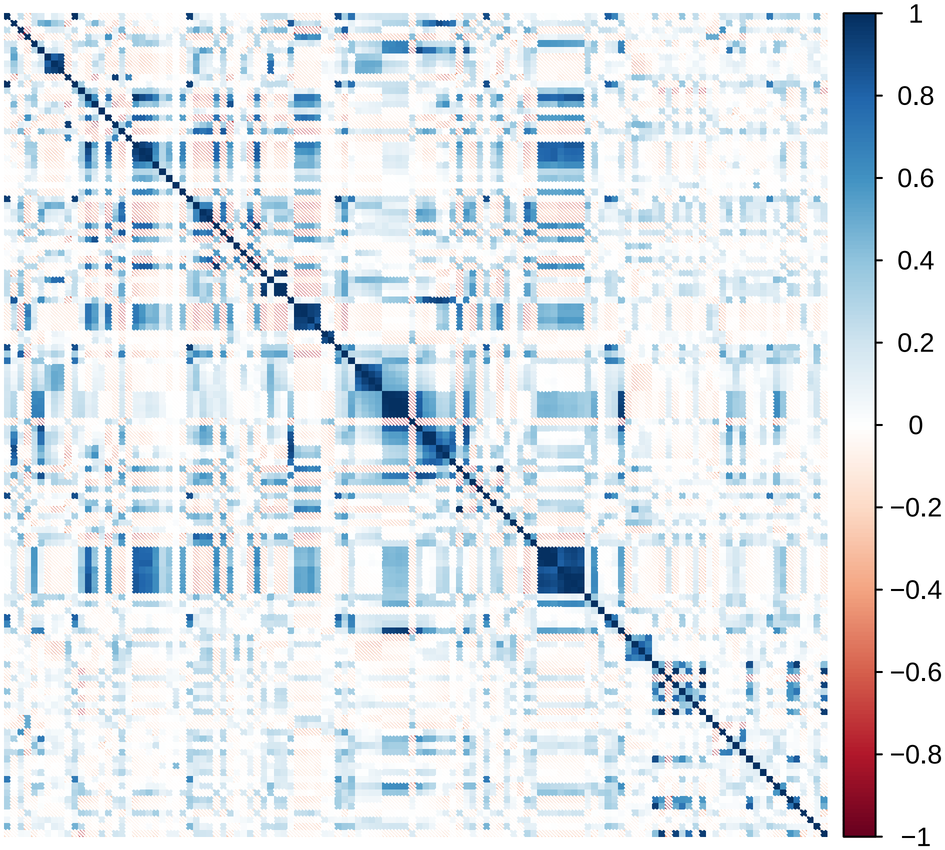
```
## [1] "1 :  7259"
## [1] "2 :  7138"
## [1] "3 :  7018"
## [1] "4 :  6899"
## [1] "5 :  6781"
## [1] "6 :  6664"
## [1] "7 :  6548"
## [1] "8 :  6433"
## [1] "9 :  6319"
## [1] "10 :  6206"
## [1] "11 :  6094"
## [1] "12 :  5983"
## [1] "13 :  5873"
## [1] "14 :  5764"
## [1] "15 :  5656"
## [1] "16 :  5549"
## [1] "17 :  5443"
## [1] "18 :  5338"
## [1] "19 :  5234"
## [1] "20 :  5131"
## [1] "21 :  5029"
## [1] "22 :  4928"
## [1] "23 :  4828"
## [1] "24 :  4729"
## [1] "25 :  4631"
```

```
## [1] "26 :  4534"
## [1] "27 :  4438"
## [1] "28 :  4343"
## [1] "29 :  4249"
## [1] "30 :  4156"
## [1] "31 :  4064"
## [1] "32 :  3973"
## [1] "33 :  3883"
## [1] "34 :  3794"
## [1] "35 :  3706"
## [1] "36 :  3619"
## [1] "37 :  3533"
## [1] "38 :  3448"
## [1] "39 :  3364"
## [1] "40 :  3281"
## [1] "41 :  3199"
## [1] "42 :  3118"
## [1] "43 :  3038"
## [1] "44 :  2959"
## [1] "45 :  2881"
## [1] "46 :  2804"
## [1] "47 :  2728"
## [1] "48 :  2653"
## [1] "49 :  2579"
## [1] "50 :  2506"
## [1] "51 :  2434"
## [1] "52 :  2363"
## [1] "53 :  2293"
## [1] "54 :  2224"
## [1] "55 :  2156"
## [1] "56 :  2089"
## [1] "57 :  2023"
## [1] "58 :  1958"
## [1] "59 :  1894"
## [1] "60 :  1831"
## [1] "61 :  1769"
## [1] "62 :  1708"
## [1] "63 :  1648"
## [1] "64 :  1589"
## [1] "65 :  1531"
## [1] "66 :  1474"
## [1] "67 :  1418"
## [1] "68 :  1363"
## [1] "69 :  1309"
## [1] "70 :  1256"
## [1] "71 :  1204"
## [1] "72 :  1153"
## [1] "73 :  1103"
## [1] "74 :  1054"
## [1] "75 :  1006"
## [1] "76 :  959"
## [1] "77 :  913"
## [1] "78 :  868"
## [1] "79 :  824"
```

```
## [1] "80 :  781"
## [1] "81 :  739"
## [1] "82 :  698"
## [1] "83 :  658"
## [1] "84 :  619"
## [1] "85 :  581"
## [1] "86 :  544"
## [1] "87 :  508"
## [1] "88 :  473"
## [1] "89 :  439"
## [1] "90 :  406"
## [1] "91 :  374"
## [1] "92 :  343"
## [1] "93 :  313"
## [1] "94 :  284"
## [1] "95 :  256"
## [1] "96 :  229"
## [1] "97 :  203"
## [1] "98 :  178"
## [1] "99 :  154"
## [1] "100 :  131"
## [1] "101 :  109"
## [1] "102 :  88"
## [1] "103 :  68"
## [1] "104 :  49"
## [1] "105 :  31"
## [1] "106 :  14"
## [1] "107 :  -2"
## [1] "108 :  -17"
## [1] "109 :  -31"
## [1] "110 :  -44"
## [1] "111 :  -56"
## [1] "112 :  -67"
## [1] "113 :  -77"
## [1] "114 :  -86"
## [1] "115 :  -94"
## [1] "116 :  -101"
## [1] "117 :  -107"
## [1] "118 :  -112"
## [1] "119 :  -116"
## [1] "120 :  -119"
## [1] "121 :  -121"
## [1] "122 :  -122"
```

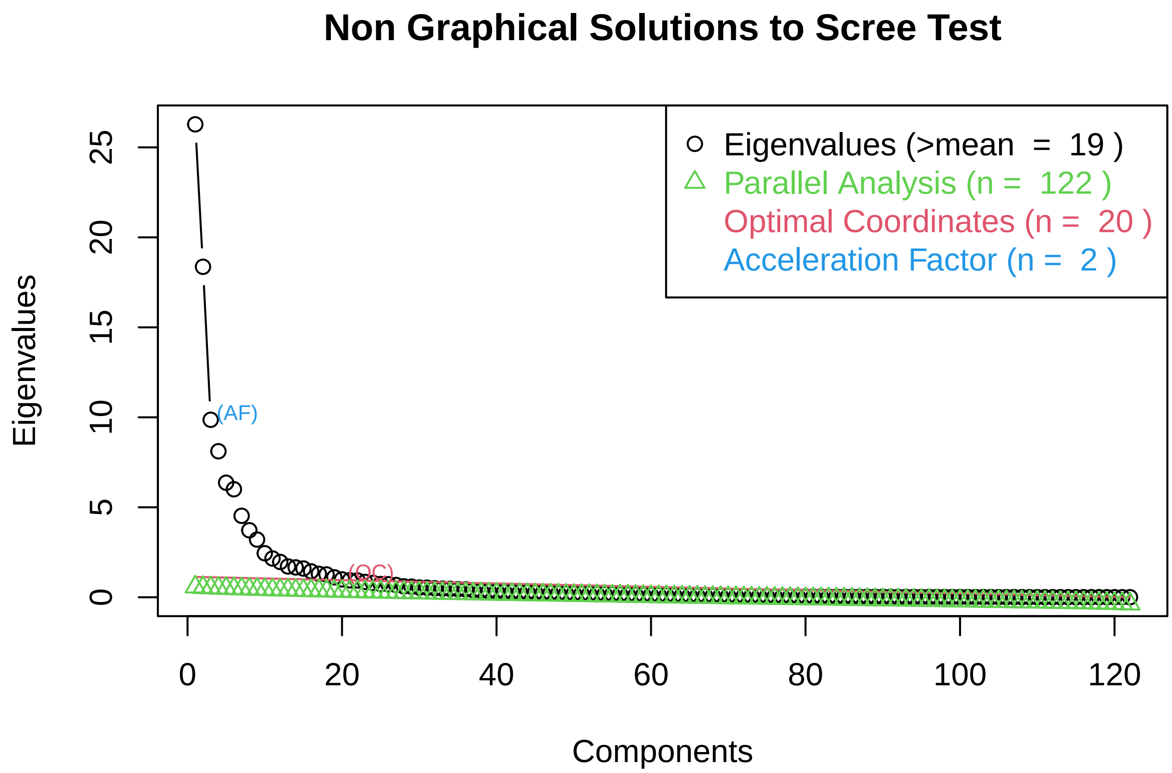
For k < 108 we have the difference is positive which means there exists no solution but we can find an approximate solution. However, for $k \geq 108$ we have that the difference is negative indicating there are infinitely many solutions and the problem is therefore not well-defined. So we must select a k < 108. Let's now compute the correlation matrix which we will use in our further analysis:

```r
R <- cor(crimeData[,-ncol(crimeData)])
corrplot(R, method="shade", tl.pos='n')
```



Using the help of the **nFactors** package we can create a plot that shows both the Scree plot and parallel analysis for the number of factors:

```
ev <- eigen(R) # Get eigenvalues
ap <- parallel(subject=nrow(crimeData),var=ncol(crimeData)-1, rep=100, model = "factors") # Conduct par
nS <- nScree(x=ev$values, aparallel=ap$eigen$qevpea)
plotnScree(nS)
```



The acceleration factor tells us where the elbow of the Scree plot is (n=2), and the optimal coordinates corresponds to an extrapolation if the preceding eigenvalue by a regression line between the eigenvalue coordinates and the last eigenvalue coordinates. We can also find the Kaiser criterion:

```
nS$Components$nkaiser
```

```
## [1] 19
```

Both the visual and analytical analyses support choosing a $k \approx 19$, so we will use k=19 for our factor analysis. We will now carry out our factor analysis:

```
fit <- fapa(R, numFactors = 19)
```

Note that the initial matrix of specific variances used is the squared multiple correlation which we calculate as $1/\text{diag}(R^{-1})$.

Let's now print out our estimated loading matrix

```
fit$loadings
```

```
##              [,1]         [,2]         [,3]         [,4]         [,5]
##  [1,] -0.269380288  0.379537758  0.553014111  0.256565132 -0.218150521
##  [2,]  0.072522440  0.468461664 -0.668836201 -0.070555792 -0.318186309
##  [3,] -0.542206097  0.074464710  0.229719572  0.029879885  0.123965853
##  [4,]  0.567137628 -0.533543164 -0.012358987 -0.070614362 -0.071754092
##  [5,]  0.254235602  0.660034580  0.108174301 -0.048901065  0.015922091
##  [6,] -0.315528520  0.674976796 -0.376252281  0.155167892 -0.107274383
##  [7,] -0.288841723  0.149412734 -0.275940959 -0.572583891 -0.227416377
##  [8,] -0.341016014  0.321682186 -0.060333821 -0.675896488 -0.184625546
##  [9,] -0.296373116  0.201203748 -0.003128732 -0.666802050 -0.153900326
```

```
## [10,] -0.189891656 -0.443181476  0.247077902  0.390018365  0.391940753
## [11,] -0.229151117  0.405077726  0.557746831  0.268260679 -0.161927355
## [12,]  0.315381560  0.386160477  0.229708848  0.142107916  0.427706059
## [13,]  0.909760783  0.231987197  0.031508946  0.169461632 -0.107210725
## [14,]  0.573880987  0.339916839 -0.121710160 -0.292181003 -0.323635685
## [15,] -0.003054331 -0.101867531 -0.247246967 -0.244429413 -0.294147371
## [16,]  0.861920639 -0.130420473  0.238296847  0.019025225 -0.008098138
## [17,] -0.341041067 -0.472433360  0.068348126  0.409958008  0.352864087
## [18,] -0.816728454  0.172009386 -0.172574056  0.186456480  0.091949967
## [19,]  0.039061353 -0.391535851  0.121165895  0.356889342  0.276899701
## [20,]  0.914191254  0.194699981  0.126995981  0.104388261 -0.076785445
## [21,]  0.837689519  0.147503190  0.322768537  0.150480014  0.007400224
## [22,]  0.728399122  0.229899173  0.391574624  0.166325632  0.012561892
## [23,]  0.624566470  0.214293208  0.086258764  0.148044659  0.017344886
## [24,]  0.259823117  0.104317324  0.062519370  0.063740217  0.045744041
## [25,]  0.406495657 -0.004955469  0.084183107  0.227834443 -0.065897721
## [26,] -0.019149547 -0.012906965  0.012595991  0.019137839 -0.011331618
## [27,]  0.610883455  0.040671755  0.182452271  0.171201688  0.015248076
## [28,] -0.415941249  0.349081526  0.495059704  0.252251254 -0.202810303
## [29,] -0.865253456  0.081529039 -0.073001160 -0.162602928 -0.002936539
## [30,] -0.723157980  0.145174605 -0.371695500  0.231946742  0.044960574
## [31,] -0.807154135  0.050965544 -0.329276750  0.293816299  0.104421910
## [32,]  0.679788100  0.191408258  0.362241997 -0.265106639 -0.103760926
## [33,] -0.783319589  0.147559948 -0.217497522  0.182243476  0.066202019
## [34,]  0.644971974  0.178252552  0.060105861 -0.176402709 -0.157746516
## [35,] -0.082298323 -0.148148994 -0.271592988  0.187607280  0.109025105
## [36,]  0.094979271 -0.052975903  0.282894877 -0.299096957 -0.003877692
## [37,] -0.661781455 -0.148779412 -0.375375211  0.157730459  0.128275885
## [38,]  0.727722397  0.119185878  0.356709920 -0.128456327 -0.089623141
## [39,] -0.628874870 -0.067576832  0.349441494  0.015782345  0.214585098
## [40,] -0.287973606  0.453553210  0.231887184 -0.369561522  0.028542688
## [41,] -0.638519254  0.086151115  0.315597992 -0.029190679  0.176482697
## [42,] -0.651339763  0.007075829  0.340021582 -0.004567703  0.201648116
## [43,] -0.088767933  0.548380478 -0.654944072  0.176264156 -0.228353044
## [44,]  0.864453531 -0.057716971 -0.287223180  0.005862793 -0.236330469
## [45,]  0.879336379 -0.104764568 -0.226098269 -0.042841463 -0.205432579
## [46,]  0.854581349 -0.004400842 -0.160644049 -0.021426311 -0.215038906
## [47,]  0.752055683 -0.088472587 -0.321675705  0.039612063 -0.231676142
## [48,] -0.050746575 -0.266095520  0.145733987 -0.264066437  0.074145160
## [49,]  0.091085488 -0.350265364  0.195772138 -0.274326006  0.048562950
## [50,] -0.389524811  0.331460783  0.494551477  0.303025322 -0.176771613
## [51,] -0.718841103  0.296293791  0.172511395  0.061297785  0.132836288
## [52,] -0.183619356  0.571225742  0.325238665  0.275141680 -0.205858638
## [53,] -0.223114705  0.311850355  0.211432429 -0.543904967 -0.088896576
## [54,] -0.262127271  0.377947222  0.203623816 -0.571912139 -0.055266041
## [55,] -0.261668690  0.437020472  0.203005381 -0.566557027 -0.034013535
## [56,] -0.296550867  0.479879751  0.161850446 -0.517008610 -0.034376789
## [57,] -0.046561204  0.864257275  0.001557305 -0.054617573  0.049796076
## [58,] -0.058621672  0.875563179 -0.017844714 -0.024285487  0.054194717
## [59,] -0.046172654  0.886594912 -0.023698180  0.003189753  0.058655756
## [60,] -0.056510940  0.889943797 -0.045259510  0.028911416  0.065460240
## [61,]  0.155511757 -0.789317835  0.282142419 -0.226478891  0.002922799
## [62,] -0.268826599  0.774762947 -0.299098697  0.194713141 -0.002016907
## [63,] -0.373717868  0.660242349 -0.480055294  0.181699119 -0.070588024
```

```
##  [64,] -0.280076214  0.633184675 -0.576677474  0.185440027 -0.100912259
##  [65,]  0.148924101  0.465677224 -0.713716577  0.178458778 -0.282551339
##  [66,]  0.249650391  0.461011785 -0.626626510  0.126933796 -0.241381248
##  [67,] -0.271293061  0.499822728 -0.577812613  0.078672273 -0.226185060
##  [68,]  0.688041193 -0.353911218 -0.245314592  0.342488850 -0.126553892
##  [69,] -0.465164056  0.735213782 -0.342372522  0.084387798 -0.078344813
##  [70,] -0.623260836  0.270527781  0.295057862 -0.134674005  0.254373464
##  [71,]  0.478623402 -0.254073841 -0.252663323  0.134950247 -0.139263175
##  [72,] -0.326957132  0.257818341  0.595903937  0.230852606 -0.155744824
##  [73,]  0.402142298  0.098254361 -0.181860964  0.007044780 -0.003556256
##  [74,]  0.626301795 -0.323897221 -0.294236590  0.363489163 -0.152320719
##  [75,] -0.521393263  0.091364095  0.047226222  0.243469196  0.047398826
##  [76,] -0.210783386 -0.364852051 -0.140423824  0.262276649  0.015671801
##  [77,]  0.192668239  0.097829876 -0.144977435 -0.338266926 -0.310157368
##  [78,] -0.826627053 -0.092256040 -0.150942499 -0.024872229 -0.021208111
##  [79,] -0.554254142  0.163044023 -0.161555933  0.166646198 -0.084919847
##  [80,]  0.684034208  0.510469583  0.152211694  0.160584480  0.037847143
##  [81,]  0.669948333  0.518607687  0.172742654  0.154118914  0.033434464
##  [82,]  0.654222139  0.510362278  0.209180621  0.146783771  0.023342039
##  [83,]  0.722966749  0.491931260  0.154271954  0.098967035 -0.038584986
##  [84,]  0.745154351  0.494010438  0.148590813  0.152112808 -0.023051776
##  [85,]  0.740978260  0.502811340  0.155907965  0.162429742 -0.028889565
##  [86,]  0.749736233  0.458269463  0.131872283  0.177139453 -0.046760087
##  [87,] -0.309394899  0.355385235  0.078510745 -0.069518035  0.014865900
##  [88,]  0.252869119  0.591677235  0.020724770  0.099202403  0.007297273
##  [89,] -0.087833566 -0.092286402  0.039533743  0.259782776  0.092704148
##  [90,] -0.285535084  0.311013029  0.522540703  0.249676526 -0.189763804
##  [91,] -0.222167843  0.322736516  0.413378933  0.214607263 -0.176960790
##  [92,]  0.055290819  0.863330165 -0.078120128  0.176844165  0.072182233
##  [93,] -0.181766277 -0.495127805 -0.257766558  0.176724095  0.089957567
##  [94,]  0.216571523 -0.275341453 -0.189189182  0.710478621  0.179391321
##  [95,] -0.072912374 -0.180311772 -0.206391581  0.703993531  0.305866518
##  [96,] -0.080315366 -0.259354635 -0.295645706  0.507025767  0.153184835
##  [97,] -0.337791769 -0.157339314  0.309594311  0.364344333 -0.616451259
##  [98,] -0.347319949 -0.428964272 -0.066192868 -0.006104840 -0.575807226
##  [99,]  0.302441833  0.163334314 -0.289236011 -0.371637328  0.608188595
## [100,] -0.312738318 -0.429112778 -0.019296635  0.034598190 -0.583756921
## [101,] -0.224268049 -0.013363739  0.422670060  0.298883908 -0.471756012
## [102,] -0.349277595 -0.292346288  0.119653541  0.012098991 -0.497344959
## [103,] -0.025480629  0.236299753  0.208634040 -0.032542286  0.013462391
## [104,] -0.347611171 -0.428543681 -0.065998465 -0.006436086 -0.575745917
## [105,]  0.172651332 -0.173962057  0.066608414  0.013527301 -0.035683406
## [106,]  0.057084182 -0.489399342  0.045084359 -0.218718591  0.043084925
## [107,] -0.236545343  0.151637950  0.146890019  0.146056848  0.099054669
## [108,] -0.182171545  0.497180493 -0.200217564  0.153609160 -0.157015837
## [109,] -0.020406433  0.326486684  0.206874646 -0.110811504  0.060018463
## [110,] -0.268985803  0.488549816 -0.015697817  0.222498777 -0.019414491
## [111,] -0.289506034 -0.147619415  0.310666122  0.368672214 -0.552145526
## [112,] -0.197749236 -0.016551695  0.129167353  0.079768563 -0.092246766
## [113,] -0.215423016  0.103712947 -0.015940659  0.153622129  0.121460198
## [114,] -0.143836310  0.126476871  0.374220768  0.145306269 -0.281563700
## [115,] -0.133337668  0.566075455  0.151649055  0.163747661  0.264936203
## [116,]  0.114905248  0.421956449  0.314229394  0.219240768  0.182156348
## [117,] -0.434372796 -0.182873889  0.319098803  0.199776329 -0.555693481
```

```
## [118,] -0.315574155 -0.135024833  0.328538225  0.340706265 -0.622857525
## [119,]  0.299265031  0.027583939  0.056080824  0.120352886  0.097538721
## [120,] -0.121552930  0.050994318 -0.001836264 -0.147906305  0.027151623
## [121,] -0.232613719  0.269542237  0.331998518  0.095018354  0.064876850
## [122,] -0.279289769 -0.358991478 -0.036163510 -0.025058818 -0.584819527
##                [,6]         [,7]          [,8]          [,9]        [,10]
##   [1,] -0.487812772  0.041071709  0.2156686568 -0.0458199861  0.074705134
##   [2,] -0.250834057 -0.203789166 -0.0180767072 -0.0314411717 -0.042554080
##   [3,] -0.178885625 -0.379186909 -0.5055396710  0.1182237416  0.083357109
##   [4,]  0.065477954  0.239761001  0.4034659739 -0.0459930120 -0.085638185
##   [5,]  0.155039585  0.089589236 -0.0014125158  0.0606864299  0.092927042
##   [6,]  0.148118017  0.171938217  0.0892049986 -0.1676161906 -0.017397727
##   [7,] -0.210794225 -0.404211840  0.2179747239 -0.0035823169 -0.094051584
##   [8,] -0.158986045 -0.179582084  0.1773527827  0.1706445762 -0.269658049
##   [9,] -0.069032139 -0.348970325  0.2838667109  0.0964619781 -0.219906103
##  [10,]  0.435715936 -0.058638826  0.2275780260 -0.1749129349  0.125059274
##  [11,] -0.510716673  0.051264337  0.2166001180 -0.0420029589  0.059311338
##  [12,] -0.336379605  0.105639059  0.0524163450 -0.0004513231 -0.117697987
##  [13,] -0.011993897 -0.102431771 -0.1647482356  0.0675651811  0.077796545
##  [14,] -0.351245452  0.136586110 -0.1991584162  0.2737268330 -0.148051824
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##  [76,]  0.138053456  0.087953444  0.0275574461 -4.251445e-02  0.2354139826
##  [77,] -0.068224256 -0.083995019 -0.0910579647 -1.807793e-02  0.0695093559
##  [78,] -0.019224007  0.003439288  0.1078801846  8.143643e-02  0.0969502315
##  [79,] -0.073170787  0.021992277  0.0860514607  3.726178e-02  0.1288092387
##  [80,]  0.030465981  0.098852234 -0.0451981124  1.998132e-02 -0.1071189111
##  [81,]  0.016603887  0.104803500 -0.0331466411  3.247809e-02 -0.1042940064
##  [82,] -0.002741883  0.101224629 -0.0219868913  4.005427e-02 -0.0964436685
##  [83,]  0.117562077 -0.024604253 -0.0897031639 -1.484465e-02 -0.0235624448
##  [84,]  0.115731966  0.022136628 -0.1067538867 -4.672543e-03 -0.0231796479
##  [85,]  0.120556066  0.036215389 -0.1035268838 -3.075690e-02 -0.0037322648
##  [86,]  0.151208026 -0.002172218 -0.1239403108 -3.400066e-02  0.0047916409
##  [87,]  0.260883492 -0.004630522 -0.2044357772 -1.137090e-01 -0.0947837269
```

```
##  [88,]  0.185306679  0.141071652 -0.2173771977 -8.031760e-03  0.1531770442
##  [89,]  0.268672949  0.071010387 -0.0788857148  3.316391e-02  0.4241353219
##  [90,] -0.042484955  0.135564130 -0.0677626414  1.076820e-01 -0.0342508201
##  [91,] -0.047534982  0.106036236 -0.0760068136  5.700405e-02 -0.0579095828
##  [92,] -0.068408446 -0.005454338 -0.0766650423  8.434455e-04  0.0463196371
##  [93,] -0.067507790  0.097427976  0.0955622744 -7.238316e-02 -0.1043596037
##  [94,] -0.054637855 -0.026492221  0.0157162635 -1.138669e-01  0.0652793623
##  [95,] -0.157001789  0.047415668  0.0838887507 -7.976463e-02 -0.0500921295
##  [96,] -0.101133725  0.085607942  0.0566376937 -1.187756e-01 -0.1273280294
##  [97,]  0.017711346 -0.382503879  0.0867876834 -5.446192e-02  0.0489357023
##  [98,] -0.089634161  0.188520337 -0.0833394708  1.251126e-01 -0.0358800027
##  [99,] -0.052845040  0.362838680 -0.1183705283  9.709321e-02 -0.0710264345
## [100,] -0.043403255  0.149985792 -0.0398040913  8.400429e-02  0.0132382743
## [101,]  0.292615138 -0.105482814  0.0656972343 -2.951909e-01 -0.0197685031
## [102,]  0.199149315  0.305349390 -0.0139375857 -2.230533e-01 -0.0389971558
## [103,]  0.387711080  0.327138130  0.0126051623 -5.092683e-01 -0.0901259545
## [104,] -0.089695193  0.188727833 -0.0832456752  1.247149e-01 -0.0356554306
## [105,] -0.147766494  0.144980388  0.2093083580 -1.868871e-01 -0.0078163538
## [106,]  0.270360164  0.176062137 -0.0339251635  1.491090e-01  0.1113783857
## [107,] -0.213851584 -0.072894301 -0.0978184649 -9.701353e-02 -0.0254728396
## [108,] -0.279126770  0.013304327  0.2187294046 -9.202192e-02  0.0027660132
## [109,]  0.057206824 -0.015728272  0.0483538250 -7.329326e-02 -0.3083114752
## [110,] -0.372572130 -0.094507078  0.1335130077 -1.906842e-01 -0.0309827910
## [111,] -0.036626671 -0.321507991  0.0557373484 -8.559129e-02  0.0491349533
## [112,] -0.043164869  0.421377777  0.3523145920 -3.215146e-01  0.1001574942
## [113,] -0.285014225 -0.035954223 -0.0670881127  2.114363e-02 -0.0899931195
## [114,] -0.042771865  0.130039246 -0.0969062199  1.679445e-02  0.0216910514
## [115,]  0.047434092 -0.008590321  0.0543708542  5.473437e-02  0.0503313393
## [116,]  0.112167342  0.048614964  0.0605833398  1.210025e-01  0.1685667444
## [117,]  0.028163558 -0.084879922 -0.0652703827  5.996119e-02 -0.0143666913
## [118,]  0.034879625 -0.312426722  0.0345230490 -2.937931e-02 -0.0377834404
## [119,]  0.140329225  0.008072236  0.2018906996 -1.513287e-01  0.1520701895
## [120,] -0.181221754  0.106550741 -0.0240117902 -7.677997e-02 -0.0500328469
## [121,]  0.038468436  0.044408463  0.0235276746 -4.465411e-02  0.0565679237
## [122,] -0.089544708  0.223513263 -0.0642668808  7.827901e-02 -0.1259730903
##                [,16]         [,17]        [,18]         [,19]
##   [1,] -0.0450981188  0.0342794989 -0.014548518 -0.0136999151
##   [2,] -0.0104226028  0.0192195531  0.035728980  0.0150326907
##   [3,]  0.0371533152  0.0792244587  0.198891040 -0.1289408204
##   [4,] -0.0526997425 -0.0182784103 -0.182691733  0.0984214330
##   [5,]  0.1006491159 -0.0165933052  0.063330508  0.0023152239
##   [6,]  0.0110946757 -0.1443001975 -0.015049044  0.0272576624
##   [7,]  0.0542003808 -0.1425444483 -0.063442618 -0.1169986841
##   [8,]  0.0201018457 -0.1155241107 -0.066081158 -0.1019247200
##   [9,]  0.0424398569 -0.1532663018 -0.082115974 -0.1607242487
##  [10,]  0.0389293046  0.0584457139  0.008865127 -0.0579861077
##  [11,] -0.0296158173  0.0364907039 -0.016080584 -0.0188244358
##  [12,]  0.1004362217  0.0307171311  0.013866923 -0.0313973469
##  [13,] -0.0242605616  0.0019370266 -0.044262383 -0.0269605553
##  [14,] -0.0263059755 -0.0143754922  0.043187381  0.0053140216
##  [15,]  0.1261539943 -0.1348073833 -0.179406315  0.0308641633
##  [16,]  0.0319273593  0.0072671630  0.058723547 -0.0003050702
##  [17,]  0.0310743286  0.0433089831 -0.013302724 -0.0387066771
##  [18,]  0.0285330702 -0.0253601109 -0.069666309  0.0218478359
```

```
## [19,]  0.1099377981  0.1343389689  0.053181281  0.0342310901
## [20,] -0.0356868305 -0.0271491207 -0.049133297 -0.0243549062
## [21,] -0.0319204699 -0.0530404047 -0.030062457 -0.0404179196
## [22,] -0.0151341756 -0.0467090142 -0.005152240 -0.0590288286
## [23,] -0.1234065967 -0.1667885981  0.104073797 -0.1248189117
## [24,]  0.0463111910  0.0382831415 -0.028776537 -0.0591529192
## [25,] -0.0037883037 -0.0105589563  0.059769362 -0.0845208542
## [26,]  0.0942265100  0.0904190711  0.175173844 -0.1684706654
## [27,]  0.0016825229 -0.0145870516  0.045009469 -0.1125896710
## [28,] -0.0481238894  0.0167613600 -0.003817107 -0.0100165471
## [29,]  0.0005707901 -0.0876475346 -0.049525010 -0.0754700923
## [30,] -0.1384445534 -0.1091168649 -0.023167810 -0.1938203665
## [31,] -0.1374856628 -0.0593833914 -0.033797965 -0.1604767045
## [32,]  0.0163488907 -0.1038101479  0.070138485  0.0132069554
## [33,]  0.0247181413  0.0058432981 -0.089840948  0.0109153764
## [34,] -0.0047270385 -0.0212282399 -0.010696041  0.0486133153
## [35,] -0.2809352058  0.0163522498 -0.028855876 -0.1903562427
## [36,]  0.1392328098 -0.1058802039  0.093521654  0.1552180889
## [37,] -0.2303297358  0.0567835074 -0.037944520 -0.1039366057
## [38,]  0.0326172807 -0.0996135892  0.089109767  0.0296326111
## [39,] -0.0172289066 -0.0610411057 -0.081080421  0.1241918738
## [40,]  0.0259690715 -0.0707852836 -0.062996806 -0.0470737641
## [41,]  0.0410845740 -0.0715841029 -0.098522856  0.1562454890
## [42,]  0.0179153851 -0.0737474377 -0.093158758  0.1505915452
## [43,]  0.0866886029  0.0580704934  0.002958346  0.1135326774
## [44,] -0.0667961436  0.0222663897  0.020297650  0.0079370665
## [45,] -0.0504804011  0.0286021893  0.013430563  0.0449605961
## [46,] -0.0216151390  0.0057825868  0.030503864  0.0141562025
## [47,] -0.1356781363  0.0332946165  0.041894256  0.0472727653
## [48,]  0.0742676208 -0.1833942757  0.083446966  0.0451896126
## [49,]  0.0656481691 -0.1437951438  0.046402103  0.0358189637
## [50,] -0.0221191648  0.0474904577  0.021391733 -0.0062355009
## [51,]  0.0316062330  0.0536599265  0.060269420 -0.0880314223
## [52,] -0.0190980755 -0.0113201838  0.054916890 -0.0234669287
## [53,] -0.1589029966  0.2471828432 -0.094432893  0.0894185131
## [54,] -0.1603095172  0.1684297425 -0.109445989  0.0909309477
## [55,] -0.1260377994  0.0972980021 -0.104832366  0.0545798018
## [56,] -0.1159336100  0.0760188906 -0.106315703  0.0509140763
## [57,]  0.0102967778  0.0576662844  0.090805056  0.0286959856
## [58,]  0.0247130977  0.0373437411  0.088657889  0.0220834565
## [59,]  0.0361103399  0.0092936368  0.099763430  0.0167288486
## [60,]  0.0347027246  0.0020955284  0.103216490  0.0171344171
## [61,]  0.0098779696  0.1292888534 -0.046561875  0.0145567809
## [62,]  0.0189989733 -0.0908991058  0.041167225 -0.0092955353
## [63,]  0.1486518257  0.0553835203 -0.009060055  0.1096939019
## [64,]  0.1364780240  0.0642942460 -0.008672165  0.0979785323
## [65,]  0.0654559847  0.0727964741 -0.005071320  0.0475514004
## [66,]  0.0456903662  0.0619524311  0.023671956  0.0918703354
## [67,]  0.0570567677  0.0239785425 -0.046268122  0.0137689449
## [68,] -0.0065221773  0.0126139678  0.003549017  0.0116204762
## [69,]  0.1046278172 -0.0525549952  0.014636425  0.0106940206
## [70,] -0.0451886348 -0.0076556356 -0.045944028 -0.0451305765
## [71,] -0.0059355686  0.0806653228  0.064991268  0.0298834001
## [72,] -0.0577138898  0.0192497879 -0.038254928 -0.0015849293
```

```
## [73,]  0.0805525957  0.0550936225  0.072183583 -0.0305011626
## [74,]  0.0023527942  0.0117796304 -0.008551379 -0.0097009910
## [75,] -0.0191172648  0.0134352131 -0.004878690  0.1140063109
## [76,] -0.1414039975 -0.0356686275 -0.015306659  0.1153378009
## [77,]  0.0302273693  0.0431198511 -0.059228248 -0.2061059670
## [78,] -0.0904470871 -0.1277841287 -0.033829217 -0.0884829849
## [79,] -0.1039064422 -0.0190325614 -0.003407940 -0.0359454403
## [80,] -0.0215892460 -0.0619953381 -0.194215691 -0.0332443536
## [81,] -0.0340616831 -0.0654044464 -0.186266169 -0.0367810323
## [82,] -0.0376343132 -0.0694643036 -0.175980383 -0.0375670249
## [83,]  0.0210065525  0.0085083789 -0.116680447 -0.0241029027
## [84,] -0.0226981739  0.0049937940 -0.147361562 -0.0324981762
## [85,] -0.0355280843  0.0095135882 -0.147502276 -0.0149467840
## [86,] -0.0313487391 -0.0008326408 -0.147317381 -0.0387078244
## [87,]  0.0120806966 -0.0078505296 -0.154779657  0.0385214539
## [88,] -0.0789668091  0.0820173696 -0.183167354  0.0032597997
## [89,] -0.1625084087  0.1018321292 -0.030026813  0.0607701047
## [90,] -0.0033058282  0.0263454033  0.002860547  0.0342124216
## [91,] -0.0148333549  0.0058674527  0.019225306  0.0128670470
## [92,]  0.0273845912 -0.0272620886  0.101077365 -0.0142772458
## [93,] -0.0905446979 -0.1066418761 -0.109804352  0.0215393395
## [94,]  0.0100006926 -0.0510245583  0.015077447  0.0322109952
## [95,] -0.0227980139 -0.0917199924 -0.066972454  0.0617707825
## [96,] -0.0112413916 -0.1462717791 -0.158021640  0.0480439923
## [97,]  0.0737830165 -0.0058393415 -0.093265053  0.0031636564
## [98,]  0.0059008354  0.1057316866  0.026704910  0.0695571738
## [99,] -0.1084340516  0.0406703691  0.081499724 -0.0154318234
## [100,]  0.0564036547  0.0642143192  0.019648796  0.0609201025
## [101,] -0.0414009412 -0.1441593539  0.144185484  0.0006118322
## [102,] -0.0156406392 -0.2026230238  0.102900550  0.0989939674
## [103,]  0.0158003677 -0.2779090416  0.093227658  0.1624590741
## [104,]  0.0053953156  0.1058589499  0.026686483  0.0695077441
## [105,] -0.1493509670  0.2145151745 -0.207766070  0.1582131936
## [106,]  0.2031374077  0.0311153555 -0.065635100  0.0761401167
## [107,]  0.0550114328  0.2026030999 -0.112745290  0.1182074023
## [108,] -0.2442516521 -0.0953832293  0.070965695 -0.0466598786
## [109,] -0.1611542874  0.1037051647  0.253102757  0.0035225611
## [110,] -0.1712427075  0.1046887884  0.014724515  0.0501204021
## [111,]  0.0789920769 -0.0714064315 -0.062541892 -0.0545776271
## [112,]  0.3895726909  0.3914261714 -0.058493811 -0.3664601032
## [113,]  0.3406165411 -0.0740382506 -0.319577536 -0.0012882795
## [114,] -0.1292320782  0.0270919486 -0.069738640 -0.0823017571
## [115,]  0.1049182584  0.0341404074  0.146241419  0.1400579114
## [116,]  0.0453254005  0.0857532219  0.168707739  0.1738918035
## [117,] -0.0201551402 -0.0425589411  0.043508080 -0.1031887607
## [118,]  0.0265836598 -0.0162039768 -0.057515120  0.0116600248
## [119,]  0.1834727524 -0.1517166801 -0.017913507 -0.0559871027
## [120,]  0.0175356754 -0.0830375810  0.005330511  0.0380596771
## [121,]  0.0453760450 -0.0376710333  0.010273746 -0.0059764371
## [122,] -0.0156774940  0.0828149725  0.111045552  0.0495818524
```

Now let's find the specific variances:

```
# Since the variance of each variable is one (as we are using the correlation matrix) we have the speci
1-fit$h2
```

```
##   [1]  0.024123830  0.072151132  0.079991967  0.077286881  0.355450393
##   [6]  0.130637683  0.105444336  0.046465359  0.045857350  0.042121224
##  [11]  0.027137762  0.259226695  0.024224360  0.043165679  0.492227893
##  [16]  0.096699512  0.047886028  0.143159105  0.328460136  0.033102817
##  [21]  0.039011373  0.059579629  0.415442500  0.876630542  0.654064774
##  [26]  0.729350779  0.496608964  0.046121629  0.068572132  0.122316053
##  [31]  0.052583682  0.066287001  0.204999313  0.071795458  0.399908103
##  [36]  0.224395459  0.112642796  0.061865983  0.094370131  0.105633104
##  [41]  0.062130049  0.038961177  0.030534009  0.029526997  0.027579632
##  [46]  0.120898958  0.154639372  0.169965020  0.109458203  0.121575036
##  [51]  0.107713095  0.262130572  0.185184606  0.056008444  0.054359921
##  [56]  0.128661279  0.070127658  0.054675459  0.044806013  0.042735235
##  [61]  0.069761868  0.064250215  0.101809354  0.086231894  0.010888925
##  [66]  0.084606960  0.141513449  0.078195511  0.051911336  0.131918507
##  [71]  0.459919012  0.109061540  0.351032021  0.082043946  0.475508124
##  [76]  0.471858954  0.238569523  0.147768973  0.533437118  0.050646130
##  [81]  0.051554188  0.067134535  0.089482068  0.038026921  0.029330129
##  [86]  0.043035244  0.433286228  0.268325499  0.437627313  0.242548886
##  [91]  0.489614878  0.043894532  0.283089890  0.095413333  0.162906830
##  [96]  0.331575853  0.016001556  0.001486096  0.044037764  0.058723709
## [101]  0.184456634  0.068953593  0.185932787  0.001351661  0.272015913
## [106]  0.201135241  0.275668362  0.195787813  0.537577123 -0.007919846
## [111]  0.191678851  0.058182313  0.556630765  0.407982125  0.292221548
## [116]  0.362551155  0.257713782  0.093201320  0.642501842  0.834984953
## [121]  0.701146076  0.055964140
```

Let's now find and print the varimax rotation of the loadings which will be more easily interpretable:

```
rotated_loadings <- varimax(fit$loadings)$loadings
rotated_loadings
```

```
## 
## Loadings:
##        [,1]   [,2]   [,3]   [,4]   [,5]   [,6]   [,7]   [,8]   [,9]   [,10]
##   [1,]                             -0.111 -0.965                            
##   [2,]         0.401 -0.551 -0.210  0.138         0.122 -0.214         0.133
##   [3,] -0.244                             -0.199        -0.491         0.594
##   [4,]  0.209 -0.420                       0.208         0.508        -0.509
##   [5,]  0.458  0.492                      -0.102                            
##   [6,] -0.176  0.845 -0.165                             -0.116              
##   [7,] -0.353        -0.256 -0.721  0.111                                   
##   [8,] -0.289  0.132 -0.347 -0.778                             -0.194       
##   [9,] -0.245        -0.115 -0.872                             -0.163       
##  [10,] -0.121 -0.104  0.906  0.213                              0.154       
##  [11,]                             -0.107 -0.952                            
##  [12,]  0.304  0.104                      -0.172                            
##  [13,]  0.886        -0.228  0.147                                          
##  [14,]  0.439        -0.788                                    -0.129       
##  [15,] -0.104        -0.161                      -0.138                     
##  [16,]  0.729 -0.264  0.162                      -0.212  0.173        -0.156
##  [17,] -0.311         0.835  0.148                              0.242       
##  [18,] -0.568  0.326  0.104 -0.106        -0.118        -0.263  0.226  0.154
##  [19,]        -0.207  0.619  0.149                                          
##  [20,]  0.916        -0.140                                                 
##  [21,]  0.933 -0.104         0.148               -0.130                     
```

```
##  [22,]  0.915                0.136               -0.150                0.122
##  [23,]  0.635        -0.117
##  [24,]  0.302
##  [25,]  0.480                0.134
##  [26,]
##  [27,]  0.641 -0.130         0.108
##  [28,] -0.148  0.102               -0.111 -0.932        -0.101
##  [29,] -0.665  0.172  0.131 -0.326        -0.110        -0.249                0.148
##  [30,] -0.583  0.481  0.122                        0.222 -0.141  0.263  0.132
##  [31,] -0.669  0.389  0.158        -0.103         0.285 -0.132  0.274  0.164
##  [32,]  0.752 -0.165        -0.195  0.108        -0.389        -0.158
##  [33,] -0.602  0.323  0.116 -0.138                      -0.245  0.128  0.103
##  [34,]  0.528        -0.625  0.126                       0.184        -0.111
##  [35,] -0.141        -0.123  0.118                0.430  0.103  0.516
##  [36,]  0.151 -0.216  0.229 -0.433  0.105        -0.587 -0.196
##  [37,] -0.672  0.135                              0.418         0.411
##  [38,]  0.782 -0.201                             -0.408        -0.120
##  [39,] -0.363                0.217        -0.180
##  [40,]         0.232        -0.777        -0.188 -0.134 -0.144
##  [41,] -0.331                0.177        -0.168        -0.104
##  [42,] -0.355                0.202        -0.173
##  [43,] -0.144  0.536 -0.616                             -0.198  0.168  0.145
##  [44,]  0.498        -0.226  0.172         0.197         0.253        -0.231
##  [45,]  0.507 -0.130 -0.206  0.149         0.201         0.302        -0.282
##  [46,]  0.548        -0.191  0.126         0.183         0.244        -0.256
##  [47,]  0.385        -0.192  0.194         0.196         0.217        -0.266
##  [48,] -0.127 -0.228
##  [49,]        -0.337
##  [50,]                             -0.134 -0.878        -0.127         0.125
##  [51,] -0.339  0.189        -0.200        -0.274        -0.385         0.405
##  [52,]         0.444               -0.126 -0.704
##  [53,]         0.107        -0.222                             -0.131
##  [54,]         0.168        -0.204                             -0.135
##  [55,]         0.214        -0.193                             -0.149  0.126
##  [56,]         0.277 -0.111 -0.158        -0.119               -0.169  0.148
##  [57,]  0.218  0.844        -0.110        -0.103
##  [58,]  0.210  0.870                      -0.103
##  [59,]  0.226  0.882                      -0.109
##  [60,]  0.211  0.897                      -0.114
##  [61,]        -0.934
##  [62,]         0.945
##  [63,] -0.265  0.723 -0.307               -0.111        -0.238         0.141
##  [64,] -0.239  0.705 -0.354                             -0.235         0.137
##  [65,]         0.445 -0.610                       0.164 -0.169         0.152
##  [66,]         0.386 -0.675         0.101               -0.108  0.166
##  [67,] -0.256  0.559 -0.386                       0.260 -0.193 -0.104  0.240
##  [68,]  0.357 -0.333         0.370         0.159                0.152
##  [69,] -0.261  0.852 -0.183                             -0.183         0.101
##  [70,] -0.325  0.392  0.323 -0.182        -0.149               -0.177 -0.172
##  [71,]  0.235 -0.329 -0.236  0.207         0.134                0.147  0.135
##  [72,]                             -0.103 -0.889               -0.165
##  [73,]  0.278        -0.265 -0.130         0.123                0.438
##  [74,]  0.314 -0.280         0.400         0.171                0.108
##  [75,] -0.347                             -0.311        -0.288  0.189  0.148
```

```
##  [76,] -0.275 -0.229  0.131                             -0.126  0.240
##  [77,]               -0.336  0.149                0.248        -0.610
##  [78,] -0.668                                           -0.178         0.247
##  [79,] -0.375  0.321                                     -0.218  0.135
##  [80,]  0.906  0.288
##  [81,]  0.909  0.289
##  [82,]  0.908  0.272
##  [83,]  0.853  0.253 -0.119                        0.135        -0.130
##  [84,]  0.904  0.244                               0.145
##  [85,]  0.904  0.246                               0.125
##  [86,]  0.895  0.206                               0.169
##  [87,]         0.309  0.237 -0.359                      -0.214 -0.202
##  [88,]  0.460  0.441                0.107          0.289 -0.137 -0.278
##  [89,]                                                          0.165
##  [90,]                             -0.126 -0.838
##  [91,]         0.163               -0.122 -0.673
##  [92,]  0.305  0.908
##  [93,] -0.336 -0.356                                            0.654
##  [94,]  0.132         0.223  0.251 -0.108                       0.698
##  [95,]                0.170  0.324                              0.762
##  [96,] -0.134                0.121                              0.731
##  [97,] -0.144                      -0.909 -0.237
##  [98,] -0.227 -0.110               -0.254  0.137
##  [99,]  0.130                       0.912  0.211
## [100,] -0.184 -0.117               -0.345  0.136
## [101,]                             -0.674 -0.335
## [102,] -0.142                      -0.280
## [103,]                                    -0.210
## [104,] -0.227 -0.109               -0.254  0.137
## [105,]        -0.132                                                      -0.807
## [106,] -0.158 -0.321                       0.111          0.777
## [107,]                                    -0.182        -0.701  0.108  0.108
## [108,]         0.574                                    -0.442 -0.116 -0.277
## [109,]  0.108  0.244                                           -0.156 -0.124
## [110,]         0.360                      -0.165        -0.888        -0.157
## [111,]                             -0.809 -0.255
## [112,]                                    -0.118                      -0.127
## [113,]         0.214  0.190                                     0.189
## [114,]        -0.116                      -0.709  0.115        -0.142
## [115,]  0.115  0.576        -0.115        -0.137 -0.184         0.178
## [116,]  0.349  0.265                      -0.256 -0.304         0.146
## [117,] -0.195         0.106        -0.566 -0.309                       0.140
## [118,]                             -0.843 -0.262
## [119,]  0.201                                    -0.105  0.271  0.120
## [120,]                              0.163
## [121,]         0.115                      -0.411
## [122,] -0.145                      -0.238  0.130
##         [,11]  [,12]  [,13]  [,14]  [,15]  [,16]  [,17]  [,18]  [,19]
##   [1,]
##   [2,]                0.177 -0.114                      -0.156 -0.481
##   [3,]         0.106                0.223          0.124         0.345
##   [4,]                                           -0.136        -0.313
##   [5,]  0.171                      -0.268          0.167 -0.114
##   [6,] -0.154         0.151                               0.105
```

```
##  [7,] -0.114                      -0.115         0.213        -0.228
##  [8,]                                            0.269
##  [9,]                                            0.281
## [10,]                              0.107        -0.107
## [11,]                                     0.173
## [12,]  0.131                              0.752
## [13,]                                     0.112                -0.298
## [14,]               -0.194                0.154                -0.186
## [15,] -0.505                      -0.140 -0.275  0.110  0.115 -0.197
## [16,]                                                          -0.377
## [17,]                              0.143 -0.110 -0.143
## [18,]                0.298               -0.139                 0.343
## [19,]  0.174                                    -0.264 -0.257 -0.121
## [20,]                                                          -0.268
## [21,]                                                   0.117 -0.106
## [22,]                                                   0.114
## [23,]  0.129                              0.209         0.220 -0.121
## [24,]         0.101
## [25,]                0.113               -0.139 -0.139
## [26,]         0.452        -0.210
## [27,]                                     0.102                -0.106
## [28,]
## [29,] -0.166         0.253               -0.154  0.186  0.112  0.258
## [30,] -0.139                       0.125 -0.154         0.223  0.105
## [31,]                0.102         0.180 -0.110         0.126  0.181
## [32,]                                            0.212  0.167 -0.166
## [33,] -0.110         0.306         0.147                        0.222
## [34,]               -0.364                0.189
## [35,]  0.204                                                0.101
## [36,]                                            0.179        -0.130
## [37,]  0.125                             -0.123                 0.166
## [38,]                                            0.118  0.163 -0.179
## [39,]                      -0.112                               0.801
## [40,]  0.101                       0.197         0.242          0.205
## [41,]                             -0.163                        0.829
## [42,]                      -0.111 -0.126                        0.840
## [43,] -0.118         0.194                              -0.198 -0.271
## [44,]                             -0.112        -0.113        -0.659
## [45,]                             -0.116                       -0.612
## [46,]                             -0.102                       -0.562
## [47,]        -0.118               -0.136        -0.115        -0.638
## [48,]               -0.857
## [49,]               -0.873
## [50,]                                                           0.136
## [51,]         0.100                0.245         0.122          0.461
## [52,]
## [53,]                                            0.837
## [54,]                                            0.907          0.111
## [55,]                                            0.879          0.152
## [56,]                                            0.798          0.186
## [57,]  0.139                                     0.320
## [58,]  0.123                                     0.289
## [59,]  0.119                                     0.255
## [60,]  0.111                                     0.221
```

```
##  [61,]               -0.103                             -0.124
##  [62,]
##  [63,] -0.117         0.186                             -0.221
##  [64,] -0.125         0.201                             -0.227 -0.149
##  [65,] -0.106         0.198                      -0.133 -0.210 -0.435
##  [66,]                0.119                0.121 -0.128 -0.155 -0.422
##  [67,] -0.126         0.208 -0.149                      -0.196 -0.136
##  [68,] -0.140                                    -0.246        -0.627
##  [69,]                0.111        -0.115                       0.123
##  [70,]                              0.111         0.142  0.122  0.561
##  [71,]                                           -0.149 -0.111 -0.409
##  [72,]                                                          0.138
##  [73,]  0.257                0.183 -0.270  0.121        -0.122 -0.221
##  [74,] -0.169               -0.133               -0.244        -0.631
##  [75,]                0.195         0.237                       0.210
##  [76,] -0.164         0.163 -0.140  0.408 -0.132 -0.114        -0.145
##  [77,] -0.197                      -0.321                      -0.155
##  [78,] -0.224         0.176               -0.236         0.214  0.313
##  [79,]                0.117         0.173 -0.189         0.138  0.140
##  [80,]
##  [81,]
##  [82,]
##  [83,]                                     0.165        -0.128
##  [84,]                                     0.125        -0.114
##  [85,]                              0.100  0.122        -0.111 -0.106
##  [86,]                                     0.130        -0.116 -0.116
##  [87,]                0.176 -0.260  0.112               -0.199  0.126
##  [88,]                      -0.140  0.246               -0.148
##  [89,]                              0.712
##  [90,]                                                          0.120
##  [91,]                                                          0.101
##  [92,]  0.106                              0.102
##  [93,]                                                         -0.109
##  [94,]                              0.255        -0.283        -0.302
##  [95,]                              0.104  0.118 -0.255
##  [96,] -0.147                                    -0.212
##  [97,]                                    -0.240
##  [98,]                                    -0.912
##  [99,]                                     0.209
## [100,]                                    -0.849
## [101,]  0.203  0.100        -0.390        -0.124
## [102,]         0.133        -0.483        -0.721                0.147
## [103,]         0.175        -0.822         0.117                0.123
## [104,]                                    -0.912
## [105,]         0.122                             -0.119
## [106,]                                                  -0.144
## [107,]               -0.119         0.153               -0.277  0.194
## [108,]                0.162        -0.115                0.347
## [109,]  0.498                      -0.189         0.117         0.161
## [110,]                                                          0.110
## [111,]                                    -0.227
## [112,] -0.121  0.916                      -0.110                0.138
## [113,] -0.390                0.146 -0.232               -0.222  0.173
## [114,]
```

```
## [115,]  0.263                       0.215  0.191        -0.107  0.277
## [116,]  0.203                       0.398  0.144                0.131
## [117,]                                    -0.470                0.117
## [118,]                                    -0.307
## [119,] -0.106  0.175        -0.121         0.357
## [120,] -0.127        -0.103        -0.198         0.165
## [121,]                                     0.172                0.209
## [122,]                                    -0.911
##
##                   [,1]   [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10]
## SS loadings     19.753 14.022 6.268 4.464 4.660 8.311 1.964 4.183 4.351 2.543
## Proportion Var   0.162  0.115 0.051 0.037 0.038 0.068 0.016 0.034 0.036 0.021
## Cumulative Var   0.162  0.277 0.328 0.365 0.403 0.471 0.487 0.522 0.557 0.578
##                 [,11] [,12] [,13] [,14] [,15] [,16] [,17] [,18] [,19]
## SS loadings     1.718 1.336 2.577 1.619 2.129 5.828 4.484 1.286 7.917
## Proportion Var 0.014 0.011 0.021 0.013 0.017 0.048 0.037 0.011 0.065
## Cumulative Var 0.592 0.603 0.624 0.637 0.655 0.703 0.739 0.750 0.815
```

The rotated matrix is more easily interpretable as it "encourages" large (in absolute) values and many near-zero values, this makes it more interpretable as it reduces the number of factors a given feature relies on. In the output above small values aren't printed, hence the empty values. Let's try and interpret some of the factors, starting with factor 1 we can see that multiple features have large values for this factor, let's find the features with the largest absolute values:

```r
colnames(crimeData)[which(abs(rotated_loadings[,1]) > 0.9)]
```

```
## [1] "x.V25" "x.V26" "x.V27" "x.V85" "x.V86" "x.V87" "x.V89" "x.V90"
```

These attributes correspond to the median family income, per capita income, per capita income for caucasians, owner occupied housing lower quartile, median and upper quartile, median rent for rental housing and upper quartile rent for medium housing. This indicates that the first factor is some weighted average of most of the factors with a large emphasis on income and accommodation costs, perhaps it is trying to model the general economic state of the community.

```r
colnames(crimeData)[which(abs(rotated_loadings[,2]) > 0.9)]
```

```
## [1] "x.V66" "x.V67" "x.V97"
```

Factor 2 has a particularly high weighting for the following features: percentage of kids born never to be married, total number of people known to be foreign born, percent of people foreign born. And so perhaps has more of a focus on demographics as opposed to economic data as in factor 1.

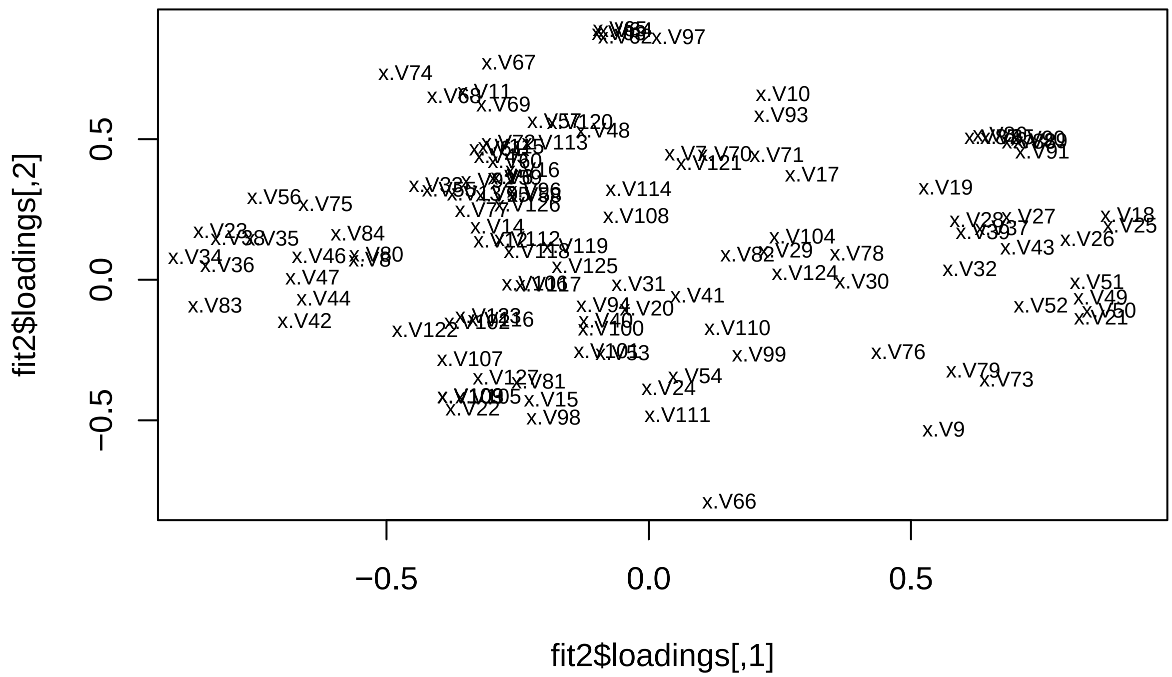
```r
colnames(crimeData)[which(abs(rotated_loadings[,3]) > 0.8)]
```

```
## [1] "x.V15" "x.V22"
```

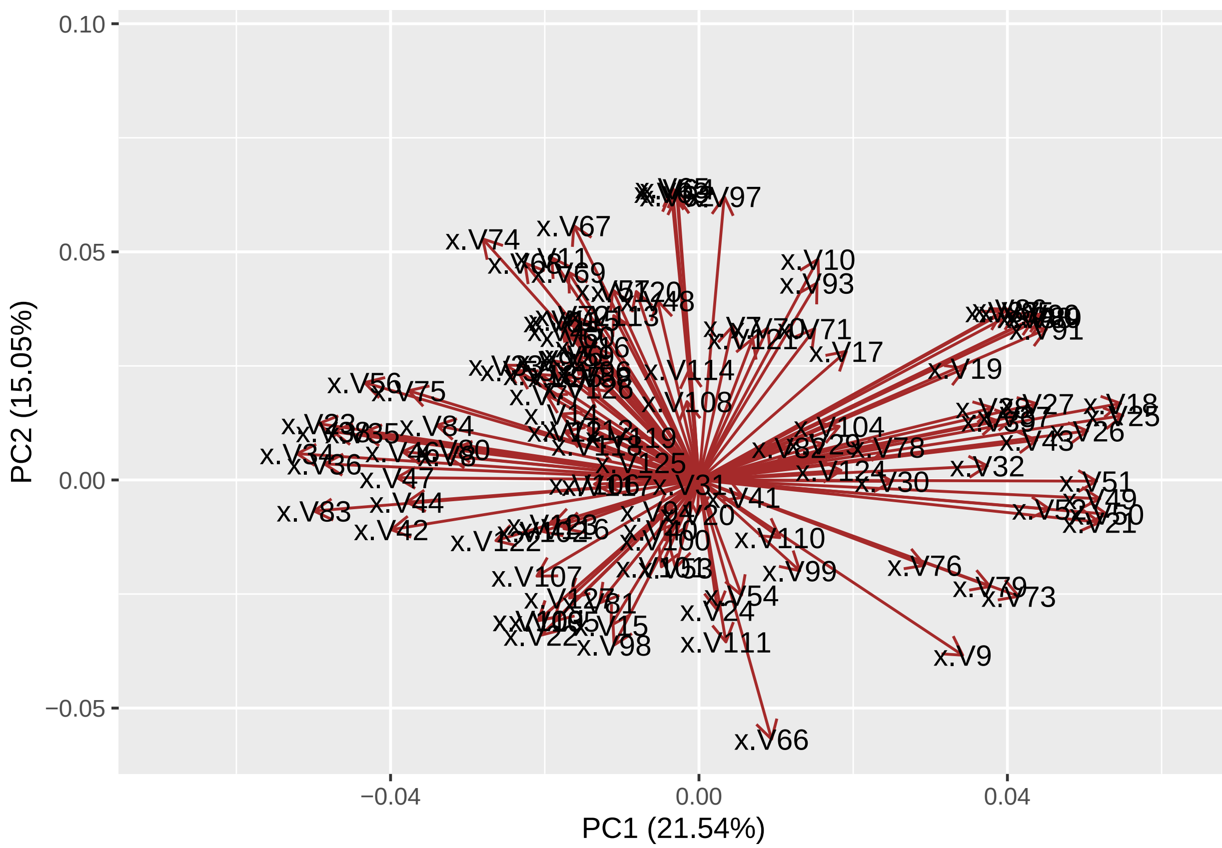
Factor 3 has particularly high weightings for the percentage of population that is 65 and over in age and percentage of households with social security income in 1989. This factor could perhaps be modeling something such as the level of vulnerability of the community.

Let's now let k=2 and compare our two-dimensional reduction of the data to our PCA in the previous portfolio on this same dataset:

```r
fit2 <- fapa(R, numFactors = 2)
plot(fit2$loadings,type="n") # set up plot
text(fit2$loadings,labels=names(crimeData),cex=.7)
```




```
pc_crimeData <- prcomp(~ . -y , crimeData, scale. = TRUE, retx=TRUE)
autoplot(pc_crimeData, data=crimeData[,-ncol(crimeData)], loadings = TRUE, alpha = 0, loadings.colour =
         loadings.label.colour='black', loadings.label = TRUE, loadings.label.size = 4,
         loadings.label.repel=TRUE)
```



Comparing the two plots we see that the contributions of the features to the first two principal components matches up well with the contributions of the features to the factors, this means that the results of the PCA and FA are very similar.

In this scenario it may make more sense to use FA as opposed to PCA as we don't simply want a dimensionality reduction of the dataset but we are trying to create some interpretation of the dataset. By using FA we infer latent variables which can be thought of as "abstract concepts" inferred from the data. For this dataset for example we would like to infer what concepts such as a communities finances and demographics cause correlation in our data.

Task 2

First we load in the data and assemble it all together into one matrix:

```
f1 <- readWave ('portfolio_2_data/audio1.wav')
X1 <- f1@left

f2 <- readWave ('portfolio_2_data/audio2.wav')
X2 <- f2@left

f3 <- readWave ('portfolio_2_data/audio3.wav')
X3 <- f3@left

X0 <- cbind ( X1 , X2 , X3 )
X <- scale(X0, center=TRUE, scale=FALSE)
```

We next use **fastICA** to perform independent component analysis and save the estimated source matrix as S:

```
results <- fastICA(X, n.comp = 3)
S <- results$S
```

We then save the estimated signals as audio files so we can listen to them!

```
savewav(S[ ,1], f = f1@samp.rate, channel = 1, filename = "portfolio_2_results/signal1.wav")

savewav(S[ ,2] , f = f1@samp.rate, channel = 1, filename = "portfolio_2_results/signal2.wav")

savewav(S[ ,3], f = f1@samp.rate, channel = 1, filename = "portfolio_2_results/signal3.wav")
```

Upon listening can confirm that the estimated signals are extremely similar to the actual sources! (just differ by a little bit of noise).