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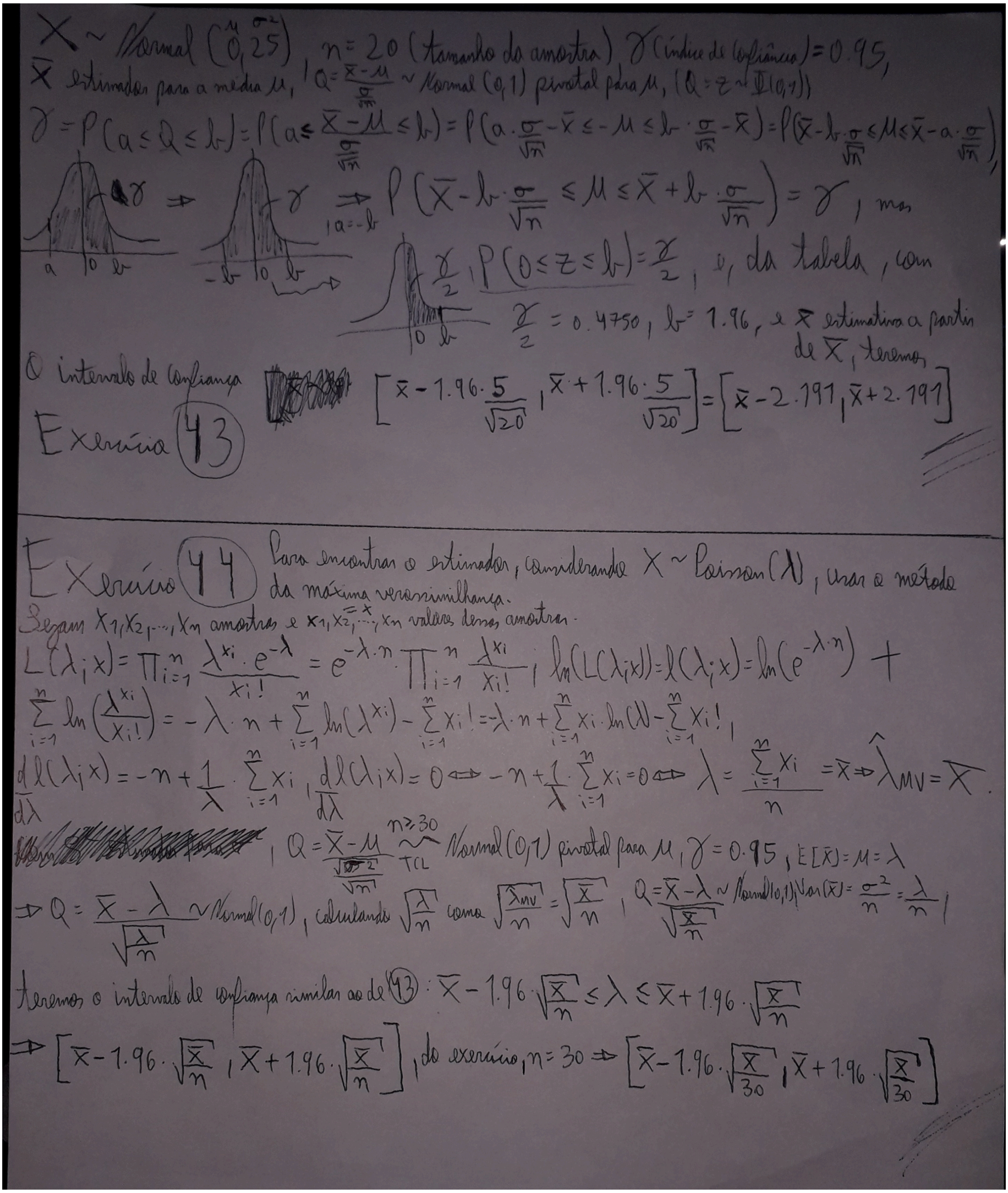
```
library("IRdisplay")
display_jpeg(file="/content/Trab2.jpg")
```



$X \sim \text{Normal}(\overset{\mu}{0}, \overset{\sigma^2}{25})$, $n = 20$ (tamanho da amostra) γ (índice de confiança) $= 0.95$,

$\bar{X}$ estimador para a média μ, $Q = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim \text{Normal}(0,1)$ pivotal para μ, $(Q = Z \sim \Phi(0,1))$

$$\gamma = P(a \leq Q \leq b) = P\left(a \leq \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq b\right) = P\left(a \cdot \frac{\sigma}{\sqrt{n}} - \bar{X} \leq -\mu \leq b \cdot \frac{\sigma}{\sqrt{n}} - \bar{X}\right) = P\left(\bar{X} - b \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} - a \cdot \frac{\sigma}{\sqrt{n}}\right),$$

$\gamma \Rightarrow \gamma \underset{a=-b}{\Rightarrow} P\left(\bar{X} - b \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + b \cdot \frac{\sigma}{\sqrt{n}}\right) = \gamma$, mas

a 0 b -b 0 b

$\frac{\gamma}{2}$, $P(0 \leq Z \leq b) = \frac{\gamma}{2}$, e, da tabela, com

0 b

$\frac{\gamma}{2} = 0.4750$, $b = 1.96$, e $\bar{x}$ estimativa a partir de $\bar{X}$, teremos

O intervalo de confiança $\left[\bar{x} - 1.96 \cdot \frac{5}{\sqrt{20}}, \bar{x} + 1.96 \cdot \frac{5}{\sqrt{20}}\right] = \left[\bar{x} - 2.191, \bar{x} + 2.191\right]$

Exercício 43

Exercício 44 Para encontrar o estimador, considerando $X \sim \text{Poisson}(\lambda)$, usar o método da máxima verossimilhança.

Sejam $X_1, X_2, \dots, X_n$ amostras e $x_1, x_2, \dots, x_n$ valores dessas amostras.

$$L(\lambda; x) = \prod_{i=1}^{n} \frac{\lambda^{x_i} \cdot e^{-\lambda}}{x_i!} = e^{-\lambda \cdot n} \cdot \prod_{i=1}^{n} \frac{\lambda^{x_i}}{x_i!}, \quad \ln(L(\lambda; x)) = \ell(\lambda; x) = \ln(e^{-\lambda \cdot n}) + \sum_{i=1}^{n} \ln\left(\frac{\lambda^{x_i}}{x_i!}\right) = -\lambda \cdot n + \sum_{i=1}^{n} \ln(\lambda^{x_i}) - \sum_{i=1}^{n} x_i! = -\lambda \cdot n + \sum_{i=1}^{n} x_i \cdot \ln(\lambda) - \sum_{i=1}^{n} x_i!,$$

$$\frac{d}{d\lambda} \ell(\lambda; x) = -n + \frac{1}{\lambda} \cdot \sum_{i=1}^{n} x_i, \quad \frac{d\,\ell(\lambda; x)}{d\lambda} = 0 \Leftrightarrow -n + \frac{1}{\lambda} \cdot \sum_{i=1}^{n} x_i = 0 \Leftrightarrow \hat{\lambda} = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x} \Rightarrow \hat{\lambda}_{MV} = \bar{X}.$$

$Q = \frac{\bar{X} - \mu}{\frac{\sqrt{\sigma^2}}{\sqrt{n}}} \underset{TCL}{\overset{n \geq 30}{\sim}} \text{Normal}(0,1)$ pivotal para μ, $\gamma = 0.95$, $E[\bar{X}] = \mu = \lambda$

$\Rightarrow Q = \frac{\bar{X} - \lambda}{\sqrt{\frac{\lambda}{n}}} \sim \text{Normal}(0,1)$, calculando $\sqrt{\frac{\lambda}{n}}$ como $\sqrt{\frac{\hat{\lambda}_{MV}}{n}} = \sqrt{\frac{\bar{X}}{n}}$, $Q = \frac{\bar{X} - \lambda}{\sqrt{\frac{\bar{X}}{n}}} \sim \text{Normal}(0,1)$, $Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{\lambda}{n}$,

teremos o intervalo de confiança similar ao de 43: $\bar{X} - 1.96 \cdot \sqrt{\frac{\bar{X}}{n}} \leq \lambda \leq \bar{X} + 1.96 \cdot \sqrt{\frac{\bar{X}}{n}}$

$\Rightarrow \left[\bar{X} - 1.96 \cdot \sqrt{\frac{\bar{X}}{n}}, \bar{X} + 1.96 \cdot \sqrt{\frac{\bar{X}}{n}}\right]$, do exercício, $n = 30 \Rightarrow \left[\bar{X} - 1.96 \cdot \sqrt{\frac{\bar{X}}{30}}, \bar{X} + 1.96 \cdot \sqrt{\frac{\bar{X}}{30}}\right]$

```
is_in_IC <- function(IC, value) {
    lower_bound <- IC[1]
    upper_bound <- IC[2]
    return(value >= lower_bound && value <= upper_bound)
}
```

```
# 43
# Confidence Interval for the mean. Built with Normal(0, 25) (sd = 5) and Normal(0,1), Gama = 0.95, X_ for the mean
IC1 <- function(samples) {
    samples_mean <- mean(samples)
    lower_bound <- samples_mean - 2.191
    upper_bound <- samples_mean + 2.191
    interval <- c(lower_bound, upper_bound)
    return(interval)
}

p_correct_IC1 <- function(number_IC) {
    real_mean = 0
    correct_IC = 0
    for(i in 1:number_IC) {
        samples <- rnorm(n = 20, mean = real_mean, sd = 5)
        ic <- IC1(samples)

        if(is_in_IC(ic, real_mean))
            correct_IC <- correct_IC + 1
    }
    return(correct_IC / number_IC)
}

for(i in 1:5)
    print(p_correct_IC1(100))
```

```
[1] 0.95
[1] 0.98
[1] 0.95
[1] 0.97
[1] 0.94
```

```
# 44
# Confidence Interval for the lambda. Built with Normal(0,1), Gama = 0.95, X_ for the lambda
IC2 <- function(samples) {
    samples_mean <- mean(samples)
    lower_bound <- samples_mean - 1.96 * sqrt(samples_mean / 30)
    upper_bound <- samples_mean + 1.96 * sqrt(samples_mean / 30)
    interval <- c(lower_bound, upper_bound)
    return(interval)
}

p_correct_IC2 <- function(number_IC) {
    real_lambda = 3.25
    correct_IC = 0

    for(i in 1:number_IC) {
        samples <- rpois(n = 30, lambda = real_lambda)
        ic <- IC2(samples)

        if(is_in_IC(ic, real_lambda))
            correct_IC <- correct_IC + 1
    }
    return(correct_IC / number_IC)
}

for(i in 1:5)
    print(p_correct_IC2(100))
```

```
[1] 0.97
[1] 0.95
[1] 0.95
[1] 1
[1] 0.94
```