Secure & Dependable Systems
Spring 2017
Assignment 04

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Problem 4.1 *Verification*:

Factorial:

```
fun EuclideanAlgorithm(m : int, n : int) : int =
    x := m
    y := n
    while x ≠ y
        if x < y
            y := y − x
        else
            x := x − y
    return x
```

Precondition:

`P(n) := n > 0`

Postcondition:

`Q (n, product) := n! == product`

Loop invariant:

`I := product == (factor-1)!`

Partial Correctness (Somewhat informal):

This algorithm is correct because the loop invariant will always be true since $n!$ is defined as $n*(n-1)!$ It will always terminate because factor starts at 1 and increases by one each time round the loop, the factor must therefore reach n.

Euclidean:

```
fun factorial(n : ℕ) : ℕ =
    product := 1
    factor := 1
    while factor ≤ n
        product := product · factor
        factor := factor + 1
    return product
```

Precondition:

`P(m, n) := m > 0 , n > 0`

Postcondition:

`Q (m, n, x) := x == GCD(m, n)`

Loop invariant:

```
I = GCD(x, y) == GCD(m, n)
```

Partial Correctness:

Base Case: When the code first reaches the loop test, x is still equal to m, and y is still equal to n, so the loop invariant trivially holds.

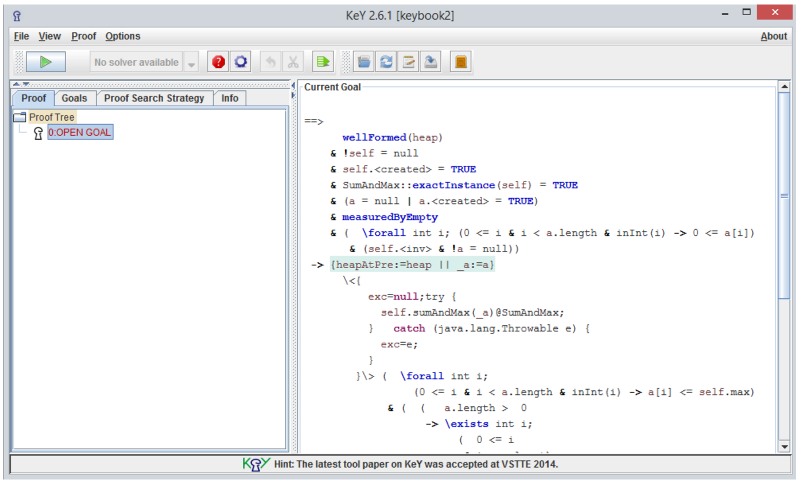
Induction: Assume that the loop invariant holds at the loop test, and also that the loop test passes. Let x_f and y_f be the values of m and n at the bottom of the loop. Consider two cases. If $(x < y)$, then $y_f = y - x$, y_f is positive, and $GCD(x, y_f) = GCD(x, y)$, because any number that divides x and y also divides $y - x$. The other case is similar, except that x_f might be 0. In both cases, $GCD(x_f, y_f) = GCD(m, n) = GCD(m, n)$, and $x_f \geq 0$, and $y_f \geq 0$, so the loop invariant holds.**QED**.

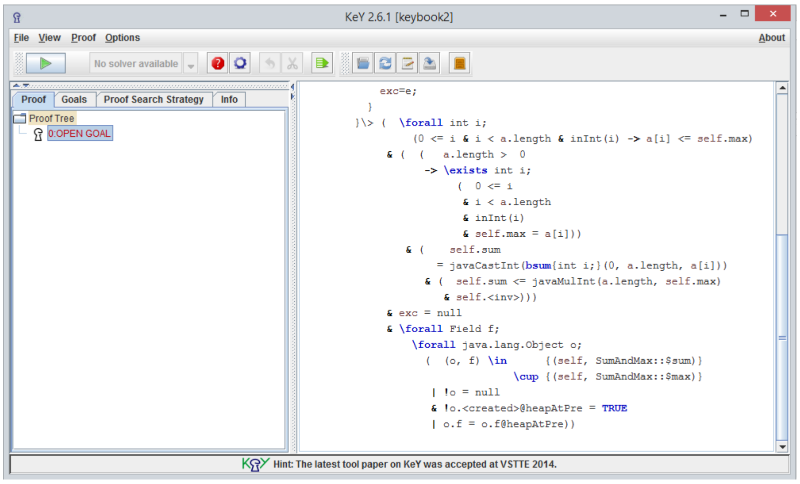
Problem 4.2 *Dynamic Logic: Practice*:

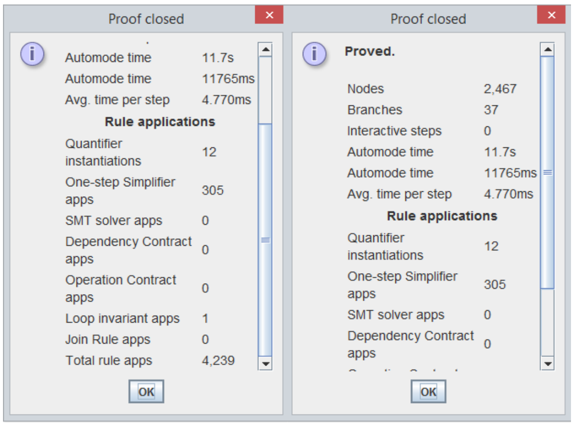
I downloaded the latest binary build for KeY on my system. I loaded a built in Java function, SumAndMax function. The function, as the name suggests, computes and returns the sum and the maximum of a given n element array. The pre and post conditions are presented below.

pre \forall int i; (0 <= i & i < a.length & inInt(i) -> 0 <= a[i]) & (self.<inv> & !a = null)
post \forall int i; (0 <= i & i < a.length & inInt(i) -> a[i] <= self.max) & ((a.length > 0 -> \exists int i; (0 <= i & i < a.length & inInt(i)

The following succession of snapshots is walks us through the program, the displaying the code used, the proof tree and a summary of the process.







Problem 4.3 *Soundness:*

while:

`while` $\phi[\delta]$ `do` $\delta \coloneqq R[\delta]$

Here, $l[\delta]$ is the loop invariant, $\phi[\delta]$ and $R[\delta]$ are the loop condition and function representing the update of the variable δ performed in the loop body, respectively.

$$\forall_{\delta:\ l[\delta]} f[\delta] = \begin{cases} \delta & \text{if } \neg\phi[\delta] \\ f[R[\delta]] & \text{if } \phi[\delta] \end{cases} \tag{1}$$

$$\forall_{\delta:\ l[\delta]} l[R[\delta]] \tag{2}$$

$$\forall_{\delta:\ l[\delta]} \wedge \left\{ \begin{array}{c} \neg\phi[\delta] \implies \pi[\delta] \\ \phi[\delta] \wedge \pi[R[\delta]] \implies \pi[\delta] \end{array} \right\} \implies \forall_{\delta:\ l[\delta]} pi[\delta] \tag{3}$$

and finally:

$$\forall_{\delta:\ l[\delta]} l[f[\delta]] \tag{4}$$

To summarize: (4) follows from the semantics (1) and the termination condition (3).

if:

$$\frac{\{B \wedge P\} S \{Q\}\ ,\ \{\neg B \wedge P\} T \{Q\}}{\{P\} \text{ if } B \text{ then } S \text{ else } T \text{ endif } \{Q\}}$$

A postcondition Q common to the then and else part is also a postcondition of the whole if...endif statement. In the then and the else part, the unnegated and negated condition B can be added to the precondition P, respectively. The condition, B, must not have side effects.

if `B` then `S` else `T` endif is equivalent to :

bool b $\coloneqq$ true; while `B` $\wedge$ `b` do `S`; `b` $\coloneqq$ false done; `b` $\coloneqq$ true; while $\neg$ `B` $\wedge$ `b` do `T`; b $\coloneqq$ false done.