This is just an extension/more practice from the previous class, covering more linear transformations.

Some more examples of linear transformations are shears, reflections, rotations, projections, and scaling. There are more examples but these are common ones to remember.

Horizontal Shears: $\begin{bmatrix} 1 & K \\ 0 & 1 \end{bmatrix}$ the horizontal components stay the same $T(\vec{e_1}) = \vec{e_1}$, anything with a vertical component gets affected

Vertical Shears: $\begin{bmatrix} 1 & 0 \\ K & 1 \end{bmatrix}$ the vertical components stay the same $T(\vec{e_2}) = \vec{e_2}$, anything with a horizontal component gets affected

Rotations by θ counterclockwise: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ if you forget this it's okay, just know how to re-derive this using your picture and the trig unit circle

An important Fact: a linear transformation is determined by what it does to a basis of its domain.

It would be nice if we always knew what $T(\vec{e_1})$ to $T(\vec{e_m})$ was for a transformation from $\mathbb{R}^m \to \mathbb{R}^n$, because then we could say the matrix of T is just

$A = \begin{bmatrix} T(\vec{e_1}) & T(\vec{e_2}) & \dots & T(\vec{e_m}) \end{bmatrix}$ but that won't always be given. We may be given what $T(\vec{v_1}), T(\vec{v_2}), \dots, T(\vec{v_m})$ is for some basis of our domain $(\vec{v_1}, \vec{v_2}, \dots, \vec{v_m})$.

Instead, we can find the matrix of T by expressing an arbitrary vector $\vec{x}$ as a linear combination of the basis we are given and by applying our properties of a linear transformation.

$\vec{x} = c_1\vec{v_1} + c_2\vec{v_2} + \dots + c_m\vec{v_m}$, $T(\vec{x}) = c_1 T(\vec{v_1}) + c_2 T(\vec{v_2}) + \dots + c_m T(\vec{v_m})$, rewrite and get A

You could also instead solve for $T(\vec{e_1}), T(\vec{e_2}), \dots, T(\vec{e_m})$ instead of $T(\vec{x})$ for an arbitrary vector $\vec{x}$, but why do Gauss-Jordan m times instead of once?