Sometimes we will have a system $A\vec{x}=\vec{b}$ which is inconsistent (meaning there isn't an $\vec{x}$ to satisfy this system).

Since we can't find $\vec{x}$, we can find vectors $\vec{x}^*$ to make $A\vec{x}^*$ as close as possible to $\vec{b}$.

These are the least squares solutions of $A\vec{x}=\vec{b}$.

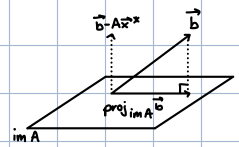
How do we get these? $A\vec{x}^*$ is the vector in the image of A closest to $\vec{b}$. That is, $A\vec{x}^* = \text{proj}_{(\text{im} A)}\vec{b}$.

We could make an orthormal basis of im A and use our projection formula, but we can do something easier.

$\vec{b} - A\vec{x}^*$ is orthogonal to im A. So $(\vec{b}-A\vec{x}^*) \in (\text{im } A)^\perp$

This also means $(\vec{b}-A\vec{x}^*) \in \ker(A^T)$. So $A^T(\vec{b}-A\vec{x}) = \vec{0}$

Rewriting we get: $A^TA\vec{x}^* = A^T\vec{b}$



$\text{proj}_{\text{im}A}\vec{b} = A\vec{x}^*$

We know what A and $\vec{b}$ are, and can get A^T easily. Now to find $\vec{x}^*$ all we need to do is Gauss-Jordan Elimination!

Now that we know how to find the least squares solution of a system, how do we know how good our approximate answer is?

We use the Residual Sum of Squares (RSS) which is $\|\vec{b}-A\vec{x}^*\|$.