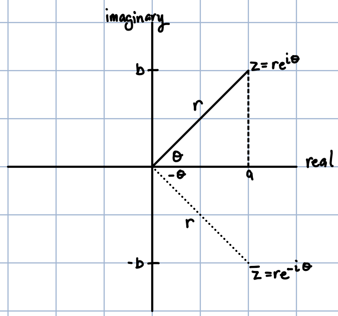
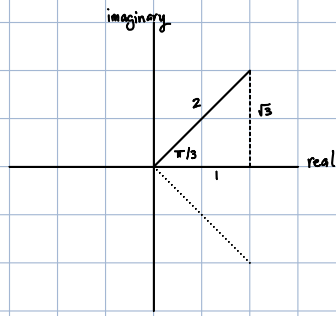
Sometimes we get complex eigenvalues when diagonalizing, and what do we do? If you found your eigenvalues were $1 \pm \sqrt{3}\, i$, your matrix D would be $\begin{bmatrix} 1+\sqrt{3}i & 0 \\ 0 & 1-\sqrt{3}i \end{bmatrix}$ and D^t is $\begin{bmatrix} (1+\sqrt{3}i)^t & 0 \\ 0 & (1-\sqrt{3}i)^t \end{bmatrix}$. It isn't obvious what $(1+\sqrt{3}i)^t$ is, so we convert this to polar form $(re^{i\theta})$ to get our closed form solution.



doing this for $1 \pm \sqrt{3}\, i$

* not drawn to scale



$1+\sqrt{3}i = 2e^{i\pi/3}$

$1-\sqrt{3}i = 2e^{-i\pi/3}$

Now we can write $(1+\sqrt{3}i)^t$ as $(2e^{i\pi/3})^t = 2^t e^{i\pi/3\, t}$. Using Euler's formula, $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, as well as our trig properties $\sin(-a) = -\sin(a)$ and $\cos(-a) = \cos(a)$, we can write D^t as $2^t \begin{bmatrix} \cos\left(\frac{\pi t}{3}\right) + i\sin\left(\frac{\pi t}{3}\right) & 0 \\ 0 & \cos\left(\frac{\pi t}{3}\right) - i\sin\left(\frac{\pi t}{3}\right) \end{bmatrix}$

Now we can write $\vec{x}(t) = SD^tS^{-1}\vec{x}(0)$ in a closed form assuming we solved for S and S^{-1} and were given an initial condition $\vec{x}(0)$.

Since $\vec{x}(0)$ is a vector, it's often easier to multiply right to left and avoid matrix multiplication.

A key fact to remember (that we don't prove, but you can take as a fact).

An $n \times n$ matrix A has n eigenvalues when listed with algebraic multiplicity.

The sum of the n eigenvalues is equal to $\operatorname{tr} A$.

The product of the n eigenvalues is equal to $\det A$.