

Gauss-Jordan Elimination is basically the same as what you've done before, except we write less.

We can write a given linear system as $A\vec{x} = \vec{b}$, where A is the coefficient matrix.

$$\begin{array}{l|l} x_1 - \frac{1}{2}x_2 - x_3 = 0 & \\ -x_1 + x_2 = 0 & \\ -\frac{1}{2}x_2 + x_3 = 0 & \end{array} \quad \begin{array}{c} \left[\begin{array}{ccc} 1 & -\frac{1}{2} & -1 \\ -1 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{array} \right] \\ A \end{array} \quad \begin{array}{c} \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right] \\ \vec{b} \end{array} \quad \begin{array}{c} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 \end{array} \right] \\ \text{Augmented Matrix} \end{array}$$

The number of columns in A = the number of variables in the system. The number of rows in A = the number of equations.

We are done eliminating when A is in reduced row-echelon form (RREF).

A matrix is in reduced row-echelon form (RREF) if:

1) The first non-zero entry in each row is 1 (leading 1 for that row)

2) If a column has a leading 1, every other entry in the column is 0

3) If a row has a leading 1, every row above it has a leading 1 further to the left

↳ Although not immediately obvious, this means any row of all zeros should be at the bottom

Example: Are the following matrices in RREF?

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$
Yes	No	Yes	No	Yes	No
	rule #2		rule #3		rule #1