A linear transformation $T: \mathbb{R}^m \to \mathbb{R}^n$ is invertible if for every $\vec{y}$ in $\mathbb{R}^n$, there exists exactly one $\vec{x}$ in $\mathbb{R}^m$ such that $T(\vec{x}) = \vec{y}$

If matrix A is invertible, then we know this about RREF(A):

- there are no free variable, all variables are defined. This is because we need only one $\vec{x}$ to make $A\vec{x}=\vec{y}$.
- all colums have a leading 1. This ensures each variable is defined.
- no row of all zeros, all rows have a leading one. This is since a row of all zeros means $A\vec{x}=\vec{y}$ is sometimes undefined.
- all of this implies A must be a square, specifically the Identity matrix

Putting it all together, A is invertible if and only if $RREF(A) = I_n$. We also know this now:

$(\vec{v}_1, \dots, \vec{v}_n)$ is a basis of $\mathbb{R}^n \iff \text{rref}\begin{bmatrix}\vec{v}_1 & \cdots & \vec{v}_n\end{bmatrix} = I_n \iff \begin{bmatrix}\vec{v}_1 & \cdots & \vec{v}_n\end{bmatrix}$ is invertible

To Find the inverse of matrix A, set up the system $\left[A \mid I_n\right] \xrightarrow[\text{reduce}]{\text{row}} \left[I_n \mid A^{-1}\right]$