Some review about invertible matrices:

If A is an $n \times n$ matrix, then A is invertible if and only if:

rref $A = I_n$

rank $A = n$

nullity $A = 0$

im $A = \mathbb{R}^n$

Ker $A = \vec{0}$

the columns of A form a basis of $\mathbb{R}^n$

For every $\vec{y}$ in the codomain, there is exactly one $\vec{x}$ in the domain such that $A\vec{x} = \vec{y}$

Now onto some facts about the determinant.

The determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\det(A) = ad - bc$.

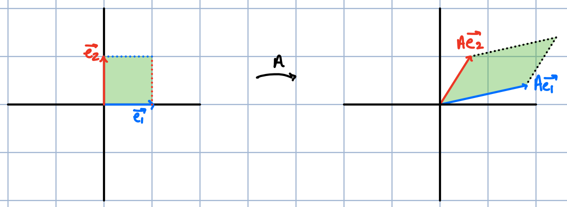
A is invertible if and only if $\det(A) \neq 0$.

If A and B are $n \times n$ matrices, $\det(A) \cdot \det(B) = \det(AB)$

If A is an invertible $n \times n$ matrix, then $\det(A^{-1}) = \dfrac{1}{\det(A)}$

For a triangular matrix, the determinant is just the product of the diagonal entries.

A geometric interpretation of the determinant for a 2×2 matrix:



$|\det A|$ = area of the output parallelogram

To find the determinant of an $n \times n$ matrix, find the determinant using minors.

Pick the row or column with the most zeros to do less work. Remember the "checkerboard" pattern

$$A = \begin{bmatrix} + & - & + & \cdots \\ - & + & & \\ + & & \ddots & \\ \vdots & & & + \end{bmatrix}$$

Example: Find the determinant of A. $A = \begin{bmatrix} 3 & -2 & 6 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ (→ second row)

Expand along the second row: $\det A = -0 \cdot \det \begin{bmatrix} -2 & 6 \\ 0 & 3 \end{bmatrix} + 1 \cdot \det \begin{bmatrix} 3 & 6 \\ 2 & 3 \end{bmatrix} - 0 \cdot \det \begin{bmatrix} 3 & -2 \\ 2 & 0 \end{bmatrix}$

$\det A = 0 + (9 - 12) - 0$

$\det A = -3$