The Method of Elimination is a process to solve linear systems.

Method of Elimination has 3 operations:

1) Swap two rows

2) Add/Subtract a multiple of one row to another

3) Multiply/Divide a row by a nonzero number

Leading Variable: The First variable in each row after you are done eliminating

Free Variable: All non-leading variables

Consistent System: there is at least 1 solution to the system of equations

Inconsistent System: there is no solution to the system of equations

Example: Solve the following system using the method of elimination

$$\left|\begin{array}{l} x_1 = \frac{1}{2}x_2 + x_3 \\ x_2 = x_1 \\ x_3 = \frac{1}{2}x_2 \end{array}\right| \quad \left|\begin{array}{rrrcl} x_1 & -\frac{1}{2}x_2 & -x_3 & = & 0 \\ -x_1 & +x_2 & & = & 0 \\ & -\frac{1}{2}x_2 & +x_3 & = & 0 \end{array}\right| \begin{array}{l} \\ +\mathrm{I} \\ \\ \end{array} \quad \left|\begin{array}{rrrcl} x_1 & -\frac{1}{2}x_2 & -x_3 & = & 0 \\ & \frac{1}{2}x_2 & -x_3 & = & 0 \\ & -\frac{1}{2}x_2 & +x_3 & = & 0 \end{array}\right| \begin{array}{l} \\ \times 2 \\ \\ \end{array}$$

$$\left|\begin{array}{rrrcl} x_1 & -\frac{1}{2}x_2 & -x_3 & = & 0 \\ & x_2 & -2x_3 & = & 0 \\ & -\frac{1}{2}x_2 & +x_3 & = & 0 \end{array}\right| \begin{array}{l} +\frac{1}{2}\mathrm{II} \\ \\ +\frac{1}{2}\mathrm{II} \end{array} \quad \left|\begin{array}{rrrcl} x_1 & & -2x_3 & = & 0 \\ & x_2 & -2x_3 & = & 0 \\ & & 0 & = & 0 \end{array}\right|$$

We are done eliminating because every row has a leading variable, except the last, but that can't be simplified any further.

Leading Variables: x_1, x_2

Free Variable: x_3

$x_1 = 2x_3$

$x_2 = 2x_3$

$x_3 = t$ where t is any real number

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

This is a consistent system that has infinitely many solutions, since we have a free variable.

Example 2: Inconsistent system

$$\left|\begin{array}{rcl} x + y & = & 2 \\ x + y & = & 7 \end{array}\right| \begin{array}{l} \\ -\mathrm{I} \end{array} \quad \left|\begin{array}{rcl} x + y & = & 2 \\ 0 & = & 5 \end{array}\right|$$

If an equation is ever $0 =$ something that is not zero, then we can stop.

This is an inconsistent system and has no solution.