A matrix $A$ is diagonalizable if we can write $A = SDS^{-1}$, where $S$ is invertible and $D$ is diagonal.

A matrix $A$ is similar to matrix $B$ if $A = SBS^{-1}$ for some invertible matrix $S$.

Going back to Discrete Dynamical Systems, we can write our solution $\vec{x}(t) = A^t \vec{x}(0)$ more simply as $\vec{x}(t) = c_1(\lambda_1)^t \vec{v}_1 + \dots + c_n(\lambda_n)^t \vec{v}_n$

where $(\vec{v}_1, \dots, \vec{v}_n)$ is an eigenbasis for $A$, and $(\lambda_1, \dots, \lambda_n)$ are the associated eigenvalues.

Instead we can write $\vec{x}(t) = SD^tS^{-1}\vec{x}(0)$ where $S = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix}$ and $D = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$

This comes from the idea of coordinates; if $A = SDS^{-1}$, then $A^t = \underbrace{(SDS^{-1})(SDS^{-1})\cdots(SDS^{-1})}_{t \text{ of these terms}}$, since the $S$ and $S^{-1}$ cancel and $\underbrace{D \cdot D \cdots}_{t \text{ terms}}$ is $D^t$.

This is useful since diagonal matrices are easy to multiply; the result is just a diagonal matrix where each entry is a product of the diagonal entries.

Another thing we can realize is that we need to have $S$ to diagonalize $A$, where the columns of $S$ form an eigenbasis for $A$.

So $A$ being diagonalizable is the same as $A$ having an eigenbasis!

What happens when we have complex eigenvalues? Is $D^t$ still easy to solve? These are questions to think about for the future.