To solve problems that ask you to find the matrix of a transformation (in $\mathbb{R}^2$ for ease of writing)

1) Pick a convenient basis. Find 2 vectors $\vec{v_1}$ and $\vec{v_2}$ that you can transform easily, i.e. $T(\vec{v_1})$, $T(\vec{v_2})$ are easy to find

2) Set up your problem. You want to find the matrix A, which you get by figuring out what T does to an arbitrary vector $\vec{x}$

   a) express $\vec{x}$ as a linear combination of your basis and solve for c_1 and c_2. $c_1\vec{v_1} + c_2\vec{v_2} = \vec{x} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = ?$

   b) use the properties of a linear transformation to solve for $T(\vec{x})$. $T(c_1\vec{v_1} + c_2\vec{v_2}) = T(\vec{x}) \Leftrightarrow c_1T(\vec{v_1}) + c_2T(\vec{v_2}) = T(\vec{x})$

   simplify the left side of the equation, and you'll get $A\vec{x}$. Now you've found A! $A\vec{x} = T(\vec{x})$

To prove something is | isn't a linear transformation:

1) Does $T(\vec{0}) = \vec{0}$? If no, not a linear transformation. If yes, continue checking 2 and 3.

2) Does $T(\vec{x}) + T(\vec{y}) = T(\vec{x} + \vec{y})$? use arbitrary vectors $\vec{x}$ and $\vec{y}$, never specific vectors with values.

3) Does $T(k\vec{x}) = kT(\vec{x})$? use an arbitrary vector $\vec{x}$ and scalar k, never specific vectors or specific values.

For questions involving bases, how do you know you have a basis? How do you know you don't? What does that mean for the system? Linear combinations?

Answering these questions will give you insight on how to go about solving these problems.

Review the other guides for deeper insights into topics, and good luck on the mini exam ☺