Given n vectors $(\vec{v_1}, \dots, \vec{v_n})$ that form a basis of $\mathbb{R}^n$, this means

1) you can express any vector $\vec{x}$ in $\mathbb{R}^n$ as a linear combination $c_1\vec{v_1} + \dots + c_n\vec{v_n}$

   ↳ this means the system $c_1\vec{v_1} + \dots + c_n\vec{v_n}$ is consistent for every vector $\vec{b}$ in $\mathbb{R}^n$

      ↳ this means we can never row reduce to have a row of all zeros, so...

2) the matrix $\begin{bmatrix} \vec{v_1} & \vec{v_2} & \dots & \vec{v_n} \end{bmatrix}$ (this matrix has each vector as a column) row reduces to the Identity Matrix I_n

   ↳ this means there is only <u>one</u> way to express any vector $\vec{x}$ as a linear combination $c_1\vec{v_1} + \dots + c_n\vec{v_n}$

The Identity matrix I_n is a square matrix with n rows and n colums, with a leading one across the diagonal and zeros everywhere else

To multiply two matrices, always check the dimensions align. If A is $p \times m$ and B is $m \times n$, then AB is a $p \times n$ matrix, but BA is not defined.

Example: $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 4 \end{bmatrix}$ Solve AB and BA if possible.

AB $\underbrace{(2 \times \overbrace{1) \times (1}^{\text{must match}} \times 2)}_{\text{resulting matrix}} = 2 \times 2$ matrix

BA $\overbrace{(1 \times \underbrace{2) \times (2}_{\text{match}} \times 1)}^{\text{resulting matrix}} = 1 \times 1$ matrix

$$B = \begin{bmatrix} -3 & 4 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ -6 & 8 \end{bmatrix} AB$$

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$B \begin{bmatrix} -3 & 4 \end{bmatrix} \begin{bmatrix} -3+8 \end{bmatrix} \quad BA = \begin{bmatrix} 11 \end{bmatrix}$$