Ch₁

```
In [1]:
        #第1章のプログラムは、事前に下記が実行されていることを仮定する。
        import numpy as np
        import matplotlib.pyplot as plt
        from matplotlib import style
        style_use("seaborn-ticks")
In [2]:
        B = np.random.randn(5,5)
        A = B.T@B
In [3]:
        for i in range(5):
          x = np_random_randn(5)_reshape(-1,)
          print(x.T@A@x)
       6.44550819001953
       2.8611258593266715
       26.50609897398167
       39.920400454824595
       10.068269950769912
       2次形式の全ての値は非負となっている
       2
In [4]:
        def k(x, y, lam):
          return D(np.abs((x - y) / lam))
ln [5]: n = 250
        x = 2 * np.random.normal(size=n)
        y = np.sin(2 * np.pi * x) + np.random.normal(size=n) / 4 # データ生成
        def D(t):
                      # 関数定義 D
          return np.maximum(0.75 * (1 - t**2), 0)
        def k(x, y, lam): # 関数定義 K
          return D(np.abs((x - y) / lam))
        def f(z, lam): # 関数定義 f
          S = 0
          T = 0
          for i in range(n):
            S = S + k(x[i], z, lam) * y[i]
            T = T + k(x[i], z, lam)
          return S / T
        plt_figure(num=1, figsize=(15, 8), dpi=80)
        plt_xlim(-3, 3)
        plt_ylim(-2, 3)
        plt_xticks(fontsize=14)
        plt_yticks(fontsize=14)
        plt_scatter(x, y, facecolors="none", edgecolors="k", marker="o")
```

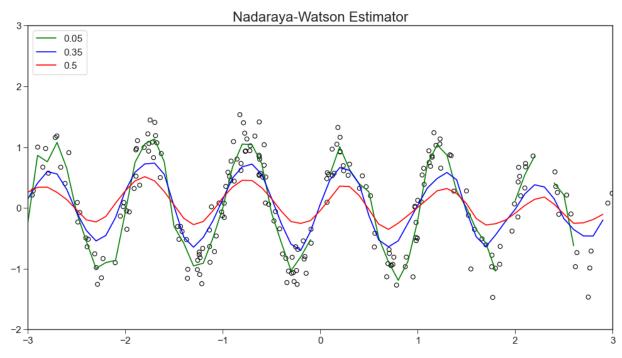
```
xx = np.arange(-3, 3, 0.1)
yy = [[] for _ in range(3)]
lam = [0.05, 0.35, 0.50]
color = ["g", "b", "r"]
for i in range(3):
    for zz in xx:
        yy[i].append(f(zz, lam[i]))
    plt.plot(xx, yy[i], c=color[i], label=lam[i])

plt.legend(loc="upper left", frameon=True, prop={"size": 14})
plt.title("Nadaraya-Watson Estimator", fontsize=20)
```

<ipython-input-5-07b6850729b4>:20: RuntimeWarning: invalid value encountered in double_sca lars

return S / T

Out[5]: Text(0.5, 1.0, 'Nadaraya-Watson Estimator')



8

```
| def K(x, y, sigma2):
        return np.exp(-np.linalg.norm(x - y)**2 / 2 / sigma2)

| def F(z, sigma2): # 関数定義 f
| S = 0
| T = 0
| for i in range(n):
| S = S + K(x[i], z, sigma2) * y[i]
| T = T + K(x[i], z, sigma2)
| return S / T
```

```
In [7]: n = 100

x = 2 * np.random.normal(size=n)

y = np.sin(2 * np.pi * x) + np.random.normal(size=n) / 4 # データ生成

#最適な lambda の値の計算

m = int(n / 10)

sigma2\_seq = np.arange(0.001, 0.01, 0.001)

SS\_min = np.inf

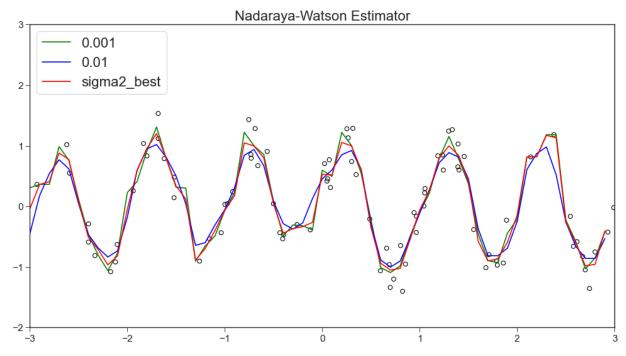
for sigma2 in sigma2\_seq:
```

```
SS = 0
  for k in range(10):
     test = range(k*m, (k+1)*m)
     train = [x for x in range(n) if x not in test]
     for i in test:
       u, v = 0, 0
       for i in train:
         kk = K(x[i], x[j], sigma2)
         u = u + kk * y[i]
         v = v + kk
       if v = 0:
         z = u / v
         SS = SS + (y[j] - z)**2
  if SS < SS_min:
     SS_min = SS
     sigma2_best = sigma2
print("Best sigma2 =", sigma2_best)
```

Best sigma2 = 0.003

```
In [8]:
         plt_figure(num=1, figsize=(15, 8), dpi=80)
         plt_scatter(x, y, facecolors="none", edgecolors="k", marker="o")
         plt_xlim(-3, 3)
         plt_ylim(-2, 3)
         plt_xticks(fontsize=14)
         plt_yticks(fontsize=14)
         xx = np_a range(-3, 3, 0.1)
         yy = [[] for _ in range(3)]
         sigma2 = [0.001, 0.01, sigma2\_best]
         labels = [0.001, 0.01, "sigma2_best"]
         color = ["g", "b", "r"]
         for i in range(3):
            for zz in xx:
              yy[i].append(F(zz, sigma2[i]))
            plt.plot(xx, yy[i], c=color[i], label=labels[i])
         plt.legend(loc="upper left", frameon=True, prop={"size": 20})
         plt.title("Nadaraya-Watson Estimator", fontsize=20)
```

Out[8]: Text(0.5, 1.0, 'Nadaraya-Watson Estimator')



```
In [9]:
           def string_kernel(x, y):
             m, n = len(x), len(y)
             S = 0
             for i in range(m):
                for j in range(i, m):
                  for k in range(n):
                    if x[(i-1):j] == y[(k-1):(k+j-i)]:
                       S = S + 1
             return S
In [10]:
           C = ["a", "b", "c"]
           m = 10
           w = np.random.choice(C, m, replace=True)
           for i in range(m):
             x = x + w[i]
           n = 12
           w = np.random.choice(C, n, replace=True)
           y = ""
           for i in range(n):
             y = y + w[i]
In [11]:
           x,y
Out[11]: ('baacccbbac', 'babbaccacbba')
In [12]:
           string_kernel(x,y)
Out[12]: 61
         15
In [13]:
           def k(s, p):
             return prob(s, p) / len(node)
           def prob(s, p):
             if len(node[s[0]]) == 0:
                return 0
             if len(s) == 1:
                return p
             m = len(s)
             S = (1 - p) / len(node[s[0]]) * prob(s[1:m], p)
             return S
In [14]:
           node = [[] for _ in range(5)]
           node[0] = [1, 3]
           node[1] = [3]
           node[2] = [0, 4]
           node[3] = [2]
           node[4] = [2]
           k([0, 3, 2, 4, 2], 1/3)
Out[14]: 0.0032921810699588485
           2**2 / (5*3**5)
In [15]:
Out[15]: 0.0032921810699588477
```

この時、パスとして存在し得ないものを関数kの引数としても値を返してしまう例として以下のようなあり得ないウォークを指定しても1->3->5

In [16]: k([0,2,4], 1/3)

Out[16]: 0.0074074074074074086

非ゼロの値を返してしまう.

解決策としては隣接リストnodeを参照して次のノードへの遷移がありうるものかどうかの判定を行う必要がある.

In []: