Assignment 02

1 $f(n) = n^2 + f(n-1) + n$
$f(1) = 2$

$O(n^3)$ because there are 3 for loops nested within each other.

2. $n^2 + 3n^3 = f(n)$ $c = 3$ $3n^3 = c \cdot g(n)$

$3n^3 + n^2 \geq 3n^3 \quad \forall n \ni n \geq 0$

$n^2 \geq 0$

$\therefore f(n) \geq c \cdot g(n)$

$\therefore c \cdot g(n)$ is a lower band on $f(n)$

$\therefore f(n) \in \Omega(n^3)$

$n \geq 0$ because
$3n^3 + n^2 = 0$
$\hookrightarrow n^2 = 0$
$3n + 1 = 0$
$\hookrightarrow n = -1/3$

$n^2 + 3n^3 = f(n)$ $g(n) = n^3$ $c = 4$

$3n^3 + n^2 \leq 4n^3$

$\frac{3n^3 + n^2}{n^3} \leq 4$

$3 + \frac{1}{n} \leq 4$ $\frac{1}{n} \leq 1 \quad \forall n$

$\therefore f(n) \leq c \cdot g(n) \quad \forall n \geq 0$

$\therefore c \cdot g(n)$ is an upper band on $f(n)$

$\therefore f(n) \in O(n^3)$

$\therefore f(n) \in \Theta(n^3)$

3 For $2^{n+1} + 2^n$, every integer constant yields

$c_1 2^n \geq 2^{n+1}$ which satisfies $O(2^n)$

if $c_2 \leq 2$, then $c_2 2^n \leq 2^{n+1}$ satisfying $\Omega(2^n)$
$(c_2 2^n = 2^{n+1}$ if $c_2 \in N)$

So, to generalize:

$a^{n+1} \in \Theta(a^n)$ because

$b \cdot a^n \geq a^{n+1}$ iff $b \geq a \rightarrow O(a^n)$
$c \cdot a^n \leq a^{n+1}$ iff $c \leq a \rightarrow \Omega(a^n)$

$\frac{b a^n}{a^n} \geq \frac{a^n}{a^n} \cdot a$ $\qquad$ $\frac{c a^n}{a^n} \leq \frac{a^n}{a^n} \cdot a$

Because of this, $a^{n+1} \in \Theta(a^n)$

4 n^2

For each node in the graph, we check for an edge between it and all other nodes < itself. Worst case, all nodes are checked and the graph is complete.

5 See excel workbook in repo

5. The recorded timings indicate that the .sort() function in <algorithm> of the C++ standard library has O(n*lg(n)). This is because n* lg(n) is an upper bound on the graph of the recorded timings of the sort method; for f(n) = record timings and g(n) = n*lg(n), for every value of n, f(n) < g(n). Because of this, f(n) < c_1 * g(n) for all c_1 >= 1, proving the time complexity is O(n*lg(n)). This means that an estimation of the worst runtimes could be much closer to the values recorded for n*lg(n), and my trial was much lower that that scenario.