AERO-222: Introduction to Aerospace Computation - Spring 2023
Homework #4 - Due Date: Thursday, April 13, 2023

Show all work and justify your answers!

Instructions

- *This homework contains both handwritten and coding problems and shall be submitted according to the following guidelines.*
- *Hardcopy:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall include screenshots of any hand-written work.*
  - *For coding problems, the hardcopy shall include any relevant derivations and emphasize the final results (i.e. boxed, highlighted, etc.). INCLUDE ALL CODING RESULTS (including plots, final values) IN THE HARDCOPY.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.pdf"
  - *If preferable, you can put all of your work into a single Jupyter notebook (.ipynb) with photos of your hand-written work as well. Markdown allows for images.*
- *Coding Submission:*
  - *Due on CANVAS at 11:59 PM on the day of the deadline.*
  - *Shall be submitted as a single file according to the provided template with the following naming scheme:* "LastnameHW#.py" or "LastNameHW#.ipynb".
  - *The script shall print out all outputs asked for in the problem.*
- *Late submissions will be accepted with a 10 point deduction per day late.*

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1. **Numerical Differentiation (By Hand) (25 pts).** Show all steps to derive the MOST accurate formula of the second derivative, f_i'', using all of the following points

$$f_{i-1}, \qquad f_i, \qquad f_{i+1}, \qquad f_{i+2}, \qquad f_{i+3},$$

via matrix inversion.

2. **Numerical Differentiation (Coding Problem) (25 pts).** The derivative of the function,

$$f(x) = 3\cos(5x) - 2x^3 + x^2 - 4x + 16$$

is the function, $f'(x) = -15\sin(5x) - 6x^2 + 2x - 4$. Discretize the function $f(x)$ using $N = 100$ points, $[x_k, f_k]$, that are uniformly distributed in $x \in [0, 4]$. Evaluate the first numerical derivative using the 5-point difference formula. Plot the absolute error between true and numerical derivatives. Pay attention to the two extremes: you cannot always use the 5-point **central** difference formula.

3. **Richardson Extrapolation (By Hand) (25 pts).** Using the function given in problem #2, estimate the 3-point central second derivative at $x = 3$, using $h_1 = 0.1$ and $h_2 = 0.01$. Use Richardson extrapolation to refine your estimate and provide your final result (Note: $h_1 \neq 2h_2$). Use Python to plot the absolute error with respect to the true solution.

4. **An Aerospace Application (Coding Problem) (25 pts).** An airfoil is placed in a wind tunnel and heated to 80°C before lowering the surface temperature to 20.7°C using convective cooling. A thermocouple is placed on the surface of the airfoil and measures the surface temperature at discrete time steps. The following temperatures are recorded,

t (sec)	0	5	10	15	20	25
T (°C)	80	44.5	30	24.1	21.7	20.7

Use numerical differentiation to compute the first derivative $T'(t_k)$ at each time step given in the table above. Specifically, apply the three-point forward, backward or central finite-difference method wherever appropriate in order to estimate the airfoil's temperature gradient, $T'(t_k)$. Provide a table of values.