AERO 626 Homework #8

(50 points)

Consider a one-dimensional, nonlinear, additive-noise model of the form

$$x_k = x_{k-1} - 0.01\sin(x_{k-1}) + w_{k-1}$$
$$z_k = 0.5\sin(2x_k) + v_k$$

where w_{k-1} is a zero-mean, white, Gaussian sequence with constant variance $P_{ww} = 0.01^2$ and v_k is a zero-mean, white, Gaussian sequence with constant variance $P_{vv} = 0.02$. The initial distribution of the state is Gaussian with mean and variance given by $m_{x,0} = 1.5$ and $P_{xx,0} = 0.15^2$, respectively.

1. Simulate the evolution of the true state of the system and measurements according to the prescribed model for a sequence of 500 steps. The random truth should be generated at $k = 0$; measurements should be simulated $\forall\ k > 0$. Provide a plot of the true trajectory and measurements for one set of randomly generated values. Plot the true trajectory as a solid line, and plot the measurements using only markers.

2. Implement an EKF. Document the derivatives required for implementing the EKF. Plot the posterior state estimation error and 3σ interval as functions of k.

3. Implement a UKF. Document the parameters and factorization method employed. Provide a plot of the posterior state estimation errors and 3σ intervals as functions of k.

4. Develop and implement a bootstrap particle filter (BPF) without resampling using $N = 10,000$ particles. Plot the posterior state estimation error and 3σ interval as functions of k. Plot $\hat{N}_{\text{eff}}$ as a function of k. Include your code at the end of your submission.

5. Develop and implement a BPF with basic resampling using $N = 10,000$ particles and a threshold of $\hat{N}_{\text{eff}} < 0.1N$. Plot the posterior state estimation error and 3σ interval as functions of k. Plot $\hat{N}_{\text{eff}}$ as a function of k. How many times is resampling performed? Include your code at the end of your submission.

6. Compare the results of each filter by computing the root-mean-square error (RMSE) and mean absolute error (MAE) using the posterior estimation errors, and comment on the performance of each filter.

The remaining question is optional.

7. Perform a Monte Carlo simulation with 500 trials for the EKF, UKF, and BPF with resampling. Make a histogram of the RMSE and MAE across all of the trials. Comment on the performance of each filter.