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************************** Question 1 ****************************
Q1 a) Equations of motion of the Non-Linear system:
cart acceleration, ddx =
-(l1*m1*sin(theta1)*dtheta1^2 + l2*m2*sin(theta2)*dtheta2^2 - F + g*m1*cos(theta1)
*sin(theta1) - g*m2*cos(theta2)*sin(theta2))/(m1*sin(theta1)^2 + m2*sin(theta2)^2 +
M)

pendulum 1 acceleration, ddtheta1 =
-(g*sin(theta1) + (cos(theta1)*(l1*m1*sin(theta1)*dtheta1^2 + l2*m2*sin(theta2)
*dtheta2^2 - F + g*m1*cos(theta1)*sin(theta1) - g*m2*cos(theta2)*sin(theta2)))/
(m1*sin(theta1)^2 + m2*sin(theta2)^2 + M))/l1

pendulum 2 acceleration, ddtheta2 =
-(g*sin(theta2) + (cos(theta2)*(l1*m1*sin(theta1)*dtheta1^2 + l2*m2*sin(theta2)
*dtheta2^2 - F + g*m1*cos(theta1)*sin(theta1) - g*m2*cos(theta2)*sin(theta2)))/
(m1*sin(theta1)^2 + m2*sin(theta2)^2 + M))/l2

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Q1 b)State Space representation of our Linearized system:
A Matrix:
[0, 1,                  0, 0,                  0, 0]
[0, 0,          -(g*m1)/M, 0,           (g*m2)/M, 0]
[0, 0,                  0, 1,                  0, 0]
[0, 0, -(g + (g*m1)/M)/l1, 0,      (g*m2)/(M*l1), 0]
[0, 0,                  0, 0,                  0, 1]
[0, 0,     -(g*m1)/(M*l2), 0, -(g - (g*m2)/M)/l2, 0]

B Matrix:
       0
     1/M
       0
1/(M*l1)
       0
1/(M*l2)

C Matrix:
     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0
     0     0     0     1     0     0
     0     0     0     0     1     0
     0     0     0     0     0     1

D Matrix:
     0

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Q1 c)Checking conditions on controllability :
-(g^6*l1^2 - 2*g^6*l1*l2 + g^6*l2^2)/(M^6*l1^6*l2^6)

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Q1 d) Substitute the values in the matrices, obtain LQR controller, simulate response
and certify local/global stability:
 Substituting M =1000 Kg, m1=m2=100 Kg, l1=20m, l2=10m, g=9.8m/s
A matrix:
         0    1.0000         0         0         0         0
         0         0   -0.9800         0    0.9800         0
         0         0         0    1.0000         0         0
         0         0   -0.5390         0    0.0490         0
         0         0         0         0         0    1.0000
         0         0   -0.0980         0   -0.8820         0

B matrix:
   1.0e-03 *

         0
    1.0000
         0
    0.0500
         0
    0.1000

Rank of the controllability matrix :
     6

It is full rank matrix, so the system at equilibrium is controllable.
The Open loop poles are:
   0.0000 + 0.9313i
   0.0000 - 0.9313i
  -0.0000 + 0.7441i
  -0.0000 - 0.7441i
   0.0000 + 0.0000i
   0.0000 + 0.0000i

Desgining Linear Quadratic Regulator...
Q matrix :
        2000           0           0           0           0           0
           0          10           0           0           0           0
           0           0     1500000           0           0           0
           0           0           0          10           0           0
           0           0           0           0     2000000           0
           0           0           0           0           0          10

R matrix :
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   1.0000e-04

LQR optimal gain matrix :
   1.0e+05 *

    0.0447    0.1426    1.1610    0.5137    0.9684   -1.0504

Closed Loop System Matrix (Ac):
Closed Loop poles:
  -2.7678 + 2.8972i
  -2.7678 - 2.8972i
  -0.1197 + 0.7599i
  -0.1197 - 0.7599i
  -0.2757 + 0.3873i
  -0.2757 - 0.3873i

all poles have real negative components, closed loop the system is stable by the ↙
Lyapunov's indirect method.
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*********************** end of Question 1 ************************
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************************** Question 2 ****************************
Q2 e) Check observability conditions:
Checking Observability condition ..
Rank with only x(t)6
Rank with theta1, theta2: 4
Rank with x, theta2: 6
Rank with x, theta1, theta2: 6
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 Qn f) Designing Luenberger Observer..
##### For the Linearized System :
The real part of leftmost pole in the LQR closed loop system is
   -2.7678

Lets choose the desired observer poles to be lesser than that
   -3.8500   -4.7500   -3.5000   -3.4000   -3.1500   -2.9000

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 Qn g) Designing LQG Controller..
Kalman Observer Gain:
    0.1448
    0.0105
    0.0063
    0.0008
   -0.0066
   -0.0011

Warning: Simulation will start at a nonzero initial time.
> In DynamicSystem.checkLsimInputs (line 92)
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In DynamicSystem/lsim (line 67)
In controller (line 305)
****End****
>>
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