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Final Project Report

Consider a crane that moves along an one-dimensional track. It behaves as a friction-less cart with mass M actuated by an external force F that constitutes the input of the system. There are two loads suspended from cables attached to the crane. The loads have mass m_1 and m_2, and the lengths of the cables are l_1 and l_2, respectively. The following figure depicts the crane and associated variables used throughout this project.

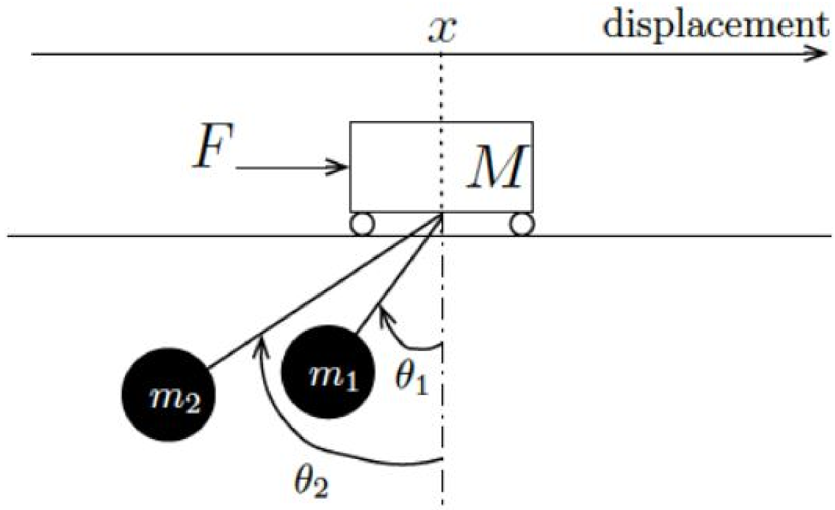


Figure 1: Crane with loads attached to it

1 Equations of motion and non-linear state space representation

Before we start, we define some acronyms and variable names.

- LQR = Linear Quadratic Regulator
- LQG = Linear Quadratic Gaussian controller
- F = External Force Applied on the crane in positive x direction. This acts as the input to the system.
- m_1, m_2 = Mass suspended as a load with the help of a cable
- l_1, l_2 = Lengths of the cables with which the masses are suspended.
- θ_1, θ_2 = Angles with which the masses m_1, m_2 are suspended from the center of mass of the crane.

- x = Displacement of the crane
- L = Lagrange Equation
- K = Kinetic Energy
- P = Potential Energy

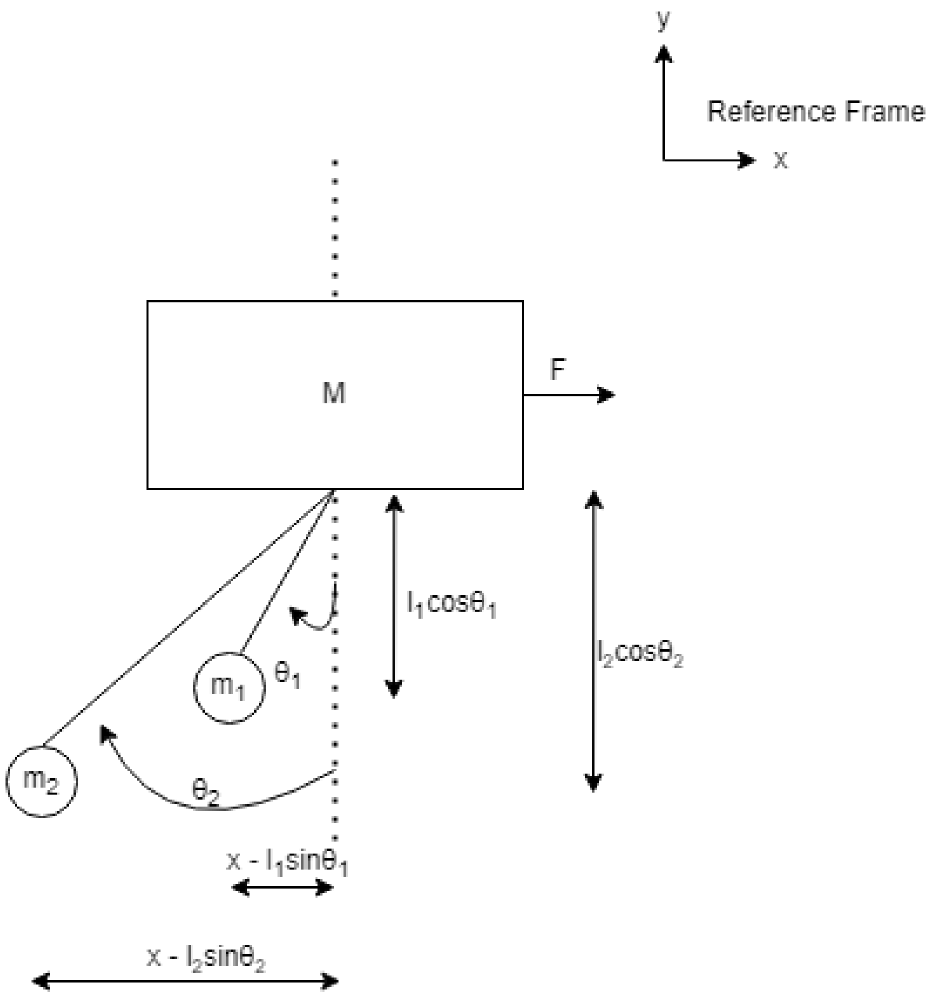


Figure 2: Separating the components using a free body diugram

We use the Euler Lagrange technique to formulate the motion of our system. The Euler Lagrange technique uses the potential energy and kinetic energy and is given by:

$$L = K - P \tag{1}$$

We first compute the kinetic energy according to our free body diagram:

$$K = \frac{1}{2}mv^2 \tag{2}$$

To get the velocity component, we first get the the positions.

$$x_1(t) = (x - l_1 sin(\theta_1))i - (l_1 cos(\theta_1))j$$

$$x_2(t) = (x - l_2 sin(\theta_2))i - (l_2 cos(\theta_2))j$$

where i and j represent the unit vector directions along x and y axis.
Now we differentiate the positions wrt time to get the velocities.

$$\dot{x_1(t)} = v_1(t) = (\dot{x} - l_1\dot{\theta}_1 cos(\theta_1))i + (l_1\dot{\theta}_1 sin(\theta_1))j$$

$$\dot{x_2(t)} = v_2(t) = (\dot{x} - l_2\dot{\theta}_2 cos(\theta_2))i + (l_2\dot{\theta}_2 sin(\theta_2))j$$

We now plug the above equations in 2

$$K = \frac{1}{2}mv^2 \tag{3}$$

Total kinetic energy is given by

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}m_1(\dot{x} - l_1\dot{\theta}_1 cos(\theta_1))^2 + \frac{1}{2}m_1(l_1\dot{\theta}_1 sin(\theta_1))^2 + \frac{1}{2}m_2(l_2\dot{\theta}_2 sin(\theta_2))^2 \tag{4}$$

We now compute the potential energy: $P = mgh$

$$PE = -m_1 g l_1 cos\theta_1 - m_2 g l_2 cos\theta_2 \tag{5}$$

We now substitute 4 and 5 in 1

$$K = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}m_1(\dot{x} - l_1\dot{\theta}_1 cos(\theta_1))^2 + \tfrac{1}{2}m_2(\dot{x} - l_2\dot{\theta}_2 cos(\theta_2))^2 + \tfrac{1}{2}m_1(l_1\dot{\theta}_1 sin(\theta_1))^2 + \tfrac{1}{2}m_2(l_2\dot{\theta}_2 sin(\theta_2))^2 - (-m_1 g l_1 cos\theta_1 - m_2 g l_2 cos\theta_2) \tag{6}$$

$$K = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}m_1(\dot{x} - l_1\dot{\theta}_1 cos(\theta_1))^2 + \tfrac{1}{2}m_2(\dot{x} - l_2\dot{\theta}_2 cos(\theta_2))^2 + \tfrac{1}{2}m_1(l_1\dot{\theta}_1 sin(\theta_1))^2 + \tfrac{1}{2}m_2(l_2\dot{\theta}_2 sin(\theta_2))^2 + m_1 g l_1 cos\theta_1 + m_2 g l_2 cos\theta_2) \tag{7}$$

We now compute the partial derivative of the the above equation wrt $\dot{x}$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = M\dot{x} + m_1(\dot{x} - l_1\dot{\theta}_1 cos(\theta_1)) + m_2(\dot{x} - l_2\dot{\theta}_2 cos(\theta_2)) \tag{8}$$

Taking the derivative wrt time, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{x}} = M\ddot{x} + m_1(\ddot{x} - l_1\ddot{\theta}_1 cos(\theta_1) + l_1\dot{\theta}_1^{\,2} sin(\theta_1)) + m_2(\ddot{x} - l_2\ddot{\theta}_2 cos(\theta_2) + l_2\dot{\theta}_2^{\,2} sin(\theta_2))$$

We infer that $\frac{\partial \mathcal{L}}{\partial x} = 0$,

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} = M\ddot{x} + m_1(\ddot{x} - l_1\ddot{\theta}_1 cos(\theta_1) + l_1\dot{\theta_1}^2 sin(\theta_1)) + m_2(\ddot{x} - l_2\ddot{\theta}_2 cos(\theta_2) + l_2\dot{\theta_2}^2 sin(\theta_2)) = F \tag{9}$$

We now compute the derivative of the Lagrangian equation wrt $\dot{\theta}_1$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} = m_1(\dot{x} - l_1\dot{\theta}_1 cos(\theta_1))(-l_1 cos(\theta_1)) + m_1(l_1\dot{\theta}_1 sin(\theta_1))(l_1 sin(\theta_1)) \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} = -m_1\ddot{x}l_1 cos(\theta_1) + m_1 l_1^2\ddot{\theta}_1 + m_1\dot{x}l_1\ddot{\theta}_1 sin(\theta_1) \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = m_1 l_1^2\dot{\theta}_1 + m_1\dot{x}l_1 cos\theta_1 \tag{12}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} - \frac{\partial \mathcal{L}}{\partial \theta_1} = -m_1\ddot{x}l_1 cos(\theta_1) + m_1 l_1^2\ddot{\theta}_1 + m_1\dot{x}l_1\ddot{\theta}_1 sin(\theta_1) - m_1 l_1^2\dot{\theta}_1 + m_1\dot{x}l_1 cos\theta_1 \tag{13}$$

Similarly, we now write down the derivative of the Lagrangian above equation wrt $\dot{\theta}_2$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} - \frac{\partial \mathcal{L}}{\partial \theta_2} = -m_2\ddot{x}l_2 cos(\theta_2) + m_2 l_2^2\ddot{\theta}_2 + m_2\dot{x}l_2\ddot{\theta}_2 sin(\theta_2) - m_2 l_2^2\dot{\theta}_2 + m_2\dot{x}l_2 cos\theta_2 \tag{14}$$

Now we try to get $\ddot{x}$, $\ddot{\theta}_1$ and $\ddot{\theta}_2$ from 9, 13 and 14 respectively.

$$(M + m_1 + m_2)\ddot{x} = F + m_1(\ddot{x}cos(\theta_1) - gsin(\theta_1)) + m_2(\ddot{x}cos(\theta_2) - gsin(\theta_2))cos(\theta_2) - m_1 l_1\dot{\theta_1}^2 sin(\theta_1) - m_2 l_2\dot{\theta_2}^2 sin(\theta_2) \tag{15}$$

$$\ddot{x} = \frac{F + m_1 gcos(\theta_1)sin(\theta_1) - m_2 gcos(\theta_2)sin(\theta_2) - m_1 l_1\dot{\theta_1}^2 sin(\theta_1) - m_2 l_2\dot{\theta_2}^2 sin(\theta_2)}{(M + m_1 sin(\theta_1)^2 + m_2 sin(\theta_2)^2} \tag{16}$$

$$l_1\ddot{\theta}_1 = \ddot{x}cos(\theta_1) - gsin(\theta_1) \tag{17}$$

$$\ddot{\theta}_1 = \frac{\ddot{x}cos(\theta_1) - gsin(\theta_1)}{l_1} \tag{18}$$

$$l_2\ddot{\theta}_2 = \ddot{x}cos(\theta_2) - gsin(\theta_2) \tag{19}$$

$$\ddot{\theta}_2 = \frac{\ddot{x}cos(\theta_2) - gsin(\theta_2)}{l_2} \tag{20}$$

We consider the state as

$$X = \begin{bmatrix} x \\ \dot{x} \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} \tag{21}$$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} \tag{22}$$

where,

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \frac{F+m_1 g cos(\theta_1) sin(\theta_1) - m_2 g cos(\theta_2) sin(\theta_2) - m_1 l_1 \dot{\theta_1}^2 sin(\theta_1) - m_2 l_2 \dot{\theta_2}^2 sin(\theta_2)}{(M+m_1 sin(\theta_1)^2 + m_2 sin(\theta_2)^2} \\ \frac{\ddot{x} cos(\theta_1) - g sin(\theta_1)}{l_1} \\ \frac{\ddot{x} cos(\theta_2) - g sin(\theta_2)}{l_2} \end{bmatrix} \tag{23}$$

To conclude part A, we obtained the equations of motion for the system and the corresponding nonlinear state-space representation.

2 Linearized system around the equilibrium point x = 0 and $\theta_1 = \theta_2 = 0$

For a time varying system,

$$\begin{bmatrix} \ddot{x}(t) \\ \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{F-m_1 g cos(\theta_1) sin(\theta_1) - m_2 g cos(\theta_2) sin(\theta_2) - m_1 l_1 \dot{\theta_1}^2 sin(\theta_1) - m_2 l_2 \dot{\theta_2}^2 sin(\theta_2)}{(M+m_1 sin(\theta_1)^2 + m_2 sin(\theta_2)^2} \\ \frac{\ddot{x} cos(\theta_1) - g sin(\theta_1)}{l_1} \\ \frac{\ddot{x} cos(\theta_2) - g sin(\theta_2)}{l_2} \end{bmatrix} \tag{24}$$

Given $\theta_1 = 0$ and $\theta_2 = 0$, we can consider for a very small angle $x_e = 0$, $\theta_1 = 0$ at E and $\theta_2 = 0$ at E where E is the equilibrium point.

We keep $cos(\theta) \simeq 1$ and $sin(\theta) \simeq \theta$ for both θs

$$\begin{bmatrix} \ddot{x}(t) \\ \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{F - m_1 \theta_1 - m_2 g \theta_2}{M} \\ \frac{F}{Ml_1} - \frac{m_1 g \theta_1}{Ml_1} - \frac{g\theta_1}{l_1} - \frac{m_2 g \theta_2}{Ml_1} \\ \frac{F}{Ml_2} - \frac{m_2 g \theta_2}{Ml_2} - \frac{g\theta_2}{l_2} - \frac{m_1 g \theta_2}{Ml_2} \end{bmatrix} \tag{25}$$

$$\begin{bmatrix} \ddot{x}(t) \\ \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{-m_1 g}{M} & \frac{-m_2 F g}{M} \\ 0 & \frac{-m_1 g}{M l_1} - \frac{g}{l_1} & \frac{-m_2 g}{M l_1} \\ 0 & \frac{-m_1 g}{M l_2} & \frac{-m_2 g}{M l_2} - \frac{g}{l_2} \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{M} \\ \frac{1}{M l_1} \\ \frac{1}{M l_2} \end{bmatrix} F \tag{26}$$

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-m_1 g}{M} & 0 & \frac{-m_2 g}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-m_1 g}{M l_1} - \frac{g}{l_1} & 0 & \frac{-m_2 g}{M l_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{m_1 g}{M l_2} & 0 & \frac{-m_2 g}{M l_2} - \frac{g}{l_2} & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{M l_1} \\ 0 \\ \frac{1}{M l_2} \end{bmatrix} F \tag{27}$$

Therefore comparing this with our general state equation we get:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-m_1 g}{M} & 0 & \frac{-m_2 g}{M} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-m_1 g}{M l_1} - \frac{g}{l_1} & 0 & \frac{-m_2 g}{M l_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{m_1 g}{M l_2} & 0 & \frac{-m_2 g}{M l_2} - \frac{g}{l_2} & 1 \end{bmatrix} \tag{28}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{1}{M l_1} \\ 0 \\ \frac{1}{M l_2} \end{bmatrix} \tag{29}$$

Thus, the state space representation of the linearized system has been obtained.

3 Conditions for which the system is controllable

To check if the system is controllable we use the above equation obtained and check whether the Grammian is invertible. This can be done by computing the rank or determinant as follows.

$$det([B\ AB\ A^2B\ A^3B\ ....\ A^{n-1}B]) = 0 \tag{30}$$

where n is the dimension of A. In our case, n=6, therefore

$$det([B\ AB\ A^2B\ A^3B\ A^4B\ A^5B]) = 0 \tag{31}$$

We utilize A & B from 28 and 29 to check for controllability. The system will be controllable if the determinant is nonzero.

$$det([B\ AB\ A^2B\ A^3B\ A^4B\ A^5B]) = -\frac{g^6(l_1^2 - 2l_1l_2 + l_2^2)}{(Ml_1l_2)^6} \tag{32}$$

$$det([B\ AB\ A^2B\ A^3B\ A^4B\ A^5B]) = -\frac{g^6(l_1^2 - 2l_1l_2 + l_2^2)}{(Ml_1l_2)^6} \tag{33}$$

$$det([B\ AB\ A^2B\ A^3B\ A^4B\ A^5B]) = -\frac{g^6(l_1 - l_2)^2}{(Ml_1l_2)^6} \tag{34}$$

From 34 we see that if $l_1 == l_2$ we can claim that the determinant will be equal to zero and thus the system will be uncontrollable. To avoid this, the system needs a constraint that $l_1 \neq l_2$. These are the the conditions for the system to be controllable.

4 Considering M = 1000Kg, m1 = m2 = 100Kg, l1 = 20m and l2 = 10m

Utilizing the values that we get from 28 and 29

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -0.55 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & -0.9 & 0 \end{bmatrix} \tag{35}$$

$$B = 0.001 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0.05 \\ 0 \\ 0.1 \end{bmatrix} \tag{36}$$

Since $l_1 \neq l_2$ the system is controllable.

We can see if the system is stable or not by checking its eigenvalues.

The Open loop poles are:

- 0.0000 + 0.9313i
- 0.0000 - 0.9313i

- 0.0000 + 0.7441i
- 0.0000 - 0.7441i
- 0.0000 + 0.0000i
- 0.0000 + 0.0000i

The eignvalues obtained are imaginary and have no real part. This means that the *linearized system is oscillating and is critically stable*. Thus we can design a controller to make the system stable. So we design a LQR controller.

We know that the cost for the LQR controller is given by:

$$J = \int_0^T (x(\tau)^T Q x(\tau) + u(\tau)^T R u(\tau)) d\tau \tag{37}$$

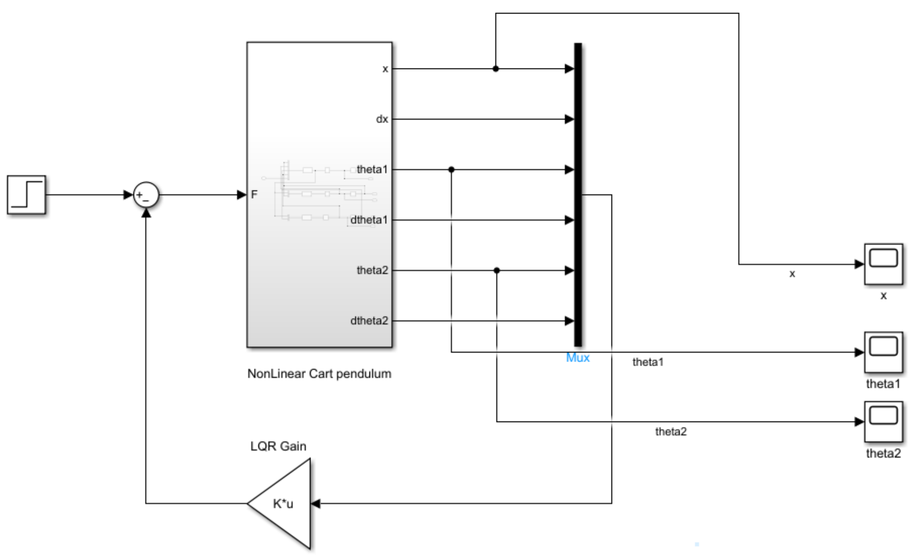


Figure 3: LQR Linear

where Q and R are the penalizing terms which have a direct effect on the system. So we tune of Q and R values as shown below. We set the initial conditions at x = 0.5 meters $\theta_1 = 15°$ and $\theta_2 = 7°$

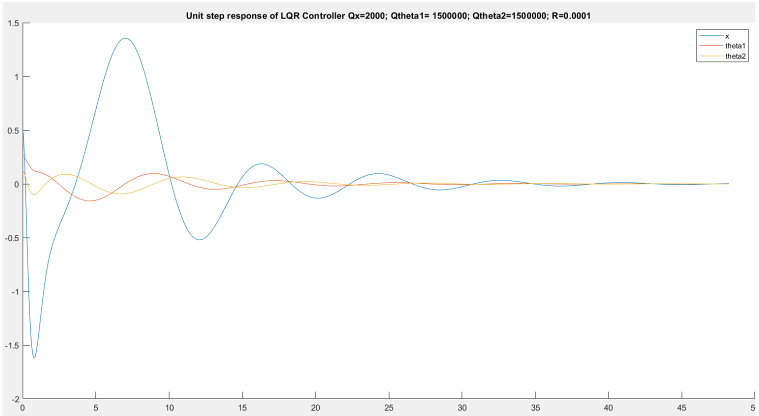
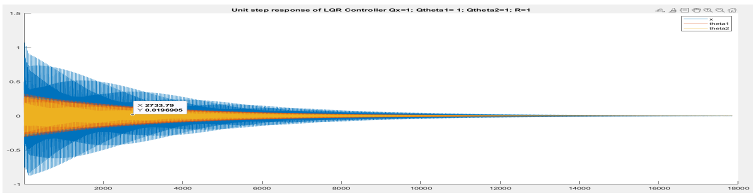
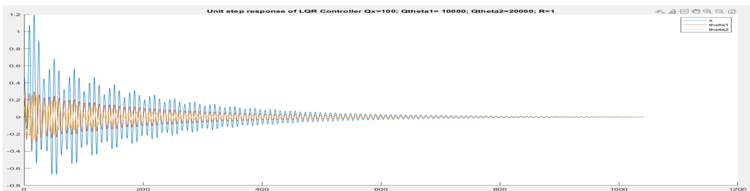


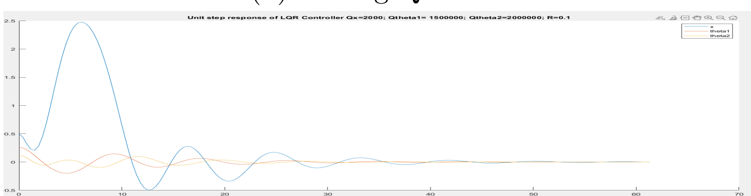
Figure 4: LQR Linearized



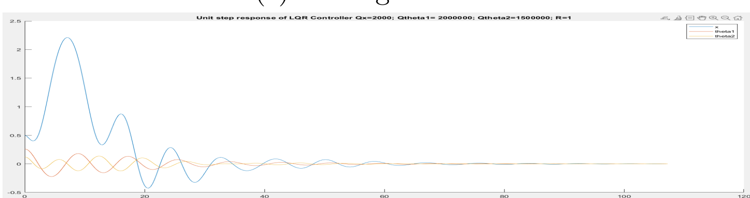
(a) Setting Q as 1



(b) Setting Q as 100



(c) Setting R as 0.1



(d) Setting R as 1

Figure 5: Tuning Q and R

We can now see from Fig. 4 that the system stabilizes. The best tuned response was inferred with a settling time of around 45 seconds

$$Q = \begin{bmatrix} 20000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1500000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2000000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} and \quad R = 0.0001 \tag{38}$$

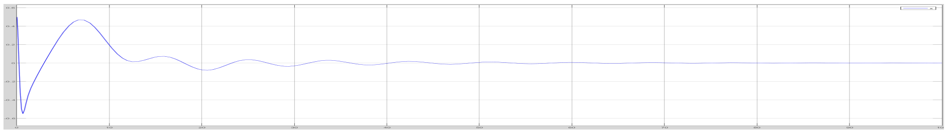
The Gain Matrix is: 1.0e+05 *[0.0447, 0.1426, 1.1610, 0.5137, 0.9684, -1.0504]

Using Lyapunov's indirect method to certify stability: Lyapunov's indirect method states to check the stability of the system, we need to check the eigen values of the closed loop dynamics.

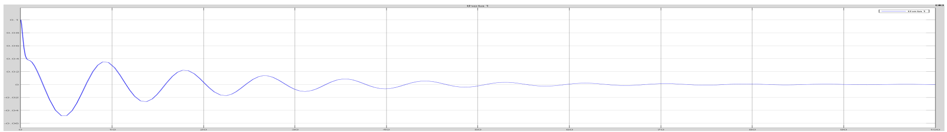
The closed loop poles of the LQR controller were:

- -2.7678 + 2.8972i
- -2.7678 - 2.8972i
- -0.1197 + 0.7599i
- -0.1197 - 0.7599i
- -0.2757 + 0.3873i
- -0.2757 - 0.3873i

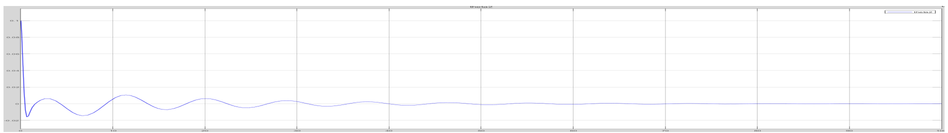
We can observe that the closed-loop poles all have a negative real term in them. This means that the system is stable. Due to the existence of imaginary values there will be some oscillations before the system stabilizes. We can see that for the equlibirum point, the sytem is stable locally.



(a) x Estimate



(b) Theta 1 Estimate



(c) Theta 2 Estimate

Figure 6: Non Linear LQR

Thus we have simulated the resulting response to initial conditions when the controller is applied to the linearized system and also to the original nonlinear system. We also tuned the Q and R parameter till we got a good response. We also used Lyapunov's indirect method to certify stability (locally or globally) of the closed-loop system

5 Check for Observability

We know that, to check if a system is observable, we compute the following,

$$Observability(A, C) = rank\left(\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{bmatrix}\right) = n \tag{39}$$

The system is observable only if the above matrix has a full rank. We now check observalibility for different cases. We provide 4 different cases

- Case 1 ($x(t)$):

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{40}$$

  The rank for this case is 6, which means the system is observable.

- Case 2 ($\theta_1(t), \theta_2(t)$):

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{41}$$

The rank for this case is 4, which means the system is not observable.

- Case 3 ($x(t), \theta_2(t)$):

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{42}$$

The rank for this case is 6, which means the system is observable.

- Case 4 ($x(t), \theta_1(t), \theta_2(t)$):

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{43}$$

The rank for this case is 6, which means the system is observable.

6 Luenberger observer

We can estimate the unavailable states of the system using a Luenberger observer. Since the dynamics of the observer are the same, we can state if a pair(A,C) is observable there exists a feedback gain matrix L. The idea behind this observer is to make the error of the estimated state and the actual state to zero.

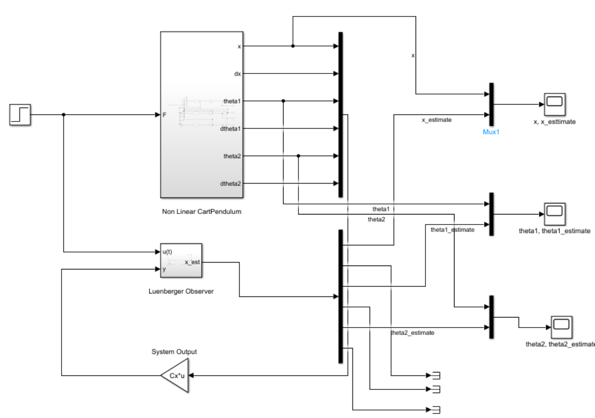
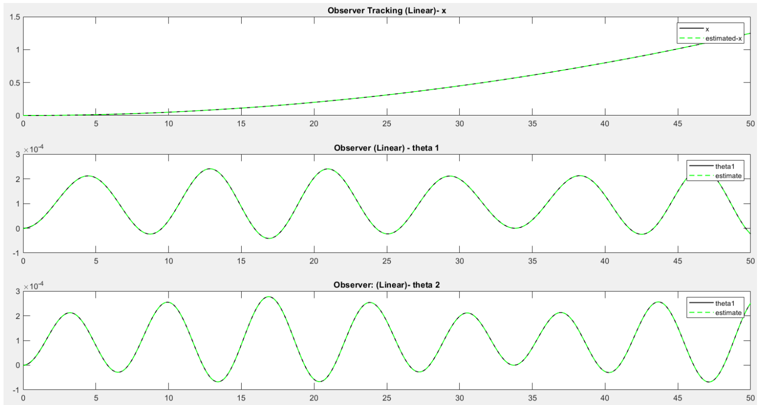


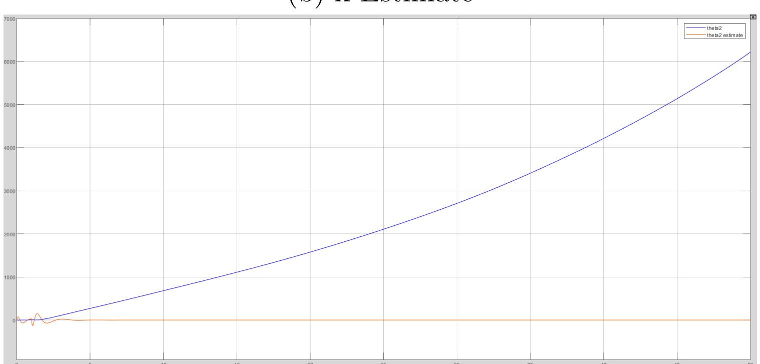
Figure 7: Open Loop Observer

Following are some of the plots for a linear and nonlinear system.

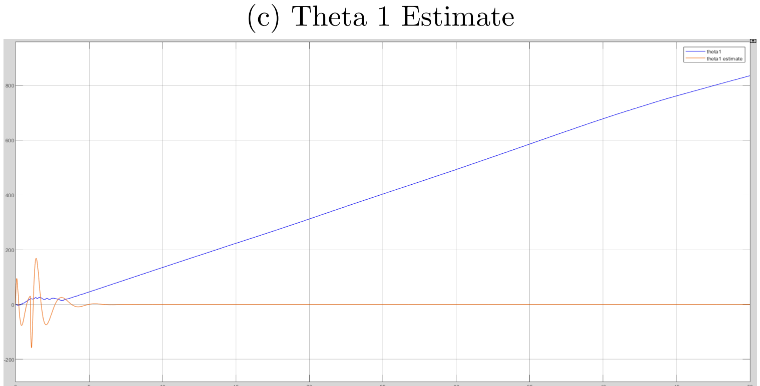


(a) x Estimate

(b) x Estimate

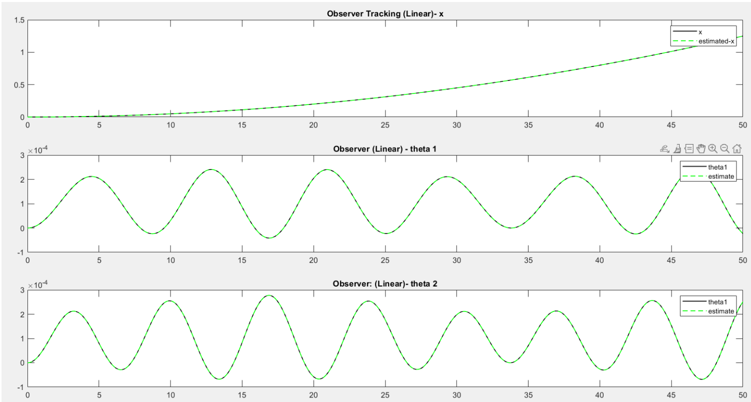


(c) Theta 1 Estimate

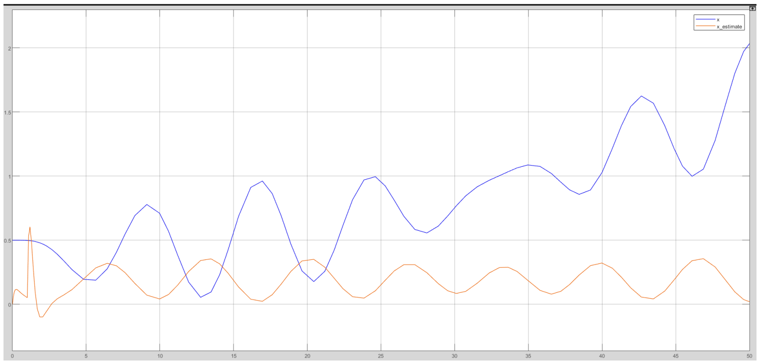


(d) Theta 2 Estimate

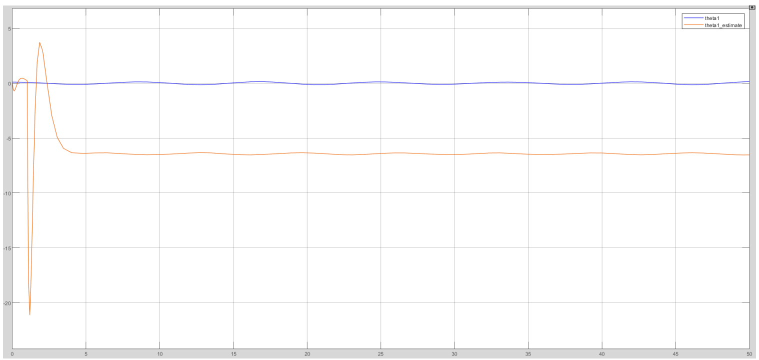
Figure 8: Non Linear Observer for Case 1



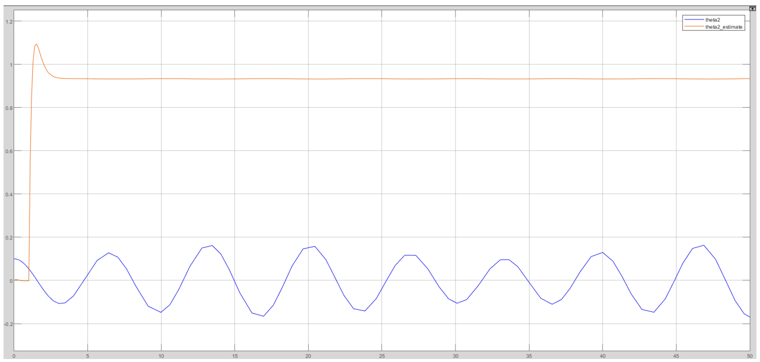
(a) x Estimate



(b) x Estimate

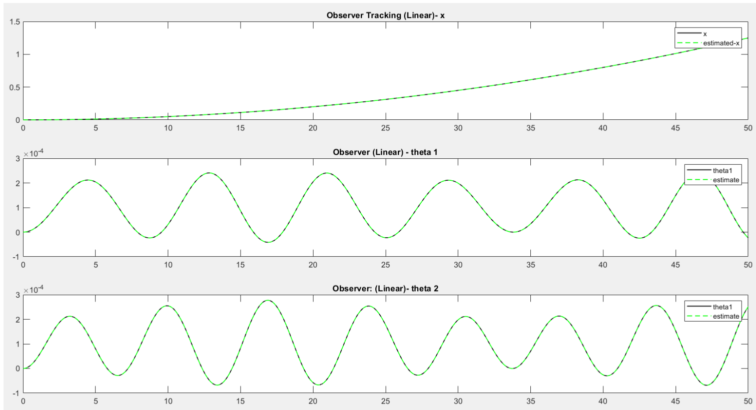


(c) Theta 1 Estimate

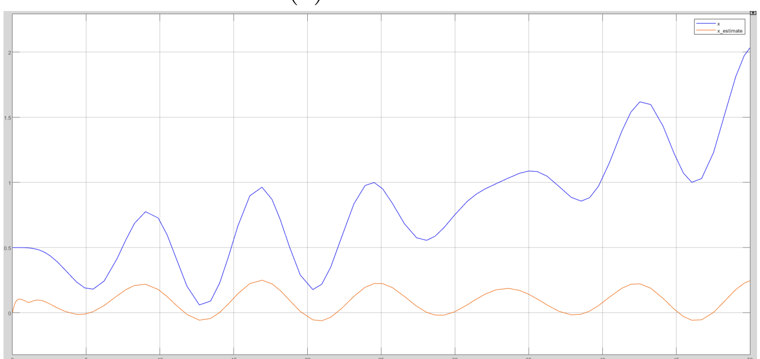


(d) Theta 2 Estimate

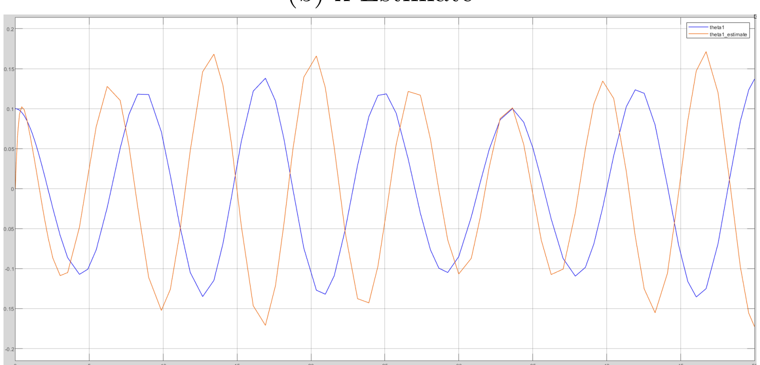
Figure 9: Non Linear Observer for Case 3



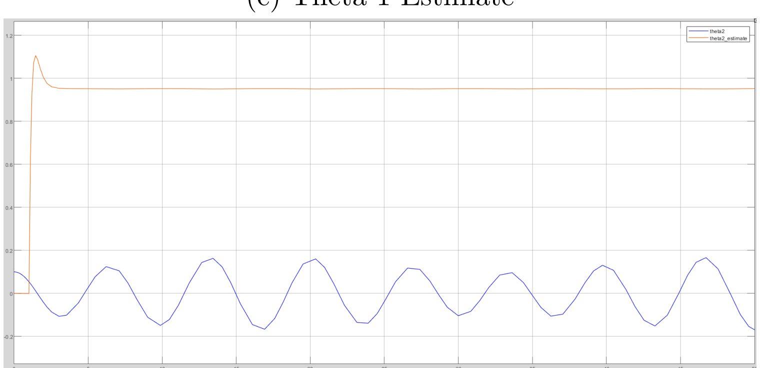
(a) x Estimate



(b) x Estimate



(c) Theta 1 Estimate



(d) Theta 2 Estimate

Figure 10: Non Linear Observer for Case 4

7 LQG

We design a Kalman Filter that accounts for the system disturbances and the measurement noise. We assume the disturbances and noise to be white gaussian noise. We know that the process noise $v(t)$ and measurement noise $w(t)$ in a Kalman filter is given by:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + v(t) \tag{44}$$

$$y(t) = C(t)x(t) + w(t) \tag{45}$$

For this, we need to a cost function which can be minimized and also a co-variance matrix for noise. We further state that the dynamics of the Kalman filter is given by:

$$\widehat{x} = (A - K_f C)\widehat{x} + \begin{bmatrix} B & K_f \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix} \tag{46}$$

$$\begin{bmatrix} \dot{x} \\ \dot{x_e} \end{bmatrix} = \begin{bmatrix} A - BK_{regulator} & B - K_{regulator} \\ 0 & A - K_f C \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} + \begin{bmatrix} u & y \end{bmatrix} \tag{47}$$

$$y = [C \quad 0]x \tag{48}$$

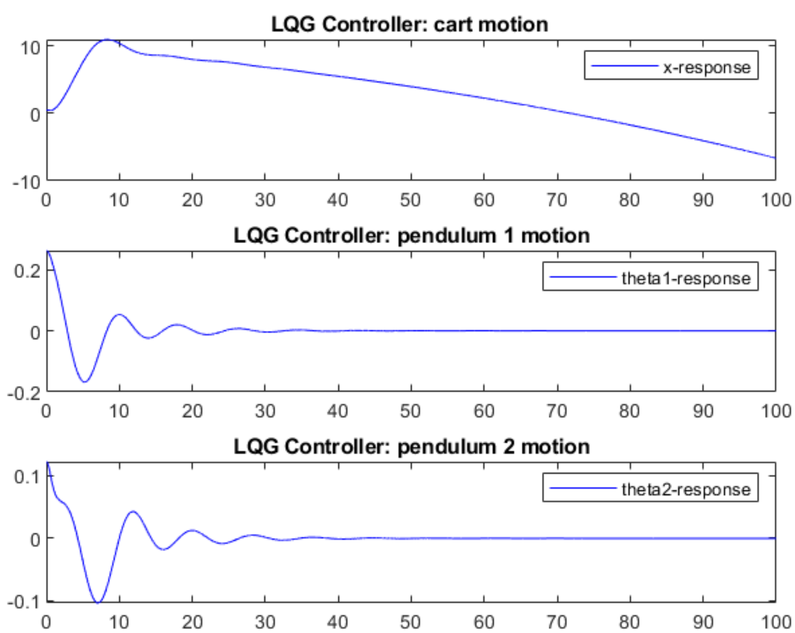


Figure 11: LQG Linear

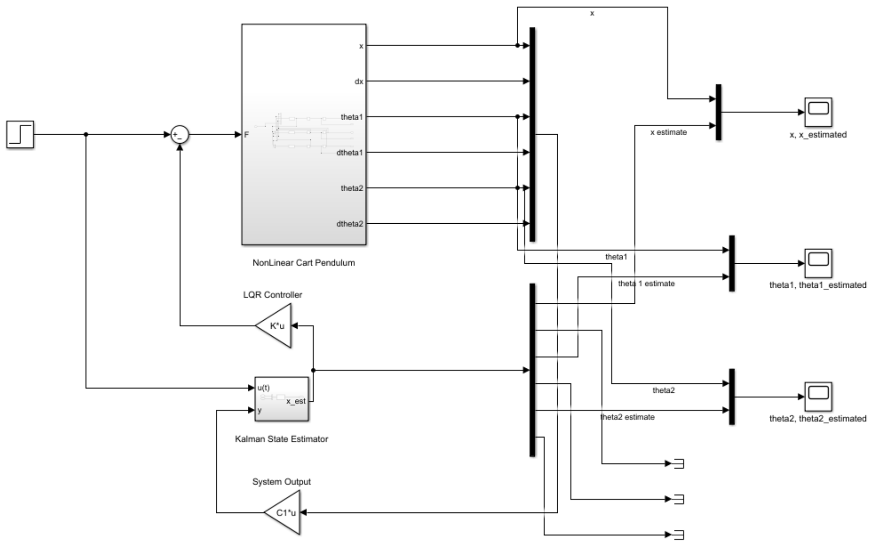
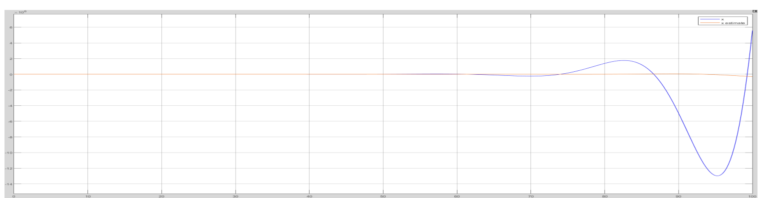
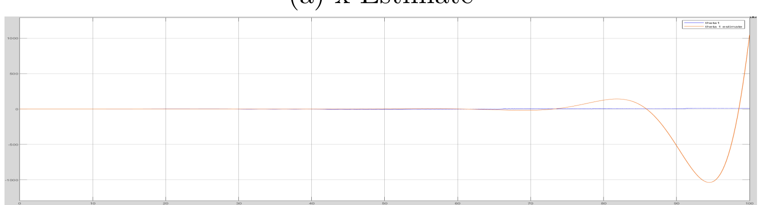


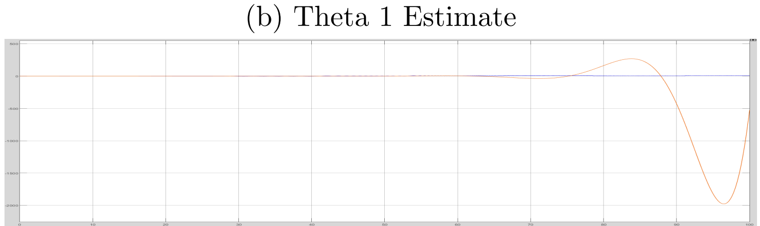
Figure 12: LQG Controller



(a) x Estimate



(b) Theta 1 Estimate



(c) Theta 2 Estimate

Figure 13: LQG Nonlinear Estimates

We tried to track a constant reference using this estimated full-state feedback controller by placing an integral component and reducing the error by considering past values for the original non linear system.

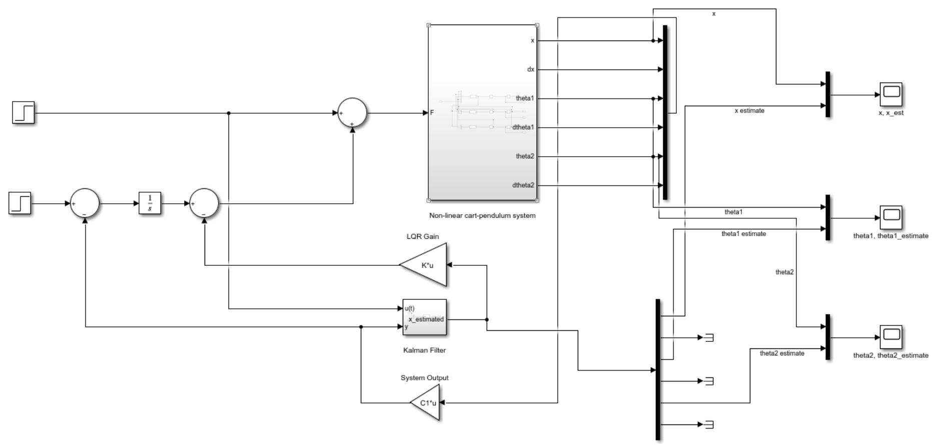
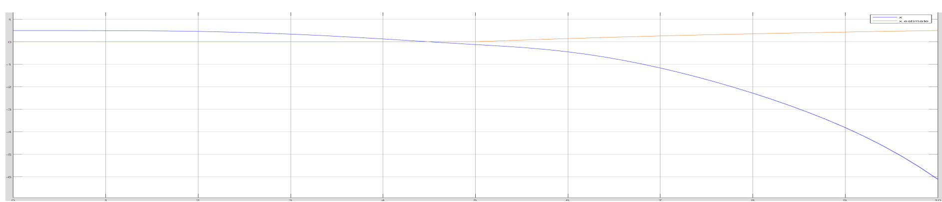
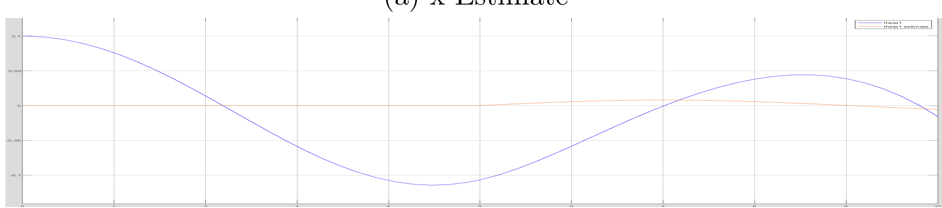


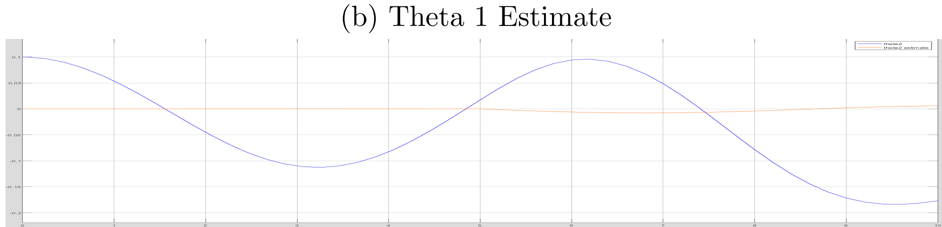
Figure 14: LQG Controller with an Integral Component



(a) x Estimate



(b) Theta 1 Estimate



(c) Theta 2 Estimate

Figure 15: LQG with a Integral Component Estimates

From the above plots it is evident that the LQG Controller with an integral component will not perform well in presence of noise. The LQG controller being a linear controller, cannot accommodate for the system disturbances and noise present in the system. Unlike full-state feedback LQR controller the LQG controller with an integral component involves full-state estimation using the available outputs. In our case we took the smallest output vector and we infer that the estimation performance was insufficient to make the system robust to noise. Another drawback of this LQG is that unlike LQR, it does not have a large gain margin.