Linear Algebra 20 June

Friday, 20 June 2025 21:00

Used for
↓
Machine Learning
&
Deep Learning

- Scalars (0th order Tensor) — (34)
- Vectors (1st order Tensors) — [34, 21, 91, 87]
- Matrix (2nd order Tensors) — [[ ] [ ] [ ]]
- Higher Order Tensors (3rd or higher order tensors)

Uses of Linear Algebra in Machine Learning →

① Generalize / represent the relationship b/w input & output cols in higher dimension data.

② Helps us in data representation numerically

- → Tabular
- → Images
- → Audio
- → Video
- → Text
- → Signal

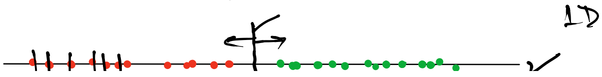
→ Help us represent all these data numerically → Then these numerical data can be given to ML/DL.

↓
Bank
↳ ML

	I/P ↓ Income	O/P Loan Approved ?
0	—	No
1	—	Yes
2	—	Yes
3	—	No
4	—	Yes
5	—	No
6	—	No
7	—	No

I/P → O/P

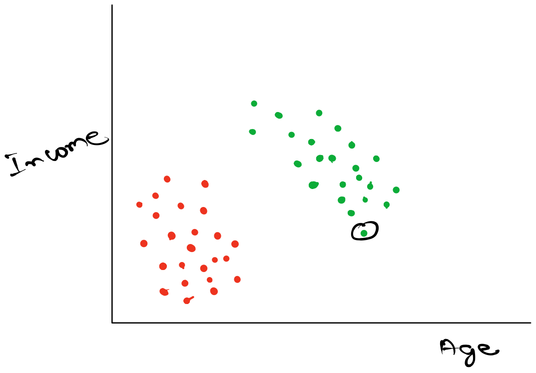
↓
ML
↓



0 ||| |||

Income

	Age	Income	Loan Approved ?
0	–	–	No
1	–	–	Yes
⋮			



Maximum number of cols we can represent as a graph visually is 3.

3 input cols
↓
3D

	Rooms	Floors	Age	Area	Price
0					
1					
2					
3					
4					
5					

↓
ML
↓

$$Price = 0.98 * Rooms^2 + 0.87 * Floors^2 + (-1.2) * Age + 1.35 * Area + \underbrace{1.8}_{\text{Base Price}}$$

Linear

$$y = m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4 + c$$

↓

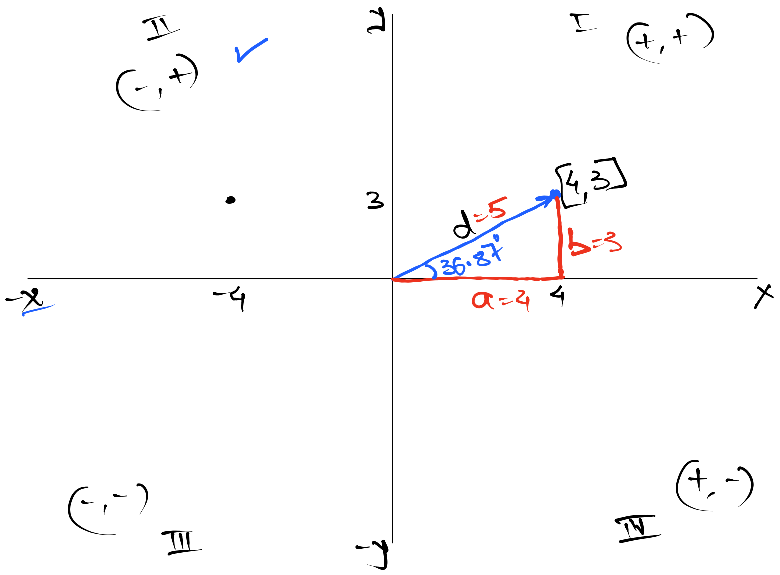
$y = mx + c$

Equation of a line

Scalars → 38 47.9 63.98 211.137

vectors → [38, 47.9, 63.98, 211.137]

vectors : Mathematical Approach



vector
[4, 3]
↓ ↓
x y

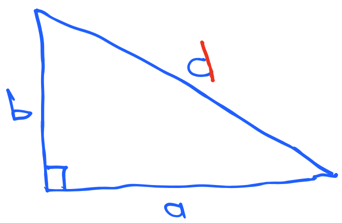
[-4, 3]

5 units

vectors have magnitude & direction

Magnitude of a vector →

Pythagoras Theorem



$$d^2 = a^2 + b^2$$

$$d = \sqrt{a^2 + b^2}$$

↓

$$d = \sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$$d = 5$$

Direction of the vector $[4, 3]$ is :

$$\tan\theta = \frac{b}{a}$$

$$\tan\theta = \frac{3}{4}$$

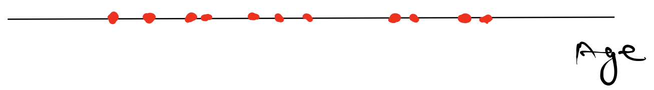
$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$= 36.87^\circ$$

Dimensions of a vector →

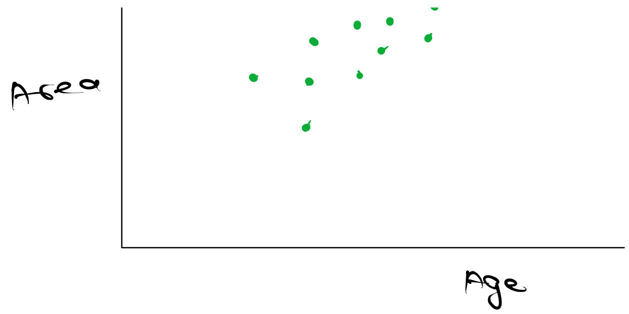
$$\begin{array}{cc} & \text{Age} \\ H_1 & [2] \\ H_2 & [3.5] \\ H_3 & [4.8] \\ H_4 & [1.7] \end{array}$$

1D vector



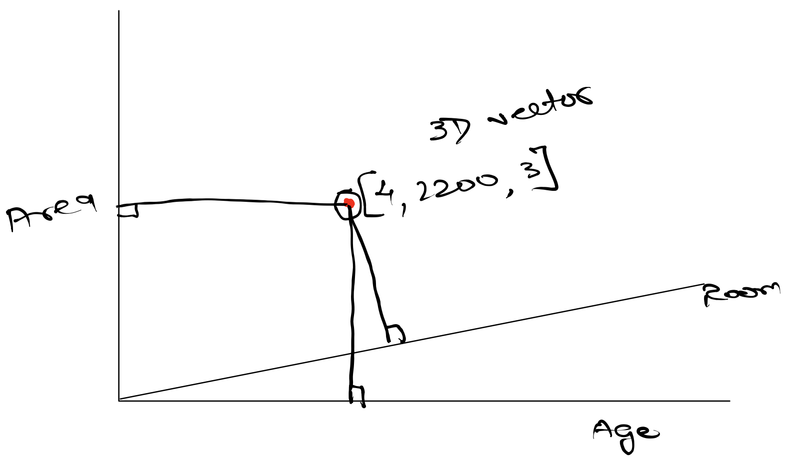
$$\begin{array}{ccc} & \text{Area} & \text{Age} \\ H_1 & [2200 & 3] \leftarrow \\ H_2 & [4000 & 2] \\ H_3 & [1700 & 1.5] \end{array}$$

2D vectors



	Area	Age	Rooms
$H_1 \to 0$	[2200	3	4]
$H_2 \to 1$	[4000	2	8]
$H_3 \to 2$			
$H_4 \to 3$			
4			

3D vector



How do we represent a vector →

1. Row vectors
2. Column vectors

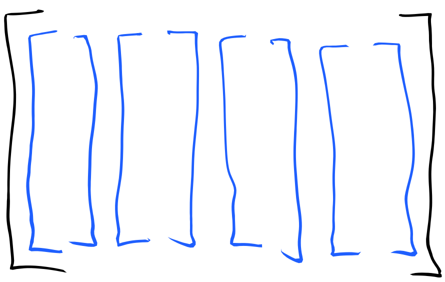
House Prices Dataset

	Rooms	Age	Area	Floors	Price
0	4	3	2200	2	$150K
1					
2					
3					

vector1(H1) = [4 3 2200 2]

Matrix = $\begin{bmatrix} [& v_1 &] \\ [& v_2 &] \\ [& v_3 &] \\ & \vdots & \end{bmatrix}$

Matrices



1. Multiplications, additions, divisions, subtractions
2. Transpose
3. Inverse

4. Determinant
5. Rank of a matrix
6. Eigenvectors & eigenvalues

Additions / Subtractions / Divisions →

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \pm \begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}$$

↓

$$\begin{bmatrix}6 & 8\\ 10 & 12\end{bmatrix} \qquad \begin{bmatrix}-4 & -4\\ -4 & -4\end{bmatrix}$$

Multiplication →

1. Normal -
2. Hadamard

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \quad \begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix}$$

Image   Filter

$$\begin{bmatrix} \; \end{bmatrix} * \begin{bmatrix} \; \end{bmatrix}$$

$$\begin{bmatrix} \; \end{bmatrix}$$

Edited Image

Transpose →

$$A = \begin{bmatrix} [2 & 7 & 8] \\ [1 & 5 & 3] \\ [9 & 0 & 6] \end{bmatrix}$$

$$A^T = \left[\begin{bmatrix} 2 \\ 7 \\ 8 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \\ 6 \end{bmatrix} \right]$$

Inverse

Determinant →

$$\begin{array}{cccc} & 1,1 & 1,2 & 1,3 \\ A = & \begin{bmatrix} 2 & 2 & -3 \\ 1 & 2 & 2 \end{bmatrix} \end{array}$$

$\lfloor 1 \quad 1 \quad 2 \rfloor$

$\det(A)$ or $||A||$

$$\det(A) = 2(2\times2-1\times2) - 2(1\times2-1\times2) + (-3)(1\times1-1\times2)$$
$$= 7$$

<u>Inverse of a matrix</u> →

$$A = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} * \operatorname{adj}(A)$$

$\det(A) = 0$

$\det(A) \neq 0$

$$\text{Price} = \underbrace{0.98}_{W_1} * \text{Rooms} + \underbrace{0.87}_{W_2} * \text{Floors} + \underbrace{(-1.2)}_{W_3} * \text{Age} + \underbrace{1.35}_{W_4} * \text{Area} + \underbrace{1.8}_{\text{Base Price}}$$

ML Algorithms

$$\hat{w} = [w_1 \quad w_2 \quad w_3 \quad w_4]$$

$$\boxed{\hat{w} = (X^T X)^{-1} X^T y}$$

X = matrix of input columns

y = vector of output column

Rank of a matrix $\rightarrow$

Len Leng wid Area

Red

40 x 10 400

$$A = \begin{bmatrix} \cancel{2}1 & \cancel{2}2 & \cancel{2}3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

Elementary operations to reduce the matrix

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -6 & -4 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank of A = 2

$$\begin{bmatrix} 2 & 3 & 0 \\ -1 & -2 & 0 \\ 6 & -7 & 0 \end{bmatrix} = 2$$