Answers to questions in

Lab 1: Filtering operations

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**Instructions**: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

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**Question 1**: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9),

(17, 121), (5, 1) and (125, 1) respectively. What do you observe?

**Answers:**

* The closer to the center we go, the more lines we get in the output graph (**why?**). Alternatively, the smaller the absolute distance from the center, the higher the frequency.
* The plot of the imaginary part is typically the same as the real part, only that it is offset half a period (1/4?) forward/backward (sin vs. cos).
* With only one point being non-zero, our plot of the magnitude of the IDFT is zero (**why?**).
* Placing a dot on the x-axis (y-axis) results in vertical (horizontal) lines.
* Other placements yield diagonal lines.

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**Question 2**: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

* Discrete Fourier transform: {f(t)} = F() =
* Fourier inversion theorem:
* Euler’s formula:

F(m, n) = {Euler’s formula}

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| --- | --- |
| A close up of a device  Description automatically generated |  |

2D and 3D visualisations of the imaginary part of the projection of the Fourier (frequency) domain position (1, 3) onto the spatial domain.

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**Question 3**: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

**Answers:**

From Eq. (4) we get:

F(m, n)

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**Question 4**: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

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**Question 5**: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:

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**Question 6**: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:

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**Question 7**: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

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**Question 8**: Why is the logarithm function applied?

Answers:

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**Question 9**: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:

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**Question 10**: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

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**Question 11**: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:

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**Question 12**: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

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**Question 13**: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

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**Question 14**: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and

100.0?

Answers:

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**Question 15**: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

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**Question 16**: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

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**Question 17**: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

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**Question 18**: What conclusions can you draw from comparing the results of the respective methods?

Answers:

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**Question 19**: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

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**Question 20**: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:

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