Extremal Network Theory and Robust GDoF Gain of Multi-Cell Cooperation over Multi-Cell TIN

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***Abstract*—We study the fundamental limits of multi-cell cooperation in downlink cellular networks under the assumption of finite precision channel state information at the transmitters (CSIT). By appealing to extremal network theory, recently introduced by Chan et al. [1], we characterize the extremal GDoF gain of multi-cell cooperation over non-cooperative multi-cell TIN in three *weak* inter-cell interference regimes: the mc-TIN regime, where multi-cell TIN is GDoF optimal under no cooperation; the mc-CTIN regime, where the GDoF region achieved through multi-cell TIN is convex without the need for time-sharing; and the mc-SLS regime, where a cooperative scheme based on simple layered superposition was shown to be optimal in small networks. We show that the extremal GDoF gain is bounded by a constant factor in the mc-TIN and mc-CTIN regimes, and scales logarithmically with the number of cells in the mc-SLS regime. The analysis is enabled by a new cooperative outer bound for cellular networks based on the aligned images approach.**

I. Introduction

In recent years, significant progress has been made on understanding the fundamental limits of interference and multi-antenna wireless networks through generalized degrees-of-freedom (GDoF) studies [2]–[5]. Such studies reveal a fundamental role of channel state information at the transmitters (CSIT)—performance gains promised by sophisticated interference management schemes that rely on infinite precision CSIT diminish, or even collapse, when CSIT is limited to finite precision. This motivated a surge of interest in robust GDoF characterizations under the assumption of finite precision CSIT, enabled by a new class of information-theoretic outer bounds known as aligned images bounds [4]–[9].

A main challenge in robust GDoF studies, and GDoF studies in general, is the sheer combinatorial complexity of general asymmetric networks, with potentially exponentially many different operational regimes of channel parameters. So far, this challenge has been avoided by focusing on small and symmetric settings [2]–[5]; or considering particular regimes of channel parameters in which simple and robust schemes are GDoF optimal, e.g. *weak* interference regimes [8]–[14]. While insights from the former approach are limited and often do not generalize to large or asymmetric settings; the latter approach has not been entirely successful in circumventing the inherent complexity of GDoF characterizations, even in special regimes where simple schemes are supposedly optimal—see [8].

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Recently, Chan et al. [1] proposed a new extremal network framework to overcome the complexity barrier in GDoF studies. In this framework, the focus is shifted towards studying extremal networks within the regimes of interest, which have the property of maximizing the GDoF gain of some schemes over others. The main application in [1] is the study of robust GDoF gains of full transmitter cooperation over simple non-cooperative schemes of power control and treating interference as noise (TIN) in networks with K transmitter-receiver pairs (i.e. $K \times K$ networks). The analysis focuses on the TIN regime [10], convex TIN (CTIN) regime [11], and simple layered superposition (SLS) regime [8], which all fall in the general *weak* interference regime. It was shown that the extremal GDoF gain is bounded by constants in the first two regimes, and scales with the logarithm of K in the third regime.

Inspired by [1], in this paper we study the extremal gain of multi-cell cooperation in $K \times KL$ downlink cellular networks, with K cells and L users per-cell. We consider the robust GDoF gain of full multi-cell cooperation over simple non-cooperative multi-cell TIN schemes (mc-TIN), with single-cell transmissions, power control and treating inter-cell interference as noise [14]. We focus on the mc-TIN regime, mc-CTIN regime and mc-SLS regime, which generalize the TIN, CTIN and SLS regimes to cellular settings (see Section III-A).

To enable extremal network analysis, we first derive a new cooperative GDoF outer bound for cellular networks under finite precision CSIT by employing the aligned images approach [6]. We then leverage this cooperative outer bound, alongside recent results on mc-TIN [9], [14], to show that the extremal GDoF gain in $K \times KL$ cellular networks mirrors that in $K \times K$ networks: it is bounded by constant multiplicative factors in the mc-TIN and mc-CTIN regimes, and scales logarithmically with the number of cells K in the mc-SLS regime. An insight that emerges is that in these three weak interference regimes of interest, additional users in each cell have no influence on extremal GDoF gains, governed by the underlying $K \times K$ network of single-user cells.

Notation: For positive integers z_1 and z_2, $\langle z_1 : z_2 \rangle$ denotes $\{z_1, z_1+1, \ldots, z_2\}$, and $\langle z_2 \rangle$ denotes $\langle 1 : z_2 \rangle$. For any real number $a \in \mathbb{R}$, we have $(a)^+ = \max\{0, a\}$. Bold symbols denote tuples, e.g. $\boldsymbol{A} = (A_1, \ldots, A_Z) = (A_i : i \in \langle Z \rangle)$, while calligraphic symbols denote sets, e.g. $\mathcal{A} = \{a_1, \ldots, a_Z\}$. For functions $f(K)$ and $g(K)$, we have $f(K) = \Theta\big(g(K)\big)$ if $\lim_{K \to \infty} f(K)/g(K) = c$, where $c > 0$.

II. System Model

Consider a K-cell cellular network in which each cell k, where $k \in \langle K \rangle$, comprises a base station denoted by BS-k and L user equipments, each denoted by UE-(l_k, k), where $l_k \in \langle L \rangle$. The set of tuples corresponding to all UEs in the network is given by $\mathcal{U} \triangleq \{(l_k, k) : l_k \in \langle L \rangle, k \in \langle K \rangle\}$.

We focus on the downlink mode, where an independent message $W_k^{[l_k]}$ is communicated to each UE-(l_k, k). The input-output relationship at the t-th channel use is given by

$$Y_k^{[l_k]}(t) = \sum_{i=1}^{K} \bar{P}^{\alpha_{ki}^{[l_k]}} G_{ki}^{[l_k]}(t) X_i(t) + Z_k^{[l_k]}(t). \quad (1)$$

In the above, $X_i(t), Y_k^{[l_k]}(t), Z_k^{[l_k]}(t) \in \mathbb{C}$ are, respectively, the symbol transmitted by BS-i, the symbol received by UE-(l_k, k), and the zero-mean unit-variance additive white Gaussian noise (AWGN) at UE-(l_k, k). The signal transmitted by BS-i is subject to the unit average power constraint $\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\left[\left|X_i(t)\right|^2\right] \leq 1$, where T is the communication duration in channel uses. The T-channel-use-long signal (or codeword) of BS-i is given by $\boldsymbol{X}_i \triangleq \big(X_i(t) : t \in \langle T \rangle\big)$.

For GDoF purposes, we define $\bar{P} \triangleq \sqrt{P}$, where P is a nominal power parameter that approaches infinity in the GDoF limit. The exponent $\alpha_{ki}^{[l_k]} > 0$ is known as the channel strength parameter between BS-i and UE-(l_k, k), while $G_{ki}^{[l_k]}(t) \in \mathbb{C}$ is a channel fading coefficient. The tuple of all fading coefficients is given by $\boldsymbol{G} \triangleq \big(G_{ki}^{[l_k]}(t) : (l_k, k) \in \mathcal{U}, i \in \langle K \rangle, t \in \langle T \rangle\big)$.

We define the array $\boldsymbol{\alpha} \in \mathbb{R}_+^{K \times K \times L}$, comprising all channel strength parameters of a given network. The (i, j, l_i)-th element of $\boldsymbol{\alpha}$ is given by $\boldsymbol{\alpha}(i, j, l_i) = \alpha_{ij}^{[l_i]}$. Note that $\boldsymbol{\alpha}$ describes the topology of a $K \times KL$ network, specifying the strengths of connections between different BS-UE pairs, and we would often refer to $\boldsymbol{\alpha}$ as the network. As seen further on, we focus on special regimes specified by imposing conditions on $\boldsymbol{\alpha}$.

A. Finite Precision CSIT

We assume that channel strength parameters are perfectly known to both the transmitters (BSs) and the receivers (UEs). Channel fading coefficients, however, are perfectly known to the receivers but only available to finite precision at the transmitters (finite precision CSIT). This implies that: a) the joint and conditional probability density functions of the channel coefficients (entries of $\boldsymbol{G}$) exist and the peak values of these distributions are bounded by a constant which is independent of P; and b) the transmitters only know the distributions of channel coefficients, but not their actual realizations [4], [6]. Therefore, the codewords $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_K$ are independent of the realizations of channel coefficients, yet may depend on their distributions as well as channel strength parameters.

B. No Multi-Cell Cooperation (IBC)

Under no multi-cell cooperation, each BS has access to the set of messages intended to UEs in the same cell only, and the network is modelled by an IBC comprising K mutually interfering Gaussian BCs (cells). For instance, each BS-k has messages $\boldsymbol{W}_k = \big(W_k^{[l_k]} : l_k \in \langle L \rangle\big)$ and encodes them into the codeword $\boldsymbol{X}_k$, independently of all other BSs. On the other end, UE-(l_k, k) in cell k sees the contributions from all codewords $\boldsymbol{X}_i$ with $i \in \langle K \rangle \setminus \{k\}$ as inter-cell interference.

Achievable rates, the capacity region and the GDoF region are all defined in a standard manner, see, e.g., [6], [10], [13]. The GDoF region of the IBC is denoted by $\mathcal{D}^{\mathrm{IBC}}$ and a GDoF tuple is denote by $\mathbf{d} = \big(d_k^{[l_k]} : (l_k, k) \in \mathcal{U}\big)$.

C. Full Multi-Cell Cooperation (MISO-BC)

Under full multi-cell cooperation, all BSs have access to all messages $\big(\boldsymbol{W}_1, \ldots, \boldsymbol{W}_K\big)$, jointly encoded into the vector codeword $\boldsymbol{X} \triangleq (\boldsymbol{X}_1, \ldots, \boldsymbol{X}_K)$, comprising K scalar codewords and of which the k-th component $\boldsymbol{X}_k$ is transmitted through BS-k. In this multi-cell cooperative setting, each BS is viewed as an antenna in a large multi-antenna transmitter, and the network is modelled by a $K \times KL$ MISO-BC with a K-antenna transmitter and KL single-antenna receivers. The GDoF region of the MISO-BC is denoted by $\mathcal{D}^{\mathrm{MBC}}$.

III. Preliminaries

Without loss of generality, we may assume that in each cell i, direct link strengths are ordered as

$$\alpha_{ii}^{[L]} \geq \alpha_{ii}^{[L-1]} \geq \cdots \geq \alpha_{ii}^{[1]} \quad (2)$$

capturing the signal-to-noise ratio (SNR) order of same-cell users. We focus on three special regimes of channel strength parameters, each described in terms of additional linear inequality conditions on $\boldsymbol{\alpha}$. The three regimes of interest fall in the *weak* (inter-cell) interference regime [2], and are included in the set of channel parameters that satisfy the signal-to-interference ratio (SIR) order given by

$$\alpha_{ii}^{[L]} - \alpha_{ij}^{[L]} \geq \alpha_{ii}^{[L-1]} - \alpha_{ij}^{[L-1]} \geq \cdots \geq \alpha_{ii}^{[1]} - \alpha_{ij}^{[1]} \geq 0 \quad (3)$$

for all $i, j \in \langle K \rangle$. The inequalities in (3) can be decomposed into two types of conditions: 1) intra-cell conditions, which govern (and preserve) the order amongst same-cell users under inter-cell interference; and 2) inter-cell conditions, which control interference levels between distinct cells.

The first type is captured by the first (left) $L-1$ inequalities in (3), which in turn guarantee that for any cell i, and against interference from any other cell j, the SIR order of same-cell users should follow their SNR order. That is, a stronger user in the SNR sense must also be stronger in the SIR sense

$$\alpha_{ii}^{[l_i]} \geq \alpha_{ii}^{[l_i-1]} \implies \alpha_{ii}^{[l_i]} - \alpha_{ij}^{[l_i]} \geq \alpha_{ii}^{[l_i-1]} - \alpha_{ij}^{[l_i-1]}. \quad (4)$$

As we will see, the above SIR order holds in all three regimes of interest. The second type of conditions are captured by the right-most inequality in (3), which implies that

$$\alpha_{ii}^{[l_i]} \geq \alpha_{ij}^{[l_i]} \quad (5)$$

holds for all $(l_i, i) \in \mathcal{U}$ and $j \in \langle K \rangle$. That is, a direct link between a BS and any of its associated UEs must be no weaker than interfering (or cross) links to the same UE. This places the cellular network in the weak inter-cell interference regime, which includes all three regimes of interest.

A. Multi-Cell TIN, CTIN and SLS regimes

We now present the three regimes of interest, starting with the mc-TIN and mc-CTIN regimes [13], [14].

Definition 1. ***(mc-TIN Regime).*** *This regime is denoted by $\mathcal{A}^{\mathrm{TIN}}$, and is given by all $\boldsymbol{\alpha} \in \mathbb{R}_+^{K\times K\times L}$ that satisfy*

$$\alpha_{ii}^{[l_i]} \geq \alpha_{ij}^{[l_i]} + \alpha_{ii}^{[l_i-1]} - \big(\alpha_{ij}^{[l_i-1]} - \alpha_{ij}^{[l_i]}\big)^+ \tag{6}$$

$$\alpha_{ii}^{[l_i]} \geq \alpha_{ij}^{[l_i]} + \alpha_{ki}^{[l_k]} \tag{7}$$

for all cells $i, j, k \in \langle K \rangle$ such that $i \notin \{j, k\}$, and for all users $l_i \in \langle 2 : L\rangle$ and $l_k \in \langle L \rangle$.

It is worth noting that (6) and (7) are stricter versions of the conditions in (4) and (5), respectively. In the mc-TIN regime, mc-TIN is GDoF optimal for cellular networks with no multi-cell cooperation (i.e. the IBC) [14]. Hence this regime can be thought of as a *very weak* inter-cell interference regime.

Definition 2. ***(mc-CTIN Regime).*** *This regime is denoted by $\mathcal{A}^{\mathrm{CTIN}}$, and is given by all $\boldsymbol{\alpha} \in \mathbb{R}_+^{K\times K\times L}$ that satisfy*

$$\alpha_{ii}^{[l_i]} \geq \alpha_{ij}^{[l_i]} + \alpha_{ii}^{[l_i-1]} - \alpha_{ij}^{[l_i-1]} \tag{8}$$

$$\alpha_{ii}^{[l_i]} \geq \max\big(\alpha_{ij}^{[l_i]} + \alpha_{ji}^{[l_j]}, \alpha_{ik}^{[l_i]} + \alpha_{ji}^{[l_j]} - \alpha_{jk}^{[l_j]}\big) \tag{9}$$

for all cells $i, j, k \in \langle K \rangle$ such that $i \notin \{j, k\}$, and for all users $l_i \in \langle 2 : L\rangle$ and $l_j \in \langle L \rangle$.

It is evident that (8) and (9) imply (4) and (5). In the mc-CTIN regime, the GDoF region achieved through mc-TIN (TINA region) is a convex polyhedron without the need for time-sharing [14]. Moreover, we have recently shown that when CSIT is limited to finite precision, mc-TIN is GDoF optimal for the IBC in the mc-CTIN regime[1] [9]. Next, we introduce the largest of the three regimes: the mc-SLS regime.

Definition 3. ***(mc-SLS Regime).*** *This regime is denoted by $\mathcal{A}^{\mathrm{SLS}}$, and is given by all $\boldsymbol{\alpha} \in \mathbb{R}_+^{K\times K\times L}$ that satisfy*

$$\alpha_{ii}^{[l_i]} \geq \alpha_{ij}^{[l_i]} + \alpha_{ii}^{[l_i-1]} - \alpha_{ij}^{[l_i-1]} \tag{10}$$

$$\alpha_{ii}^{[l_i]} \geq \max\big(\alpha_{ij}^{[l_i]}, \alpha_{ki}^{[l_k]}, \alpha_{ik}^{[l_i]} + \alpha_{ji}^{[l_j]} - \alpha_{jk}^{[l_j]}\big) \tag{11}$$

for all cells $i, j, k \in \langle K \rangle$ such that $i \notin \{j, k\}$, and for all users $l_i \in \langle 2 : L\rangle$ and $l_k \in \langle L \rangle$.

The mc-SLS regime generalizes the SLS regime, introduced by Davoodi and Jafar in [8], to the cellular setting considered here. In [8], it was shown that a SLS scheme, based on layered superposition and rate-splitting, achieves the entire GDoF region of the 3×3 MISO-BC in the SLS regime.[2] The complexity of the GDoF region achieved through SLS, and its dependency on several design variables and channel parameters, prohibits the extension of the result in [8] beyond the 3-user setting. A way to circumvent this complexity is to resort to extremal network analysis as in [1], which we adopt in this work. Before we proceed, it is worth noting that

$$\mathcal{A}^{\mathrm{TIN}} \subseteq \mathcal{A}^{\mathrm{CTIN}} \subseteq \mathcal{A}^{\mathrm{SLS}}. \tag{12}$$

B. Multi-Cell TIN

In the mc-TIN scheme, each cell adopts a power-controlled single-cell scheme—with superposition coding and successive decoding of same-cell signals—while all inter-cell interference is treated as noise [9], [14]. We assume a fixed successive decoding order in each cell, where UE-(l_i, i) decodes for messages $W_i^{[1]}, W_i^{[2]}, \ldots, W_i^{[l_i]}$, in that order. The GDoF region achievable through mc-TIN (TINA region) is denoted by $\mathcal{D}^{\mathrm{TINA}}$. In [14], it was shown that the TINA region is given by a union of convex polyhedral regions (PTIN regions), and while the TINA region is not convex in general, it is convex in the mc-CTIN regime. Moreover, under finite precision CSIT, $\mathcal{D}^{\mathrm{TINA}}$ is optimal in the mc-CTIN regime [9], that is

$$\boldsymbol{\alpha} \in \mathcal{A}^{\mathrm{CTIN}} \implies \mathcal{D}^{\mathrm{IBC}} = \mathcal{D}^{\mathrm{TINA}}. \tag{13}$$

IV. Cooperation Outer Bound

As shown in [8], characterizing the GDoF region of the general asymmetric MISO-BC under finite precision CSIT is a formidable task, even when restricting to small networks and special regimes (e.g. the SLS regime). With this in mind, we take the alternative route of appealing to extremal network theory [1], through which we evaluate the extremal GDoF gain of multi-cell cooperation over the simple non-cooperative scheme of multi-cell TIN. We start by deriving a cooperative GDoF outer bound in the mc-SLS regime. To this end, let us define cycle bounds for the MISO-BC.

Definition 4. ***(Cycles).*** *A cycle π of length $|\pi| = M$ is an ordered sequence of M users from distinct cells, given by*

$$\pi = \big((l_{i_1}, i_1) \to (l_{i_2}, i_2) \to \cdots \to (l_{i_M}, i_M)\big). \tag{14}$$

We define $\{\pi\} \triangleq \big\{(l_{i_1}, i_1), (l_{i_2}, i_2), \cdots, (l_{i_M}, i_M)\big\}$ as the set of users involved in cycle π. The m-th user in a cycle π is also denoted by $\pi(m) = (l_{i_m}, i_m)$, from which (14) is equivalently expressed as $\pi = \big(\pi(1) \to \cdots \to \pi(M)\big)$. The set of all cycles (of all lengths) is denoted by Π. Each cycle π is associated with two implicit cycles given by

$$\sigma = (i_1 \to i_2 \to \cdots \to i_M) = \big(\sigma(1) \to \sigma(2) \to \cdots \to \sigma(M)\big)$$

$$\rho = (l_{i_1} \to l_{i_2} \to \cdots \to l_{i_M}) = \big(\rho(1) \to \rho(2) \to \cdots \to \rho(M)\big)$$

encompassing BS indices and UE indices, respectively. For cycles of length M, indices are interpreted modulo M, e.g. $\pi(M + m) = \pi(m)$ and $\sigma(M + m) = \sigma(m)$ for integer m.

Definition 5. ***(MISO-BC Cycle Bounds).*** *Each cycle $\pi \in \Pi$ gives rise to $|\pi|$ bounds on the sum-GDoF of users in the set given by $\big\{(s_k, k) : s_k \in \langle l_k \rangle, (l_k, k) \in \{\pi\}\big\}$, comprising each UE-$(l_k, k)$ in $\{\pi\}$ as well as same-cell users that precede UE-(l_k, k) in the SNR order. The m-th MISO-BC cycle bound associated with π is given by*

$$\sum_{(l_k,k)\in\{\pi\}} \sum_{s_k \in \langle l_k \rangle} d_k^{[s_k]} \leq \Delta_{\pi,m}^{+}. \tag{15}$$

[1] Note that this is not always the case under perfect CSIT, see [9], [14]

[2] This extends directly to the $3L \times 3$ MISO-BC, with more transmit antennas than users, by muting additional transmit antennas [8]. Whether this holds in the other direction, for the $3 \times 3L$ MISO-BC, is not yet clear.

For each π and m, the quantity $\Delta^+_{\pi,m}$ is given by

$$\Delta^+_{\pi,m}=\begin{cases}\alpha^{[l_{i_1}]}_{i_1 i_1}, & \text{if } |\pi|=1\\ \alpha^{[l_{i_{m+1}}]}_{i_{m+1} i_m}+\sum_{m=1}^{M}\delta^{[l_{i_m} l_{i_{m+1}}]}_{i_m i_{m+1}}, & \text{if } |\pi|>1\end{cases} \quad (16)$$

where the quantity $\delta^{[l_i l_j]}_{ij}$ is defined as

$$\delta^{[l_i l_j]}_{ij} \triangleq \alpha^{[l_i]}_{ii}-\alpha^{[l_j]}_{ji}. \quad (17)$$

We are now ready to present the first result of this paper.

Theorem 1. *In the mc-SLS regime, the MISO-BC GDoF region $\mathcal{D}^{\mathrm{MBC}}$ is included in the outer bound region $\mathcal{D}^{\mathrm{MBC}}_{\mathrm{out}}$, given by the set all GDoF tuples $\mathbf{d}\in\mathbb{R}^{KL}_{+}$ that satisfy*

$$\sum_{(l_k,k)\in\{\pi\}}\sum_{s_k\in\langle l_k\rangle} d^{[s_k]}_k \leq \Delta^+_{\pi,m},\ \forall m\in\langle|\pi|\rangle, \pi\in\Pi. \quad (18)$$

Proof. The proof of the outer bound is based on the aligned images approach [6], and is relegated to the Appendix. □

A question that comes to mind is whether $\mathcal{D}^{\mathrm{MBC}}_{\mathrm{out}}$ is tight at all. As it turns out, $\mathcal{D}^{\mathrm{MBC}}_{\mathrm{out}}$ is tight in the mc-SLS regime for 2-cell networks with an arbitrary number of users, i.e. $2\times 2L$ MISO-BC. In this setting, it can be shown that $\mathcal{D}^{\mathrm{MBC}}_{\mathrm{out}}$ is achieved using a scheme based on SLS.[3] Beyond 2-cells, $\mathcal{D}^{\mathrm{MBC}}_{\mathrm{out}}$ is not tight in general—this can be seen from the 3×3 MISO-BC region in [8], which requires additional inequalities beyond the cycle bounds in Definition 5.

Although $\mathcal{D}^{\mathrm{MBC}}_{\mathrm{out}}$ is not tight in general, it is sufficient for the purpose of extremal analysis, as we will see next.

V. Extremal GDoF Gain

We define the sum-GDoF achievable by mc-TIN as

$$d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}) \triangleq \max_{\mathbf{d}\in\mathcal{D}^{\mathrm{TINA}}(\boldsymbol{\alpha})}\sum_{(l_k,k)\in\mathcal{U}} d^{[l_k]}_k \quad (19)$$

computed from the TINA region. Note that the dependency of the GDoF on $\boldsymbol{\alpha}$ is made explicit in this section. The sum-GDoF achieved through multi-cell cooperation is defined as

$$d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha}) \triangleq \max_{\mathbf{d}\in\mathcal{D}^{\mathrm{MBC}}(\boldsymbol{\alpha})}\sum_{(l_k,k)\in\mathcal{U}} d^{[l_k]}_k. \quad (20)$$

In a similar manner, we define the sum-GDoF cooperative upper bound $d^{\mathrm{MBC}}_{\mathrm{out},\Sigma}(\boldsymbol{\alpha})$ from $\mathcal{D}^{\mathrm{MBC}}_{\mathrm{out}}(\boldsymbol{\alpha})$ in Theorem 1.

Note that $d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha})$ is the optimal sum-GDoF achieved in the underlying MISO-BC with no restriction on the employed scheme, while $d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha})$ is the maximum sum-GDoF achieved while restricting to simple non-cooperative mc-TIN schemes. We are interested in extremal ratios between $d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha})$ and $d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha})$ in the regimes of interest, which capture the *potential* benefits of multi-cell cooperation in each of these regimes. For a regime of channel parameters $\mathcal{A}\subset\mathbb{R}^{K\times K\times L}_{+}$, the extremal GDoF gain from multi-cell cooperation is defined as

$$\eta_{K,L}(\mathcal{A}) \triangleq \max_{\boldsymbol{\alpha}\in\mathcal{A}}\frac{d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha})}{d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha})} \quad (21)$$

[3] The proof is lengthy and is left to an extended version of this paper [15].

which is parametrized by the network size, specified by K, L. We are now ready to present the second result of this paper.

Theorem 2. *The extremal GDoF gain of multi-cell cooperation over multi-cell TIN in the regimes of interest is as follows*

$$\eta_{K,L}(\mathcal{A})=\begin{cases}\frac{3}{2}, & \mathcal{A}=\mathcal{A}^{\mathrm{TIN}}\\ 2-\frac{1}{K}, & \mathcal{A}=\mathcal{A}^{\mathrm{CTIN}}\\ \Theta(\log(K)), & \mathcal{A}=\mathcal{A}^{\mathrm{SLS}}.\end{cases} \quad (22)$$

Theorem 2 shows that under partial CSIT, the potential GDoF gains of multi-cell cooperation over the simple non-cooperative scheme of multi-cell TIN are bounded (and small) in the mc-TIN and mc-CTIN regimes. On the other hand, there is potential for larger gains in the mc-SLS regime, which may scale logarithmically with the number of cells K. Another key observation is that in the regimes of interest, $\eta_{K,L}(\mathcal{A})$ does not depend on the number of users per-cell L. This observation is key to our proof of Theorem 2, where we invoke results for $K\times K$ networks (i.e. single-user cells) by Chan et al. [1].

A. Proof of Theorem 2

For any $\boldsymbol{\alpha}\in\mathbb{R}^{K\times K\times L}_{+}$, the sub-array $\boldsymbol{\alpha}^{[L]}\in\mathbb{R}^{K\times K}_{+}$, with the (i,j)-th element given by $\boldsymbol{\alpha}^{[L]}(i,j)=\alpha^{[L]}_{ij}$, specifies a $K\times K$ network (K-user IC under no cooperation, and K-user MISO-BC under full cooperation). From Theorems 5.1, 6.1 and 7.1 in [1], we know that extremal GDoF gains for $K\times K$ networks in the regimes of interest are given by

$$\eta_{K,1}(\mathcal{A})=\begin{cases}\frac{3}{2}, & \mathcal{A}=\mathcal{A}^{\mathrm{TIN}}\\ 2-\frac{1}{K}, & \mathcal{A}=\mathcal{A}^{\mathrm{CTIN}}\\ \Theta(\log(K)), & \mathcal{A}=\mathcal{A}^{\mathrm{SLS}}.\end{cases} \quad (23)$$

In what follows, we focus on an arbitrary regime $\mathcal{A}$ from $\{\mathcal{A}^{\mathrm{TIN}},\mathcal{A}^{\mathrm{CTIN}},\mathcal{A}^{\mathrm{SLS}}\}$. Let $\boldsymbol{\alpha}^{\star}$ be a network that satisfies

$$\frac{d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha}^{\star})}{d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}^{\star})}=\max_{\boldsymbol{\alpha}^{[L]}\in\mathcal{A}}\frac{d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha}^{[L]})}{d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}^{[L]})}=\eta_{K,1}(\mathcal{A}). \quad (24)$$

Note that $\boldsymbol{\alpha}^{\star}$ exists—it can be constructed by setting $\boldsymbol{\alpha}^{\star[L]}$ as the sub-array that maximizes $d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha}^{[L]})/d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}^{[L]})$, while setting all other coefficients to zero. It follows that

$$\eta_{K,L}(\mathcal{A})=\max_{\boldsymbol{\alpha}\in\mathcal{A}}\frac{d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha})}{d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha})}\geq\frac{d^{\mathrm{MBC}}_{\Sigma}(\boldsymbol{\alpha}^{\star})}{d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}^{\star})}=\eta_{K,1}(\mathcal{A}) \quad (25)$$

which proves a lower bound for the extremal gains in Theorem 2. It now remains to prove a matching upper bound.

To this end, we first observe that for any $\boldsymbol{\alpha}$, we have

$$d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha})=d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}^{[L]}). \quad (26)$$

That is, starting from a network $\boldsymbol{\alpha}^{[L]}$ with single-user cells, including additional (weaker) users in each cell does not increase the sum-GDoF achieved using TIN. The fact that $d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha})\geq d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}^{[L]})$ clearly holds, since $\boldsymbol{\alpha}^{[L]}$ is a sub-network of $\boldsymbol{\alpha}$, and TIN is a special case of mc-TIN. The other direction, i.e. $d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha})\leq d^{\mathrm{TIN}}_{\Sigma}(\boldsymbol{\alpha}^{[L]})$, holds by construction of the mc-TIN scheme—all L messages of cell k are recovered by UE-(L,k), which in turn bounds the achievable GDoF in each cell k by the achievable GDoF of UE-(L,k).

In addition to the above observation, we also observe that

$$d_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha}) \leq d_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha}^{[L]}) \tag{27}$$

To see this, note that $\mathcal{D}_{\mathrm{out}}^{\mathrm{MBC}}(\boldsymbol{\alpha})$ in Theorem 1 is included in the region given by all $\mathbf{d} \in \mathbb{R}_{+}^{KL}$ that satisfy

$$\sum_{k \in \{\sigma\}} \bar{d}_k \leq \Delta_{\pi,m}^{+}, \ \forall m \in \langle |\pi| \rangle, \pi \in \Pi,$$
$$\{\pi\} = \{(L,\sigma(1)), \ldots, (L,\sigma(|\pi|))\}. \tag{28}$$

where $\bar{d}_k \triangleq \sum_{s_k \in \langle L \rangle} d_k^{[s_k]}$. This holds as (28) is obtained from (18) by removing some inequalities, and only keeping inequalities that involve all L users in each participating cell. Denoting the sum-GDoF outer bound obtained from (28) as $\bar{d}_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha})$, we have $d_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha}) \leq \bar{d}_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha})$. Next, we view (28) as a GDoF region for a $K \times K$ network, with GDoF tuples given by $\bar{\mathbf{d}} \triangleq \big(\bar{d}_k : k \in \langle K \rangle\big) \in \mathbb{R}_{+}^{K}$. In this case, it can be verified that (28) coincides with $\mathcal{D}_{\mathrm{out}}^{\mathrm{MBC}}(\boldsymbol{\alpha}^{[L]})$, and we have $d_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha}^{[L]}) = \bar{d}_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha})$. Combining this with the previous observation, it follows that the inequality in (27) holds.

From (26) and (27), we obtain

$$\eta_{K,L}(\mathcal{A}) \leq \max_{\boldsymbol{\alpha} \in \mathcal{A}} \frac{d_{\Sigma,\mathrm{out}}^{\mathrm{MBC}}(\boldsymbol{\alpha})}{d_{\Sigma}^{\mathrm{TIN}}(\boldsymbol{\alpha})} \tag{29}$$

$$\leq \max_{\boldsymbol{\alpha} \in \mathcal{A}} \frac{d_{\Sigma,\mathrm{out}}^{\mathrm{MBC}}(\boldsymbol{\alpha}^{[L]})}{d_{\Sigma}^{\mathrm{TIN}}(\boldsymbol{\alpha}^{[L]})}. \tag{30}$$

Finally, from the proofs of Theorems 5.1, 6.1 and 7.1 in [1], it can be deduced that for all $\boldsymbol{\alpha} \in \mathcal{A}$, we have

$$d_{\mathrm{out},\Sigma}^{\mathrm{MBC}}(\boldsymbol{\alpha}^{[L]}) \leq \eta_{K,1}(\mathcal{A}) d_{\Sigma}^{\mathrm{TIN}}(\boldsymbol{\alpha}^{[L]}). \tag{31}$$

Combining this with the inequality in (30), we obtain the desired upper bound $\eta_{K,L}(\mathcal{A}) \leq \eta_{K,1}(\mathcal{A})$. This completes the proof of Theorem 2.

VI. Conclusion

The results presented here extend previous results by Chan et al. [1] to cellular networks, and give rise to a number of interesting questions. For instance, it is not clear whether the logarithmic extremal gain in the mc-SLS regime is a fundamental gain of the MISO-BC over the IBC, or an artefact of limiting the IBC to mc-TIN. Will this gain diminish if we replace mc-TIN with a layered superposition scheme with rate-splitting for the IBC? Another intriguing direction is to study extremal GDoF gains beyond the mc-SLS regime.

Appendix

To prove the outer bound in Theorem 1, we work with deterministic approximations of the Gaussian channel. The deterministic counterpart to the model in (1) is given by

$$\bar{Y}_k^{[l_k]}(t) = \sum_{i=1}^{K} \big\lfloor \bar{P}^{\alpha_{ki}^{[l_k]} - \alpha_{\max,i}} G_{ki}^{[l_k]}(t) \bar{X}_i(t) \big\rfloor \tag{32}$$

where both real and imaginary components of $\bar{X}_i(t)$ are drawn from $\big\langle 0 : \lceil \bar{P}^{\alpha_{\max,i}} \rceil \big\rangle$, and $\alpha_{\max,i} \triangleq \max_{(l_j,j) \in \mathcal{K}} \alpha_{ji}^{[l_j]}$. In the regime of interest, we have $\alpha_{\max,i} = \alpha_{ii}^{[L_i]}$. As shown in [6], the GDoF of the original channel in (1) is bounded above by the GDoF of the deterministic channel in (32). Next, we recall a key lemma from [1] (see also [4] for the proof).

Lemma 1. ***(Lemma 2.1 in [1], or Lemma 1 in [4])*** *For the above deterministic model, consider the output signals*

$$\bar{Y}_k(t) = \sum_{i=1}^{K} \big\lfloor \bar{P}^{\lambda_i - \alpha_{\max,i}} G_{ki}(t) \bar{X}_i(t) \big\rfloor \tag{33}$$

$$\bar{Y}_j(t) = \sum_{i=1}^{K} \big\lfloor \bar{P}^{\nu_i - \alpha_{\max,i}} G_{ji}(t) \bar{X}_i(t) \big\rfloor \tag{34}$$

where $\lambda_i, \nu_i \in [0, \alpha_{\max,i}]$, for all $i \in \langle K \rangle$, are the corresponding channel strength parameters. Let U be an auxiliary random variable and assume that $(U, \bar{\boldsymbol{X}}_1, \ldots, \bar{\boldsymbol{X}}_K)$ are independent of channel realizations in $\boldsymbol{G}$. We have

$$H\big(\bar{\boldsymbol{Y}}_k|\boldsymbol{G},U\big) - H\big(\bar{\boldsymbol{Y}}_j|\boldsymbol{G},U\big) \leq \max_{i \in \langle K \rangle} (\lambda_i - \nu_i)^{+} T \log(P) + To(\log(P)). \tag{35}$$

For brevity, we adopt the compact notation of [1] to represent differences of entropies of the type in Lemma 1. Using this compact notion, inequality (35) is expressed as

$$\mathbb{H}\left([\lambda_1, \ldots, \lambda_K]\right) - \mathbb{H}\left([\nu_1, \ldots, \nu_K]\right) \leq \max_{i \in \langle K \rangle} (\lambda_i - \nu_i)^{+} T \log(P) \tag{36}$$

where the $o(\log(P))$ term is dropped as it is inconsequential for GDoF purposes (we ignore such terms henceforth). We are now fully equipped to derive the outer bound.

Single-cell bounds in (18), associated with cycles of length $|\pi| = 1$, are directly obtained from the capacity region of the degraded Gaussian BC [16]. We hence focus on multi-cell bounds in (18), associated with cycles of length $|\pi| \geq 2$.

Consider an arbitrary cycle $\pi \in \Pi$ of length $|\pi| = M \geq 2$, and let σ and ρ be the two associated implicit cycles (see Definition 4). First, we consider a single participating cell i and its l_i participating users, where $(l_i, i) \in \{\pi\}$, and we focus on the corresponding sum-rate $\sum_{s=1}^{l_i} R_i^{[s_i]}$. Let U_i be an axillary variable independent of $\boldsymbol{W}_i = (W_i^{[1]}, \ldots, W_i^{[l_i]})$, the set of messages in cell i. Fano's inequality implies

$$T \sum_{s=1}^{l_i} R_i^{[s]} \leq \sum_{s=1}^{l_i} I\Big(W_i^{[s]}; \bar{\boldsymbol{Y}}_i^{[s]} | \boldsymbol{G}, W_i^{[1:s-1]}, U_i\Big)$$
$$= \sum_{s=2}^{l_i} H\Big(\bar{\boldsymbol{Y}}_i^{[s]} | \boldsymbol{G}, W_i^{[1:s-1]}, U_i\Big) - H\Big(\bar{\boldsymbol{Y}}_i^{[s-1]} | \boldsymbol{G}, W_i^{[1:s-1]}, U_i\Big)$$
$$+ H\Big(\bar{\boldsymbol{Y}}_i^{[1]} | \boldsymbol{G}, U_i\Big) - H\Big(\bar{\boldsymbol{Y}}_i^{[l_i]} | \boldsymbol{G}, \boldsymbol{W}_i, U_i\Big) \tag{37}$$
$$= \sum_{s=2}^{l_i} \mathbb{H}\left(\big[\alpha_{i1}^{[s]}, \ldots, \alpha_{iK}^{[s]}\big]\right) - \mathbb{H}\left(\big[\alpha_{i1}^{[s-1]}, \ldots, \alpha_{iK}^{[s-1]}\big]\right)$$
$$+ H\Big(\bar{\boldsymbol{Y}}_i^{[1]} | \boldsymbol{G}, U_i\Big) - H\Big(\bar{\boldsymbol{Y}}_i^{[l_i]} | \boldsymbol{G}, \boldsymbol{W}_i, U_i\Big). \tag{38}$$

Now we focus on bounding the sum of differences of entropies in the brief notation in (38). Using Lemma 1, we obtain

$$\sum_{s=2}^{l_i}\mathbb{H}\left(\left[\alpha_{i1}^{[s]},\ldots,\alpha_{iK}^{[s]}\right]\right)-\mathbb{H}\left(\left[\alpha_{i1}^{[s-1]},\ldots,\alpha_{iK}^{[s-1]}\right]\right)$$
$$\leq\sum_{s=2}^{l_i}\max_{j\in\langle K\rangle}\left(\alpha_{ij}^{[s]}-\alpha_{ij}^{[s-1]}\right)^{+}T\log(P) \tag{39}$$
$$\leq\sum_{s=2}^{l_i}\left(\alpha_{ii}^{[s]}-\alpha_{ii}^{[s-1]}\right)T\log(P) \tag{40}$$
$$=\left(\alpha_{ii}^{[l_i]}-\alpha_{ii}^{[1]}\right)T\log(P). \tag{41}$$

Going from (39) to (40) follows from the SIR condition (10) in the mc-SLS regime, as well as the SNR order in (2), which allows us to drop the $(\cdot)^{+}$. From (38) and (41), we obtain

$$T\sum_{s=1}^{l_i}R_i^{[s]}\leq\left(\alpha_{ii}^{[l_i]}-\alpha_{ii}^{[1]}\right)T\log(P)$$
$$+H\left(\bar{\boldsymbol{Y}}_i^{[1]}|\boldsymbol{G},U_i\right)-H\left(\bar{\boldsymbol{Y}}_i^{[l_i]}|\boldsymbol{G},\boldsymbol{W}_i,U_i\right) \tag{42}$$

which holds for any single cell $i\in\{\sigma\}$. Next, we combine the bounds obtained from (42), for all $i\in\{\sigma\}$, to obtain

$$T\sum_{m=1}^{M}\sum_{s=1}^{\rho(m)}R_{\sigma(m)}^{[s]}\leq\sum_{m=1}^{M}\left(\alpha_{\sigma(m)\sigma(m)}^{[\rho(m)]}-\alpha_{\sigma(m)\sigma(m)}^{[1]}\right)T\log(P)$$
$$+\sum_{m=1}^{M}H\left(\bar{\boldsymbol{Y}}_{\sigma(m)}^{[1]}|\boldsymbol{G},U_{\sigma(m)}\right)-H\left(\bar{\boldsymbol{Y}}_{\sigma(m)}^{[\rho(m)]}|\boldsymbol{G},\boldsymbol{W}_{\sigma(m)},U_{\sigma(m)}\right). \tag{43}$$

Next, we set $U_{\sigma(m)}=\left(\boldsymbol{W}_{\sigma(m+1)},\ldots,\boldsymbol{W}_{\sigma(M)}\right)$. Focusing on the differences of entropies in (43), we obtain

$$\sum_{m=1}^{M}H\left(\bar{\boldsymbol{Y}}_{\sigma(m)}^{[1]}|\boldsymbol{G},U_{\sigma(m)}\right)-H\left(\bar{\boldsymbol{Y}}_{\sigma(m+1)}^{[\rho(m+1)]}|\boldsymbol{G},\boldsymbol{W}_{\sigma(m+1)},U_{\sigma(m+1)}\right)$$
$$=H\left(\bar{\boldsymbol{Y}}_{\sigma(M)}^{[1]}|\boldsymbol{G}\right)+\sum_{m=1}^{M-1}\mathbb{H}\left(\left[\alpha_{\sigma(m)1}^{[1]},\ldots,\alpha_{\sigma(m)K}^{[1]}\right]\right)$$
$$-\mathbb{H}\left(\left[\alpha_{\sigma(m+1)1}^{[\rho(m+1)]},\ldots,\alpha_{\sigma(m+1)K}^{[\rho(m+1)]}\right]\right) \tag{44}$$
$$\leq T\log(P)\alpha_{\sigma(M)\sigma(M)}^{[1]}+$$
$$T\log(P)\sum_{m=1}^{M-1}\max_{j\in\langle K\rangle}\left(\alpha_{\sigma(m)j}^{[1]}-\alpha_{\sigma(m+1)j}^{[\rho(m+1)]}\right)^{+} \tag{45}$$
$$\leq T\log(P)\alpha_{\sigma(M)\sigma(M)}^{[1]}+$$
$$T\log(P)\sum_{m=1}^{M-1}\left(\alpha_{\sigma(m)\sigma(m)}^{[1]}-\alpha_{\sigma(m+1)\sigma(m)}^{[\rho(m+1)]}\right). \tag{46}$$

In the above, (45) is obtained from a direct application of Lemma 1. The bound in (46) holds due to the mc-SLS condition in (11), which implies

$$\alpha_{\sigma(m)\sigma(m)}^{[1]}-\alpha_{\sigma(m+1)\sigma(m)}^{[\rho(m+1)]}\geq\alpha_{\sigma(m)j}^{[1]}-\alpha_{\sigma(m+1)j}^{[\rho(m+1)]}$$
$$\alpha_{\sigma(m)\sigma(m)}^{[1]}-\alpha_{\sigma(m+1)\sigma(m)}^{[\rho(m+1)]}\geq 0$$

from which (46) directly follows. By combining the bounds in (46) and (43), we obtain the desired cycle bound as

$$T\sum_{m=1}^{M}\sum_{s=1}^{\rho(m)}R_{\sigma(m)}^{[s]}\leq\alpha_{\sigma(M)\sigma(M)}^{[\rho(M)]}T\log(P)$$
$$+\sum_{m=1}^{M-1}\left(\alpha_{\sigma(m)\sigma(m)}^{[\rho(m)]}-\alpha_{\sigma(m+1)\sigma(m)}^{[\rho(m+1)]}\right)T\log(P)$$
$$=\left(\Delta_{\pi}+\alpha_{\sigma(1)\sigma(M)}^{[\rho(1)]}\right)T\log(P)=\left(\Delta_{\pi,M}^{+}\right)T\log(P) \tag{47}$$

where the last two equalities follow from the definitions of Δ_π and $\Delta_{\pi,M}^{+}$. This proves one of the M GDoF bound associated with cycle π. To obtain the remaining $M-1$ bounds for the same cycle, we follow the same steps while replacing π with a shifted version π', where $\pi'(m)=\pi(m+j)$. Repeating the same for all cycles $\pi\in\Pi$ with $|\pi|\geq 2$, we obtained the outer bound in Theorem 1, which concludes the proof.

REFERENCES

[1] Y. Chan, J. Wang, and S. A. Jafar, "Toward an extremal network theory—robust GDoF gain of transmitter cooperation over TIN," *IEEE Trans. Inf. Theory*, vol. 66, no. 6, pp. 3827–3845, 2020.

[2] R. H. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, 2008.

[3] S. A. Jafar and S. Vishwanath, "Generalized degrees of freedom of the symmetric Gaussian K user interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 7, pp. 3297–3303, 2010.

[4] A. G. Davoodi and S. A. Jafar, "Generalized degrees of freedom of the symmetric K user interference channel under finite precision CSIT," *IEEE Trans. Inf. Theory*, vol. 63, no. 10, pp. 6561–6572, 2017.

[5] A. G. Davoodi, B. Yuan, and S. A. Jafar, "GDoF region of the MISO BC: Bridging the gap between finite precision and perfect CSIT," *IEEE Trans. Inf. Theory*, vol. 64, no. 11, pp. 7208–7217, 2018.

[6] A. G. Davoodi and S. A. Jafar, "Aligned image sets under channel uncertainty: Settling conjectures on the collapse of degrees of freedom under finite precision CSIT," *IEEE Trans. Inf. Theory*, vol. 62, no. 10, pp. 5603–5618, 2016.

[7] H. Joudeh and B. Clerckx, "On the separability of parallel MISO broadcast channels under partial CSIT: A degrees of freedom region perspective," *IEEE Trans. Inf. Theory*, vol. 66, no. 7, pp. 4513–4529, 2020.

[8] A. G. Davoodi and S. A. Jafar, "Optimality of simple layered superposition coding in the 3 user MISO BC with finite precision CSIT," *IEEE Trans. Inf. Theory*, vol. 65, no. 11, pp. 7181–7207, 2019.

[9] H. Joudeh and G. Caire, "Optimality of treating inter-cell interference as noise under finite precision CSIT," in *Proc. IEEE Int. Symp. Inf. Theory*, 2020.

[10] C. Geng, N. Naderializadeh, A. S. Avestimehr, and S. A. Jafar, "On the optimality of treating interference as noise," *IEEE Trans. Inf. Theory*, vol. 61, no. 4, pp. 1753–1767, 2015.

[11] X. Yi and G. Caire, "Optimality of treating interference as noise: A combinatorial perspective," *IEEE Trans. Inf. Theory*, vol. 62, no. 8, pp. 4654–4673, 2016.

[12] H. Sun and S. A. Jafar, "On the optimality of treating interference as noise for K-user parallel Gaussian interference networks," *IEEE Trans. Inf. Theory*, vol. 62, no. 4, pp. 1911–1930, 2016.

[13] H. Joudeh and B. Clerckx, "On the optimality of treating inter-cell interference as noise in uplink cellular networks," *IEEE Trans. Inf. Theory*, vol. 65, no. 11, pp. 7208–7232, 2019.

[14] H. Joudeh, X. Yi, B. Clerckx, and G. Caire, "On the optimality of treating inter-cell interference as noise: Downlink cellular networks and uplink-downlink duality," *IEEE Trans. Inf. Theory, to appear*.

[15] H. Joudeh and G. Caire, "Cellular networks with finite precision CSIT: GDoF optimality of multi-cell TIN and extremal gains of multi-cell cooperation," *arXiv:2008.08945*, 2020.

[16] T. M. Cover and J. A. Thomas, *Elements of information theory*. John Wiley & Sons, 2012.