

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

**Pearson Edexcel Level 3 GCE**

**Tuesday 6 June 2023**

Afternoon (Time: 2 hours)

Paper reference **9MA0/01**

**Mathematics**

**Advanced**

**PAPER 1: Pure Mathematics 1**

**You must have:**  
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P72804A

©2023 Pearson Education Ltd.  
N:1/1/1/1/1/



  
**Pearson**

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3} dx$$

writing each term in simplest form.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 1 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 1 is 4 marks)**

2.

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \quad x \in \mathbb{R}$$

where  $a$  is a positive constant.

Given  $(x - a)$  is a factor of  $f(x)$ ,

(a) show that

$$a(4a^2 + 5a - 6) = 0 \quad (2)$$

(b) Hence

(i) find the value of  $a$

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3 \quad (4)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 2 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 2 is 6 marks)**

3. Relative to a fixed origin  $O$

- the point  $A$  has position vector  $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point  $B$  has position vector  $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where  $a$  is a positive integer.

(a) Show that  $|\vec{OA}| = \sqrt{38}$  (1)

(b) Find the smallest value of  $a$  for which

$$|\vec{OB}| > |\vec{OA}|$$
 (2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 3 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 3 is 3 marks)**





**Question 4 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 4 is 5 marks)**

5. A continuous curve has equation  $y = f(x)$ .

The table shows corresponding values of  $x$  and  $y$  for this curve, where  $a$  and  $b$  are constants.

$x$	3	3.2	3.4	3.6	3.8	4
$y$	$a$	16.8	$b$	20.2	18.7	13.5

The trapezium rule is used, with all the  $y$  values in the table, to find an approximate area under the curve between  $x = 3$  and  $x = 4$

Given that this area is 17.59

- (a) show that  $a + 2b = 51$

(3)

Given also that the sum of all the  $y$  values in the table is 97.2

- (b) find the value of  $a$  and the value of  $b$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 5 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 5 is 6 marks)**

6.

$a = \log_2 x$

$b = \log_2(x + 8)$

Express in terms of  $a$  and/or  $b$ 

$$(a) \log_2 \sqrt{x} \quad (1)$$

$$(b) \log_2(x^2 + 8x) \quad (2)$$

$$(c) \log_2 \left( 8 + \frac{64}{x} \right) \quad (3)$$

Give your answer in simplest form.

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 6 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 6 is 6 marks)**



## Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 2 8 0 4 A 0 1 5 4 4

**Question 7 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





**Question 7 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 7 is 8 marks)**



## Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 8 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



## Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)



9. The first three terms of a geometric sequence are

$$3k + 4 \quad 12 - 3k \quad k + 16$$

where  $k$  is a constant.

(a) Show that  $k$  satisfies the equation

$$3k^2 - 62k + 40 = 0 \quad (2)$$

Given that the sequence converges,

(b) (i) find the value of  $k$ , giving a reason for your answer,

(ii) find the value of  $S_{\infty}$  (5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



## Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 7 marks)



P 7 2 8 0 4 A 0 2 3 4 4

10. A circle  $C$  has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where  $k$  is a constant.

(a) Find in terms of  $k$ ,

(i) the coordinates of the centre of  $C$

(ii) the radius of  $C$

(3)

The line with equation  $y = 2x - 1$  intersects  $C$  at 2 distinct points.

(b) Find the range of possible values of  $k$ .

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





## Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 2 8 0 4 A 0 2 5 4 4

**Question 10 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 10 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 10 is 9 marks)**

A graph showing the relationship between  $\log_{10} V$  (vertical axis) and  $t$  (horizontal axis). The origin is marked  $O$ . A straight line is plotted, passing through the points  $(0, 3)$  and  $(10, 2.79)$ .

The value,  $V$  pounds, of a mobile phone,  $t$  months after it was bought, is modelled by

where  $a$  and  $b$  are constants.

Figure 2 shows the linear relationship between  $\log_{10} V$  and  $t$ .

The line passes through the points  $(0, 3)$  and  $(10, 2.79)$  as shown.

Using these points,

- (a) find the initial value of the phone, (2)
- (b) find a complete equation for  $V$  in terms of  $t$ , giving the exact value of  $a$  and giving the value of  $b$  to 3 significant figures.

Exactly 2 years after it was bought, the value of the phone was £320

- (c) Use this information to evaluate the reliability of the model. (2)

## This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



**Question 11 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 11 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 11 is 7 marks)**

12.

$$y = \sin x$$

where  $x$  is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for  $\sin(A \pm B)$
- assume that as  $h \rightarrow 0$ ,  $\frac{\sin h}{h} \rightarrow 1$  and  $\frac{\cos h - 1}{h} \rightarrow 0$

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





**Question 12 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 12 is 5 marks)**

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height,  $H$  m, of a carriage above the ground,  $t$  seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where  $a$  and  $b$  are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when  $t = 40$

(1)

In an alternative model, the vertical height,  $H$  m, of a carriage above the ground,  $t$  seconds after the carriage starts the first circuit, is given by

$$H = 29 \cos(9t + \alpha)^\circ + \beta \quad 0 \leq \alpha < 360^\circ$$

where  $\alpha$  and  $\beta$  are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



## Question 13 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 2 8 0 4 A 0 3 5 4 4

**Question 13 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 13 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 13 is 7 marks)**

14. Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all  $n \in \mathbb{N}$

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 14 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 14 is 4 marks)**

15. A curve has equation  $y = f(x)$ , where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where  $A$  and  $B$  are constants to be found.

(5)

(b) Hence show that the  $x$  coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation  $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$  has two positive roots  $\alpha$  and  $\beta$  where  $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for  $\alpha$  and  $\beta$

Diagram 1 shows a plot of part of the curve with equation  $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$  and part of the line with equation  $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with  $x_1 = 1$  can be used to find an approximation for  $\beta$

(1)

Use the iteration formula with  $x_1 = 1$ , to find, to 3 decimal places,

(d) (i) the value of  $x_2$

(ii) the value of  $\beta$

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that  $\alpha = 0.432$  to 3 decimal places.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





## Question 15 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 2 8 0 4 A 0 4 1 4 4

### Question 15 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

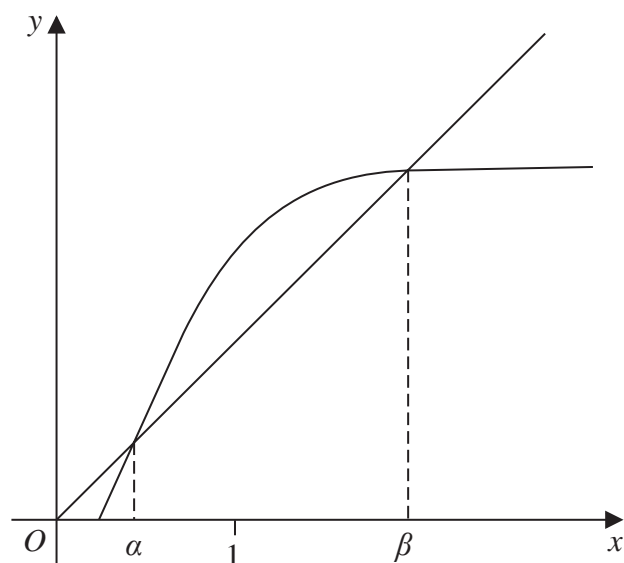
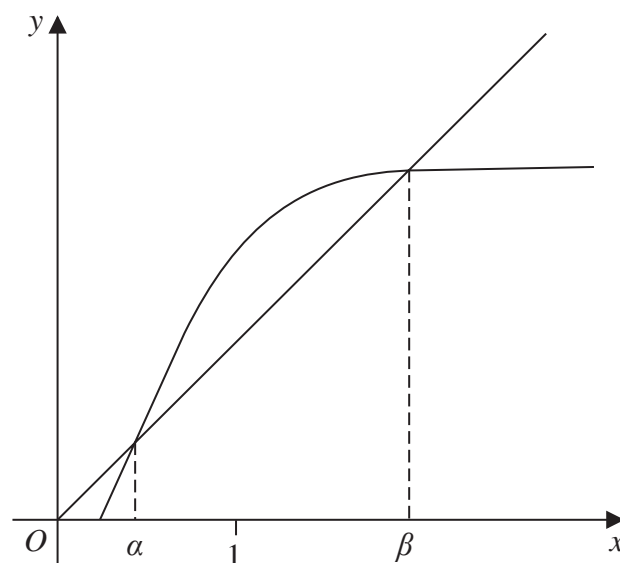


Diagram 1



copy of Diagram 1

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



## Question 15 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 2 8 0 4 A 0 4 3 4 4

**Question 15 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**(Total for Question 15 is 13 marks)****TOTAL FOR PAPER IS 100 MARKS**