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PAPER 2: Pure	Mathemat	ics 2		
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You must have: Mathematical Formula	ae and Statistic	al Tables (Gre	een), calculato	Total Marks
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Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## **Instructions**

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
   there may be more space than you need
- there may be more space than you need.
- You should show sufficient working to make your methods clear.
   Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
- use this as a guide as to how much time to spend on each question.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶







In this question you must show all stages of your working.
 Solutions relying entirely on calculator technology are not acceptable.

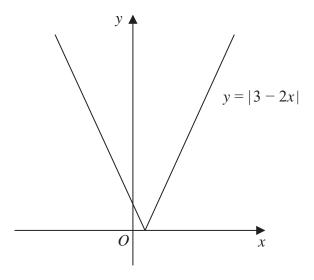


Figure 1

Figure 1 shows a sketch of the graph with equation y = |3 - 2x|Solve

$$|3-2x|=7+x$$

**(4)** 

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Question 1 continued	
	(Total for Question 1 is 4 marks)



2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

**(2)** 

(b) Solve

$$4^{x} = 100$$

giving your answer to 2 decimal places.

**(2)** 





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Question 2 continued
(Total for Question 2 is 4 marks)
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**3.** A sequence of terms  $a_1, a_2, a_3, \dots$  is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

- (a) (i) Show that this sequence is periodic.
  - (ii) State the order of this periodic sequence.

**(2)** 

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

**(2)** 



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Question 3 continued
(Total for Question 3 is 4 marks)



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4.	Given	tha

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x$$

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Question 4 continued
(Total for Question 4 is 3 marks)
(Total for Question 4 is 5 marks)



5. The table below shows corresponding values of x and y for  $y = \log_3 2x$ 

The values of y are given to 2 decimal places as appropriate.

X	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, \mathrm{d}x$$

**(3)** 

Using your answer to part (a) and making your method clear, estimate

(b) (i) 
$$\int_{3}^{9} \log_{3} (2x)^{10} dx$$
(ii) 
$$\int_{3}^{9} \log_{3} 18x dx$$

(ii) 
$$\int_{3}^{9} \log_3 18x \, \mathrm{d}x$$

**(3)** 





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Question 5 continued
(Total for Question 5 is 6 marks)
(Total for Question 5 is 6 marks)



**6.** 

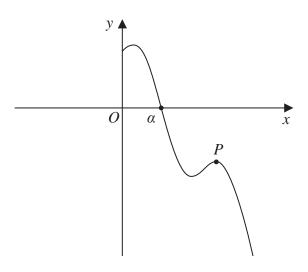


Figure 2

Figure 2 shows a sketch of part of the curve with equation y = f(x) where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and *x* is measured in radians.

The point *P*, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

**(4)** 

The curve crosses the *x*-axis at  $x = \alpha$ , as shown in Figure 2.

Given that, to 3 decimal places, f(4) = 4.274 and f(5) = -1.212

(b) explain why  $\alpha$  must lie in the interval [4, 5]

**(1)** 

(c) Taking  $x_0 = 5$  as a first approximation to  $\alpha$ , apply the Newton-Raphson method once to f(x) to obtain a second approximation to  $\alpha$ .

Show your method and give your answer to 3 significant figures.

**(2)** 

Questio	on 6 continued		

Question 6 continued

Question 6 continued
(Total for Question 6 is 7 marks)



7. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4-9x}$$

writing each term in simplest form.

**(4)** 

A student uses this expansion with  $x = \frac{1}{9}$  to find an approximation for  $\sqrt{3}$ 

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of  $\sqrt{3}$  giving a brief reason for your answer.

**(1)** 

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Question 7 continued
(Total for Question 7 is 5 marks)



8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

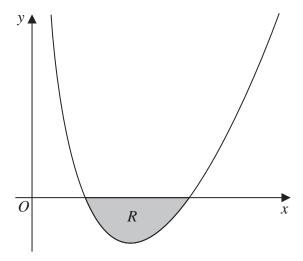


Figure 3

Figure 3 shows a sketch of part of a curve with equation

$$y = \frac{(x-2)(x-4)}{4\sqrt{x}} \qquad x > 0$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Find the exact area of R, writing your answer in the form  $a\sqrt{2} + b$ , where a and b are constants to be found.



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Question 8 continued	
(Total for Question	n 8 is 6 marks)



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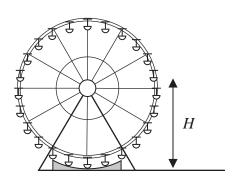


Figure 4

Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, Hm, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A\sin(bt + \alpha)^{\circ}|$$

where A, b and  $\alpha$  are constants.

Figure 5 shows a sketch of the graph of *H* against *t*, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution
- (a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of a to 3 significant figures.

**(4)** 

(b) Explain why an equation of the form

$$H = |A\sin(bt + \alpha)^{\circ}| + d$$

where d is a positive constant, would be a more appropriate model.

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Question 9 continued
(Total for Question 9 is 5 marks)
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**10.** The function f is defined by

$$f(x) = \frac{8x+5}{2x+3} \qquad x > -\frac{3}{2}$$

(a) Find  $f^{-1}\left(\frac{3}{2}\right)$ 

**(2)** 

(b) Show that

$$f(x) = A + \frac{B}{2x+3}$$

where *A* and *B* are constants to be found.

**(2)** 

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of  $g^{-1}$ 

**(1)** 

(d) Find the range of  $f g^{-1}$ 

**(3)** 

Question 10 continued



Question 10 continued

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Question 10 continued	
	(Total for Question 10 is 8 marks)



11. Prove, using algebra, that	$n(n^2+5)$	
is even for all $n \in \mathbb{N}$ .		(4)

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Question 11 continued
(Total for Orestion 11 in America)
(Total for Question 11 is 4 marks)



**12.** The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k}$$

where k is a positive constant.

(a) Show that

$$f'(x) = (12x^2 - 8x + 3k)g(x)$$

where g(x) is a function to be found.

(3)

Given that the curve with equation y = f(x) has at least one stationary point,

(b) find the range of possible values of k.

**(3)** 


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Question 12 continued	
(Total	for Question 12 is 6 marks)



## 13. Relative to a fixed origin O

- the point A has position vector  $4\mathbf{i} 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector  $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector  $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

**(3)** 

The line segment OB is extended to a point D so that  $\overrightarrow{CD}$  is parallel to  $\overrightarrow{OA}$ 

(b) Find  $|\overrightarrow{OD}|$ , writing your answer as a fully simplified surd.

**(3)** 



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**Question 13 continued** 

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Question 13 continued

Question 13 continued	
	(Total for Organization 12 in Communication
	(Total for Question 13 is 6 marks)



**14.** (a) Express 
$$\frac{3}{(2x-1)(x+1)}$$
 in partial fractions.

**(3)** 

When chemical A and chemical B are mixed, oxygen is produced.

A scientist mixed these two chemicals and measured the total volume of oxygen produced over a period of time.

The total volume of oxygen produced,  $V \text{m}^3$ , t hours after the chemicals were mixed, is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{3V}{(2t-1)(t+1)} \qquad V \geqslant 0 \qquad t \geqslant k$$

where k is a constant.

Given that exactly 2 hours after the chemicals were mixed, a total volume of 3 m<sup>3</sup> of oxygen had been produced,

(b) solve the differential equation to show that

$$V = \frac{3(2t-1)}{(t+1)} \tag{5}$$

The scientist noticed that

- there was a **time delay** between the chemicals being mixed and oxygen being produced
- there was a **limit** to the total volume of oxygen produced

Deduce from the model

- (c) (i) the time delay giving your answer in minutes,
  - (ii) the **limit** giving your answer in m<sup>3</sup>

**(2)** 

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Question 14 continued



Question 14 continued

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Question 14 continued	
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15. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that the first three terms of a geometric series are

$$12\cos\theta$$

$$5 + 2\sin\theta$$

$$6 \tan \theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

**(3)** 

Given that  $\theta$  is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of  $\boldsymbol{\theta}$ 

**(2)** 

(c) show that the sum to infinity of the series can be expressed in the form

$$k(1-\sqrt{3})$$

where k is a constant to be found.

**(5)** 

Question 15 continued



Question 15 continued

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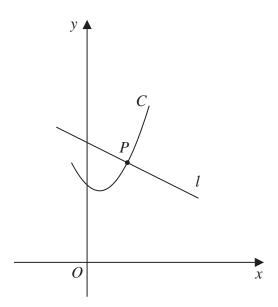


Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 2\tan t + 1 \qquad \qquad y = 2\sec^2 t + 3 \qquad \qquad -\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{3}$$

The line *l* is the normal to *C* at the point *P* where  $t = \frac{\pi}{4}$ 

(a) Using parametric differentiation, show that an equation for l is

$$y = -\frac{1}{2}x + \frac{17}{2} \tag{5}$$

(b) Show that all points on C satisfy the equation

$$y = \frac{1}{2} (x - 1)^2 + 5 \tag{2}$$

The straight line with equation

$$y = -\frac{1}{2}x + k$$
 where k is a constant

intersects C at two distinct points.

(c) Find the range of possible values for k.

**(5)** 

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Question 16 continued		



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Question 16 continued		



Question 16 continued		
	(Total for Question 16 is 12 marks)	
	TOTAL FOR PAPER IS 100 MARKS	

