

Gait sequence modelling and estimation

using Hidden Markov Models



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Declaration

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Acknowledgments

Abstract

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Chapter 1

Introduction

1.1 Background to the study

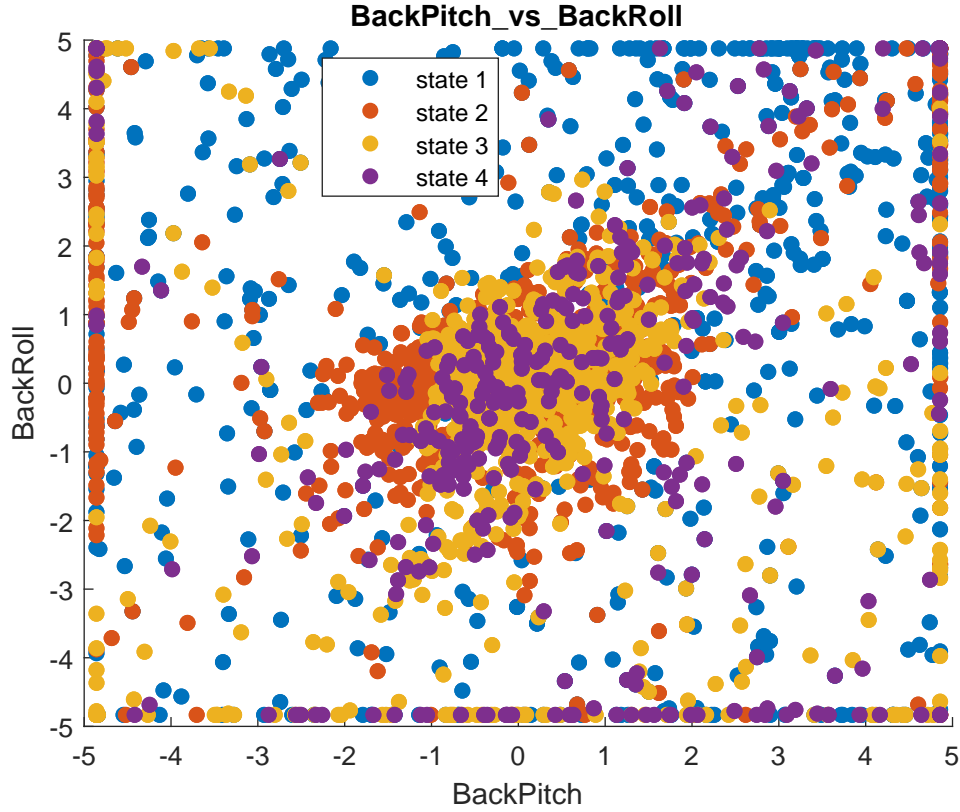
A very brief background to your area of research. Start off with a general introduction to the area and then narrow it down to your focus area. Used to set the scene [1].

Bio-inspired robotics uses nature to inform real-world engineering systems. Research has been conducted at UCT to investigate the manner in which a cheetah uses its tail for stability during high acceleration, quick turns and sudden braking, with an aim to incorporating identified mechanisms into sophisticated robot designs. One way to acquire useful data is to strap an inertial measurement unit (IMU) to an animal, and log the sensor data while certain actions are being performed. We currently have such a dataset of a dog moving, along with corresponding video data.

1.2 Objectives of this study

The objective of this project is to design, implement, and test Hidden Markov Models (HMM) for estimating gait sequence from Inertia Measurement Unit (IMU) data.

so that specific models can be formulated and their parameters estimated and interrogated. The project can be extended to include any other useful analysis of gait patterns from similar sensor measurements



1 - formulate model 2 - estimate its parameters 3 - Interrogate its parameters 4 - Useful analysis of gait patterns from IMU measurements

1.2.1 Problems to be investigated

Description of the main questions to be investigated in this study.

The main questions to be answered are the following:

1. How well can HMM model gait sequence dynamics using IMU data, in the absence of enough training samples?
2. Can dimensionality reduction cause an increase in performance of HMM models when there is not enough training data?

1.2.2 Purpose of the study

Give the significance of investigating these problems. It must be obvious why you are doing this study and why it is relevant.

1.3 Scope and Limitations

Scope indicates to the reader what has and has not been included in the study. Limitations tell the reader what factors influenced the study such as sample size, time etc. It is not a section for excuses as to why your project may or may not have worked.

1 - Does not include data collection 2 - Focus on design of HMM only 3 - Focus on analysis of the model 4 - Focus on impact of dimensionality reduction

1.4 Plan of development

Here you tell the reader how your report has been organised and what is included in each chapter.

I recommend that you write this section last. You can then tailor it to your report.

Chapter 2

Literature Review

2.1 Gait sequence modelling and estimation

2.1.1 Quadrupede gait modelling

Periodicity

2.1.2 Quadrupede gait estimation

2.2 Case study: Inertia Measurement Unit

2.3 Hidden Markov Models

Hidden Markov Models (HMMs) are doubly embedded stochastic processes with a rich underlying statistical structure. Introduced at the end of the 1960s by Baum and colleagues, they have become one of the preferred techniques in speech recognition after the implementation of Baker and Jelinek in 1970s. HMMs have been successfully applied to various other engineering problems in pattern recognition for classification and fraud detection purposes, amongst others.

The type of HMM depends on the possible connections between the states. Thus, an HMM in which a state can transition to any other state is an ergodic. Other types such

as the Left-Right model or Bakis do not allow all possible transitions between the states.

2.3.1 HMM parameters specification

An HMM is fully specified by the following parameters

1. N , the number of distinct states of the model. Together they form the set of individual states $S = \{S_1, S_2, \dots, S_N\}$.
2. T , the number of observations. A sample observation sequence is denoted as $O = \{O_1, O_2, \dots, O_T\}$.
3. $Q = q_t$, the set of states with q_t denoting the current state at time instance, t such that $q_t \in S$ and $t = 1, 2, \dots, T$.
4. K , the number of distinct observation symbols per state.
5. $V = \{v_1, v_2, \dots, v_K\}$, the feature set of K dimensions.
6. $A = \{a_{ij}\}$, the state transition probabilities. a_{ij} denotes probability of transitioning from state S_i to state S_j .
7. $\Phi = \{\phi_j(k)\}$, the probability distribution of observation symbols in state j .
8. The initial state distribution, $\pi = \pi_i$

For continuous HMM (CHMM), i.e, HMM with continuous-valued observations, Φ consists in a probability distribution function. Many applications have successfully modelled such distributions with mixtures of Gaussian distributions. As such, ϕ is approximated by a weighted sum of M multivariate Gaussian distributions η . For a given, observation sequence, ϕ and η are therefore given by equations 2.1 and 2.2,

$$\phi(O_t) = \sum_{m=1}^M \beta_{jm} \eta(\mu_{jm}, \Sigma_{jm}, O_t), \quad (2.1)$$

$$\eta(\mu, \Sigma, O) = \frac{1}{\sqrt{(2\pi)^K |\Sigma|}} \exp\left(-\frac{1}{2}(O - \mu)' \Sigma^{-1} (O - \mu)\right) \quad (2.2)$$

$$1 \leq j \leq N; 1 \leq m \leq M; \beta_{jm} \geq 0; \sum_{m=1}^M \beta_{jm} = 1$$

where β_{jm} is the mixture composition coefficient; μ_{jm} , Σ_{jm} , respectively the mean vector and covariance matrix of state j ; M is the number of mixture components and K is the dimensionality of O .

As a summary, the compact specification of a continuous valued observation HMM is defined by 2.3 and that of a discrete HMM in 2.4.

$$CHMM = \lambda_C = (A, \beta_{jm}, \mu_{jm}, \Sigma_{jm}, \pi) \quad (2.3)$$

$$DHMM = \lambda_D = (A, b_j(k), \pi) \quad (2.4)$$

Basic assumptions of HMMs theory

HMM theory is built on three basic assumptions listed below.

1. *The Markov assumption:* HMM assumes that the probability of being in the current at any instance of time t , is uniquely dependent on the previous state, at time, $t + 1$. More specifically, $a_{ij} = P[q_t = S_j | q_{t+1} = S_i]$. This assumption makes it unsuitable for long-range correlation capturing applications.
2. *The stationary assumption:* Furthermore, HMM state transition probabilities are assumed to be time-independent. Thus, the transition probabilities of two distinct time, t_1 and t_2 are identical, $P[q_{t_1} = S_j | q_{t_1-1} = S_i] = P[q_{t_2} = S_j | q_{t_2-1} = S_i]$. HMMs can therefore effectively model mechanisms with stationary observations.
3. *The output/observation independence assumption:* The current observation also known as emission symbol is statistically independent of the previous observations. It is "emitted" only by the current state, $P[O | q_1, q_2, \dots, q_T, \lambda] = \prod_{t=1}^T P[O_t | q_t, \lambda]$.

The three assumptions make an HMM model a relatively simple graphic modelling to be implemented. This simplicity naturally comes with some limitations in modelling more complex problems, which however, may be modelled with higher order HMMs. Furthermore, the three assumptions are very similar to those of a Markov chain. This is because the stochastic process of an HMM pertaining to the hidden states can be reduced to a Markov chain. In fact, an HMM is an extension of a Markov Chain. The essential difference between the two is that, with the former, there is no a one-to-one mapping between the states and the observation symbols.

2.3.2 The three basics problems for HMM design

In [1], Lawrence argued that an HMM design needs to answer three fundamental problems. They are the *training problem*, the *evaluation problem*, and the *decoding problem*. Each problem and its solution is discussed in greater details next.

The evaluation problem

The evaluation problem is about answering this question: *Given the observation sequence $O = O_1O_2O_T$, and a model λ , how do we efficiently compute $P(O|\lambda)$, the probability of the observation sequence?* The naive answer to this question is simply computing the $P(O|\lambda)$ according to equation 2.5:

$$P(O|\lambda) = \sum_{q_1}^{q_T} \pi_{q_1} b_{q_1}(O_1) a_{q_1q_2} b_{q_2}(O_2) \dots a_{q_{T-1}q_T}(O_T) \quad (2.5)$$

This approach has two issues, it is not only, computationally too expensive because of the exponential complexity with respect to T , but also, intractable for very long sequence. In practice, $P(O|\lambda)$ is computed by an algorithm called *forward-backward* procedure, which is a more efficient method.

The decoding problem

The decoding problem can be reduced to this interrogation: *Given the observation sequence $O = O_1O_2O_T$, and the model λ , how do we choose a corresponding state sequence $Q = q_1q_2\dots q_T$ which is optimal in some meaningful sense i.e, best "explains" the observations?* Simply put, this problem is about deciphering the most likely hidden states that emitted the visible observation sequences. This is done dynamically using the Viterbi algorithm.

The training problem

Given the model, λ , the training problem raises the following question: *how do we adjust the model parameters λ to maximise the $P(O|\lambda)$, the probability of the probability of the observation sequence?* This problem is usually solved by iterative learning algorithms

called expectation-maximisation. Examples of this algorithms are Baum-Welch method or any gradient based method.

When using Baum-Welch algorithm, the parameters are initialised by guesses then re-estimated iteratively to find the parameters with maximum likelihood. This method is vulnerable to local maxima issues. To avoid such cases, it is advice to run it multiple times with different initial values in order to keep the estimation with the highest likelihood value.

Overfitting, order of markov, robustness: bias-var

2.4 k-Nearest Neighbour

2.5 Dimensionality reduction

Dimensionality or dimension reduction is used pattern recognition, machine learning and statistics to find the most compact representation of the dataset by removing redundant and irrelevant information. It is achieve by extracting principal features, i.e, feature extracting or by selecting the most relevant subset of the initial feature vector, i.e, feature selection, using a supervised or an unsupervised approach.

2.5.1 Motivations for dimensionality reduction

When building a model, the need for dimensionality is supported by several reasons. Some of the important ones are presented in three points. Firstly, by reducing the feature space's dimension, we can build model with higher quality. In most classification problems, the feature domains contain variables with very little to no information for the purpose at hand. Thus, removing these features reduces the complexity of the problem which can in return, increases the model's accuracy.

Secondly, working with hundreds to thousands of features can be diffult to conceptualise and visualise. By using dimensionality reduction, we can better understand the model and present it to others by comprehensive visualisation.

The third reason is about efficiency in terms of computational time and storage. In

general, pattern recognition and machine learning algorithms computationally intense. Besides, storage capacity is limited in some engineering applications such as embedded systems. So, solving the problem only with the relevant features can alleviate these two problems. Consequently, the computational speed of the model can be increased by using dimensionality reduction.

Various dimensionality reduction have been developed in literature, the next section will present a handful of the ones used in the present work.

2.5.2 Feature selection: filters and wrappers

Filters and wrappers are two major categories of feature selection methods. The structure of both methods are illustrated by figure 2.1 and figure 2.2 , respectively. They both

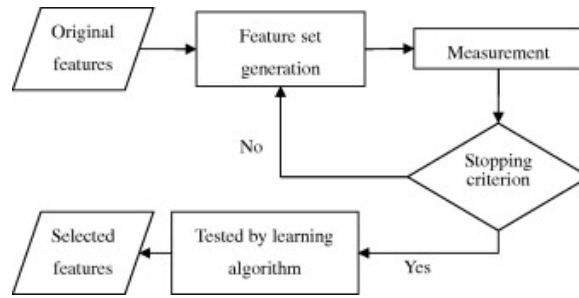


Figure 2.1: The procedure of filters in dimensionality reduction

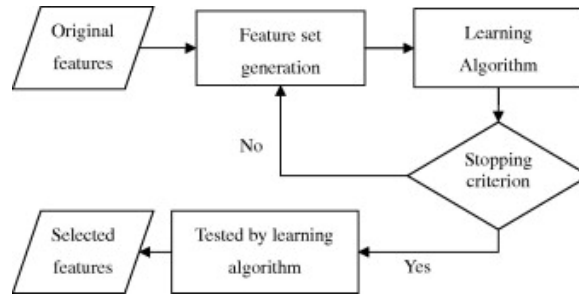


Figure 2.2: The procedure of wrappers in dimensionality reduction

require a mechanism for generating a subset of the original feature set and a stopping criterion, which can be a distance measure, a measure of similarity between the features. However, wrappers identify the best feature subset using a learning algorithm usually by fully searching the feature space, whereas, filters use a simple measurement metric based on mutual information, correlation and other distance criteria. As a result of the two different approaches, filters are fast and do not guarantee optimal classification accuracy. On the other hand, wrappers give accurate prediction results but are very slow. Both

approaches are often combined to build effective and efficient feature selection methods. One such approach is shown in figure 2.3. In this approach, the filters are used as a

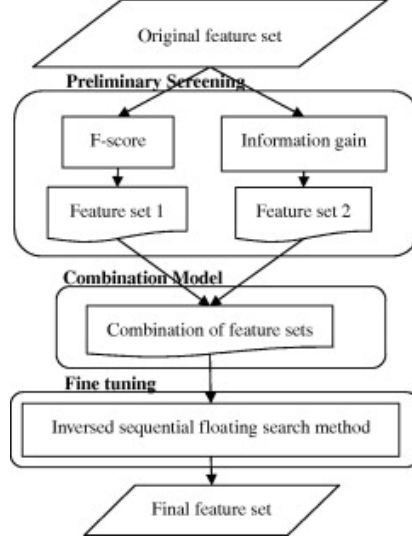


Figure 2.3: The procedure of hybrid filter-wrapper in dimensionality reduction

preliminary stage to discard irrelevant features. A feature is deemed irrelevant if it cannot discriminate between the different classes or if it contains redundant information. The output of the filter stage is a smaller set of relevant features which is fed into the wrapper to find the optimal final subset of features. Thus, the filter stage effectively reduces the search time of the wrapper. In classification problems, using the separability index matrix to determine the classification content or degree of the feature generally results in very good results. The next section gives a particular attention to this approach.

Feature ranking using separability index matrix

In this section, we dive into a systematic approach to determine the 'classifiability of a feature' as present in their paper, Jeong-Su, Sang Wan Lee and Zeungnam Bien, proposed a new criterion called separability index matrix to identify features that can discriminate between the different classes of a classification problem. The important concepts of this method are here defined together with their significance.

1. Separability Degree Matrix (SDM)

SDM_k is a $C \times C$ symmetric matrix of the separability measures between the different classes of classification problem of C distinct classes given a particular feature x_k . It is defined in 1

$$SDM_k = [J(w_i, w_j; x_k)]$$

a

where $J(w_i, w_j; x_k)$ denotes the separability or the distance value between class w_i and class w_j when the criterion function $J(\cdot)$ such as mahalanobis or euclidian, is applied to the feature x_k . SDM_k is symmetric matrix with zero diagonal values, because for a given feature x_k , $J(w_i, w_j; x_k) = J(w_j, w_i; x_k)$ and $J(w_i, w_i; x_k) = 0$. This observation can be exploited to half the computation required to calculate $SDM_v = SDM_1, SDM_2, \dots, SDM_N$, the set of all SDM for the feature set of size N .

2. Separability Index Matrix (SIM)

$SIM_k = c_{ij}$ is a $C \times C$ matrix of binary values 0, 1. if $c_{ij} = 0$, then the classes w_i and w_j are not separable by the feature x_k .

SIM_k is obtained by applying an threshold function to SDM_k . For instance,

$$\begin{aligned} SIM_k(i, j) &= 0 \quad \text{if} \quad SDM_k(i, j) < SDM_{avg}(i, j) \\ SIM_k(i, j) &= 1 \quad \text{if} \quad SDM_k(i, j) \geq SDM_{avg}(i, j) \end{aligned}$$

where SDM_{avg} is the element wise average of SDM_v . SIM_k is significant because it can be used to systematically determine the irrelevant features, i.e, features whose $SIM_k = 0$, and/or redundant ones.

3. Classifiability: $G(x_k)$

Although, conventional distance criterion reveal the separability of the feature distribution, we are often more interested in, how effectively can a particular feature distinguish one class from another. This information is denoted $G(x_k)$, the 'classifiability' of a the feature x_k '. $G(x_k)$ is computed by 2.6.

$$G(x_k) = \sum_{i=1}^C \sum_{j=1}^C (SIM_k * WM_k) \quad (2.6)$$

$$\text{with } WM_k = SIM_k / \sum_{i=1}^N SIM_i \quad (2.7)$$

where $*$ and $/$ denote respectively element wise matrix multiplication and division.

This method can be used to effectively and efficiently rank the features in a classification

for subset selection or as a preliminary step in a hybrid filter-wrapper feature selection solution.

2.5.3 Feature extraction

Principal component analysis: PCA

Linear discriminant analysis: LDA

2.5.4 Hybrid filter-wrapper methods

2.6 Sufficiency of Training Data

2.7 Techniques to increase Training Data

2.7.1 Mirroring

Chapter 3

Hidden Markov Model design

This section focuses on the design of the HMM used to test the hypotheses postulated above.

3.1 Description of available dataset

The available dataset was acquired from a moving dog using Inertia Measurement Units. Two inertial measurements units (IMU) were strapped to the front and back of a dog. Each unit has an accelerometer, a gyroscope and a magnetometer. The dataset contains calibrated measurements of a dog running, walking, and trotting then walking; together with the footfalls. The footfall is represented by a binary value that indicates the state of the dog's leg: if it is on or above the ground, at a particular instant in its gait sequence. More specifically, the value 0 means leg up and the value 1 means leg down. The four variables representing the footfalls effectively constitute the ground truth, informing us about the state in which the dog is, at a given time in its movement.

The dataset can be retrieved from nine different matlab files. Each file contains twenty four matlab variables. The variables of interest are listed in the table 3.1.

The observations are continuous and the statistical property are assumed to be stationary, i.e, they do not vary over time. In this sta

3.1. DESCRIPTION OF AVAILABLE DATASET

Observations			
Body part	Accelerometer	Gyroscope	Magnetometer
Front	accFrontX	FrontPitch	magFront_cal
	accFrontY	FrontRoll	magFront_cal2
	accFrontZ	FrontYaw	magFront_cal3
Back	accBackX	BackPitch	magBack_cal
	accBackY	BackRoll	magBack_cal2
	accBackZ	BackYaw	magBack_cal3

Table 3.1: IMU measurements and footfall variables in dataset

stationary: statistical property do not vary over time or **non-stationary:** properties vary over time

pure or corrupted?

3.1.1 Quadrupede Gait sequence modelling

One of the objectives of this project is to effectively model the gait sequence dynamic of the dog from IMU measurements using HMM. Based on the fact that quadrupedes achieve inverted pendulum-like movements like humans, their gait dynamic can be modelled as a succession of latent states observed through the 'visible' IMU measurements. The states representing the footfalls and the observations, the outputs of the accelerometer, gyroscope and the magnetometer. Similar to human gait mechanism, it is sound to assume that the current state of a quadrupede is conditionally dependent on its previous state. This inference combined with the statistical robustness of HMM makes it the best model candidate when the available dataset is not too large.

HMM model elements: states and observations properties

The problem at hand requires 16 distinct states that make up the state vector S , shown in equation 3.1

$$S = S_i = \{(LF, RF, LB, RB)\} = \{0000, 0001, 0010, \dots, 1111\}. \quad (3.1)$$

$$|S| = N = 2^4 = 16$$

$$i = 1, 2, \dots, 16$$

3.1. DESCRIPTION OF AVAILABLE DATASET

The 16 distinct states are derived from the combination of the four binary footfalls. In practice, the dataset may not reveal all the 16 states.

The stream of IMU measurement form the observation sequence. An observation instance is a row vector of K dimensions. The initial K value before any dimensionality reduction is 18, from the 18 IMU measurements. Thus, an observation sequence O is a TxK matrix of continuous-valued voltages as presented in 3.2. T is the total number of the successive measurements.

$$O_t = \{O_t^k\} = O_t^1, O_t^2, \dots, O_t^{18}. \quad (3.2)$$

$$k = 1, 2, \dots, 18. \quad (3.3)$$

$$t = 1, 2, \dots, T. \quad (3.4)$$

Splitting the 16-states HMM in two 4-states HHMs

In order to simplify the problem, it was decided to split the four legs in two sub-parts: two front legs and two back legs. This decision exploits the fact that *"fore and hind quarters of dogs behaved like two independent bipeds"* as demonstrated in.

As a result, the initial 16-states HMM becomes two distinct 4-states HHMs. These two models may be combined to reconstruct the holistic 16-states model. With the two separate HHMs, we are faced a simpler task of dsicriminating between 4 distinct states instead of 16. A further benefit of this decision is that this work may well be applied to human gait estimation using IMU measurements. From here onward, we will focus on the 4-states HMM model.

Transition between states

It was assumed that in its movement, a dog may transition from one state S_i to any other state S_j where $i, j = 1, 2, 3, 4$. For instance, if a dog has its left leg above ground and its right leg on ground, at time instance t, it may move to any of the 3 other possible positions or remain in the same state, in the next time instance, $t + 1$. Thus, we are dealing with an ergodic type HMM, where all the states transitions are allowed. The graphical model of the simplified HMM is illustrated by figure 3.1

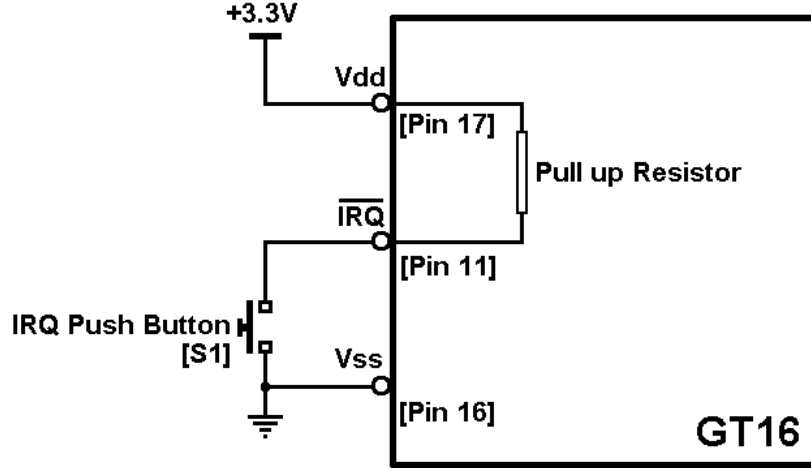


Figure 3.1: Ergodic HMM graphical model showing the hidden states, observation, and transitions between states

3.1.2 Pre-processing and increasing the dataset

Very little data pre-processing was required given that a relatively clean data was already available in Matlab files. So, this stage of the design consisted in three simple steps.

1. **Extracting and labelling the feature vector:** In this step, the TxK observation matrix was formed using the 18 different variables listed in 3.1, for a given gait sequence. Using, the footfalls as ground-truth, i.e, (LF, RF) and (LB, RB), each observation instance was correctly labelled. The labelling is useful to stratify the different into the four states whenever required in the training process.
2. **Aggregating the gait sequences:** The individual gait sequence samples - for a walking, running and trotting dog - are very small in size, it was therefore decided to aggregate them in a single longer sequence. As such, the observation sequence becomes that of a dog walking, trot, then running or a similar combination of the three actions. The impact of such a decision on the model's parameter is that, instead of fitting a single action, it fits all the three types of actions. In theory, this loss of specificity can negatively affects the model's precision. Netherveless, the design a approach can be applied to a specific action if there is enough training data.
3. **Increasing the data by mirroring:** The aggregated observation sequence amounts to 2695 different samples. In order, to further increase the dataset, the mirrored sequence was appended to the initial sequence. This decision works on the assumption that, given a gait sequence, the reverse sequence is also a valid sequence from

the subject matter. Practically speaking, each mirrored observation sample is a duplicate, no new sample is added. Moreover, the state distribution remains unchanged. However, as demonstrated in the results section 4.2, this increase in data size caused an increase in the model's performance.

As a summary, the output of the pre-processing stage is an 18x5390 matrix where each row represents an observation sample and each column a particular feature. This dataset was fed directly into the HMM model or in the dimensionality reduction module to remove irrelevant or redundant features. The design of the dimensionality reduction stage is now explained.

3.1.3 Dimension reduction

3.1.4 Model parameters estimation

As a reminder, a continuous HMM model is completely specified by its initial state distribution: π transition matrix: A ; the mean covariance matrices: μ, Σ which can be combined into Φ . If the observations are modelled with gaussian mixture distributions, one addition parameter is required for the initial mixture distribution: β . The next sub-sections discuss how each parameters was estimated in this project.

Transition matrix: A

For each of the front and back 4-state HMM, the state transition matrix A , is a 4-by-4 matrix. Two different approaches were considered in the estimation of A . The two methods make use of the expectation maximisation algorithm but differ in the input arguments considered.

1. Approach 1: Exploiting the available ground truth

This approach takes advantage of the ground truth for a labelled dataset to reduce the HMM model to a Markov Chain. This is done by making the hypothetical observation sequence identical to the state sequence. Then, using approach used in discrete HMM, the transition matrix can be estimated using maximum likelihood algorithm. For each state, the pseudocount was set to the number of occurrences of that state in the training data plus a constant value: $PseudoA = |S_i| + C$. The

3.1. DESCRIPTION OF AVAILABLE DATASET

additional constant C is to avoid having 0 in the transition matrix for states and transitions not reflected in the dataset. It can be chosen empirically. This method is very simple however, it has two limitations. It not only assumes that the number of states is known but also requires the training data to be labelled. Although, the dataset at hand satisfies the two constraints, the second approach which eliminates these possible setbacks, was also considered.

2. Approach 2: This method is the standard approach found in literature using expectation algorithm such as Welch-Baum algorithm. and described in the literature review.

Mean and covariance matrices: μ , Σ and β

The observations were modelled using mixture Gaussian distributions, characterised by a $K \times M \times N$ mean matrix: μ and a $K \times K \times M \times N$ co-variance matrix: Σ . where:

$K = \text{number of features}$, $M = \text{mixture number}$, and $N = \text{number of states}$.

The observations were grouped K classes following based on the ground-truth represented by the state sequence. The each group of observation was used as input to the EM algorithm to estimate the μ and Σ . In general, the optimal number of mixture, M is chosen empirically, in this project, it was estimated using akaike information criterion (AIC). So, gaussian mixture models were built using EM algorithm while varying the M from 1 to K , the feature number. Since AIC is a measure of information loss, the model with the minimum AIC best represents the dataset. The number of mixture M , is therefore set to the mixture number of this model. This algorithm is outlined below.

```
1 function M = best_M(data, K)
2     data = training_set
3     AIC = zeros(1, K);
4     models = cell(1, K);
5     for m = 1:K
6         model = gauss_mixture(data, m);
7         AIC(m) = model.AIC;
8     end
9     [minAIC, minAIC_Idx] = min(AIC);
10    M = minAIC_Idx;
11 end
```

The mixture components were considered evenly distributed initially. In order to avoid zero values in the covariance matrix, it was regularized with 10^{-10} . The maximum

3.1. DESCRIPTION OF AVAILABLE DATASET

number of iteration of the EM algorithm was empirically set to 1000.

Initial state distribution: π

The initial state distribution was estimated using the probability of occurrence of each state in the training data sample. Thus, $\pi = \{\pi_i\} = \frac{|S_i|}{T}$

Optimal number of features

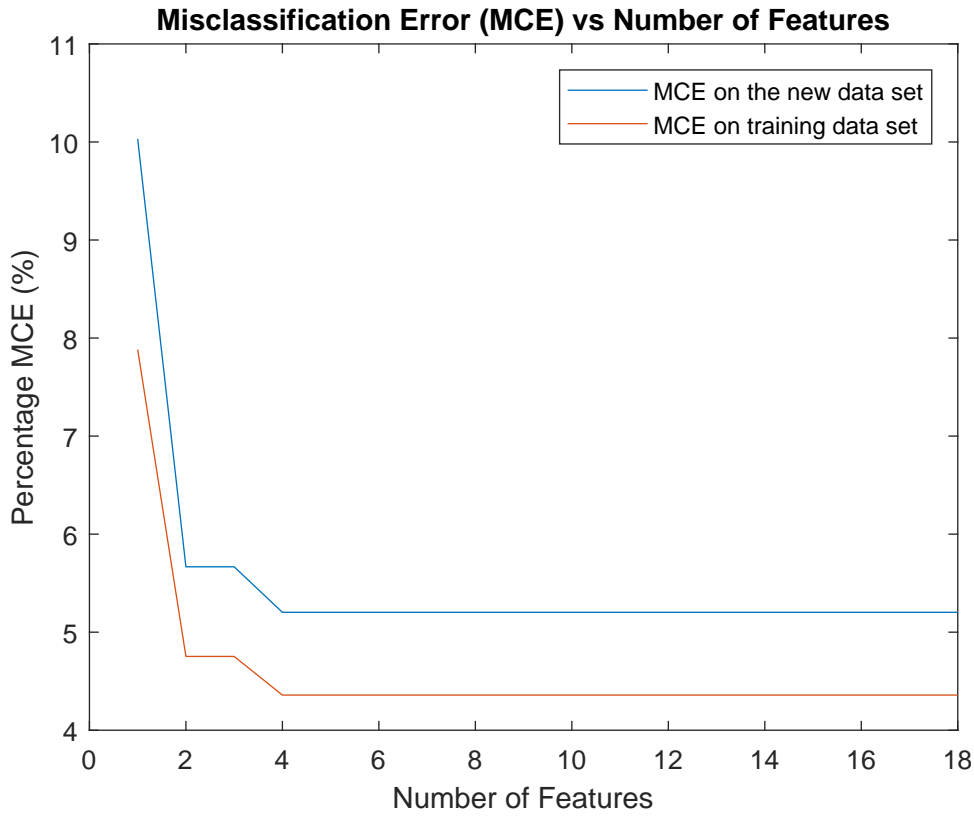


Figure 3.2: Misclassification error vs feature number using separability index and KNN

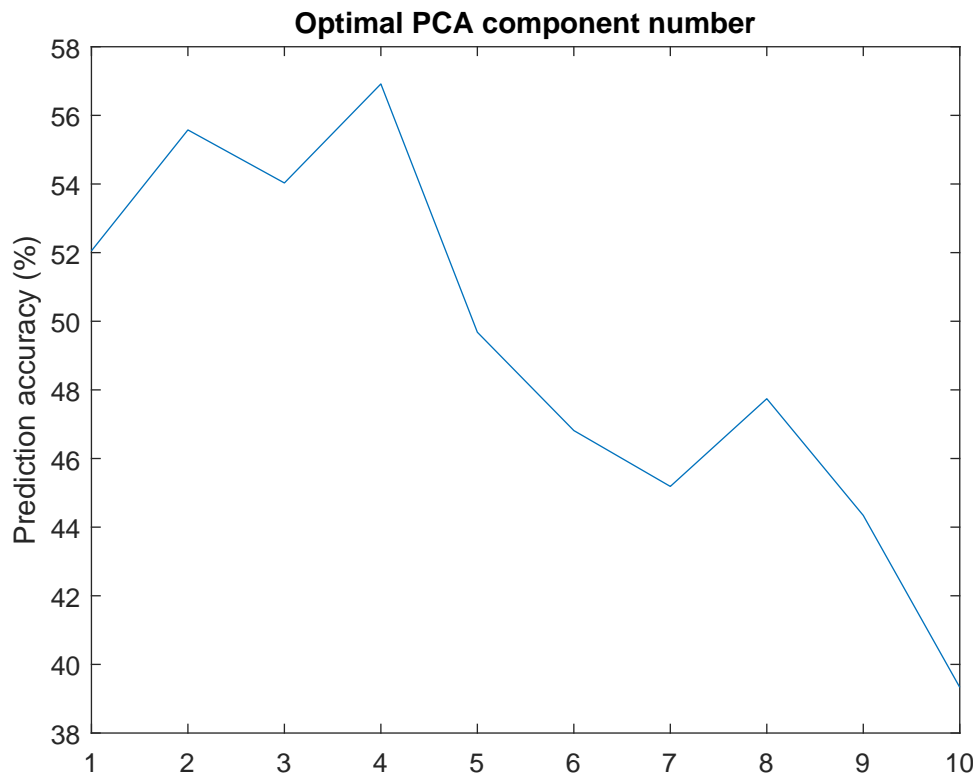


Figure 3.3: Optimal PCA component number

Optimal PCA component number

3.2 HMM implementation

3.2.1 Proving learning ability by increasing loglik for many iteration: check one of papers

Chapter 4

Results

4.1 Experiment 1: The effect of a CHMM' observation dimensionality on its performance

4.1.1 Aim of the experiment

The aim of this experiment is to investigate how the number of features impacts the accuracy of a Hidden Markov Model with continuous emission symbols (CHMM), in the absence of enough training data. Thus, the hypothesis under investigation is:

In the absence of enough training data, a CHMM with observations of high dimensionality performs poorly.

4.1.2 Experiment apparatus

To perform this experiment, the following materials are required:

- λ , a continuous Hidden Markov Model specified by $\lambda = (A, \beta_{jm}, \mu_{jm}, \Sigma_{jm}, \pi)$.
- At least two sample data sets training the model and testing it.
- A criterion to rank and select subsets of features.
- A measure to evaluate the performance of the CHMM model.

4.1. EXPERIMENT 1: THE EFFECT OF A CHMM' OBSERVATION DIMENSIONALITY ON ITS PERFORMANCE

- Finally, a way to visualise the results of the experiments

4.1.3 Experiment procedure

The experiment was performed with the steps listed below:

1. Step 0 - Preliminary data pre-processing: This step consisted in the data pre-processing as described in
2. Step 1 - Partitioning data into training and test sets: Here, the dataset was randomly sampled into training and test sets. The training set was relatively small, it was a sequence 539 observations.
3. Step 2 - Feature ranking: The features were sorted in a descending order based on their ability to discriminate the different states of the CHMM. The separability index method described in was used for this purpose.
4. Step 3 - Data subset selection: Select the optimal feature subset, starting with 1 dimension.
5. Step 4 - Model building and training: The CHMM model, λ was built and trained using with training dataset using the optimal feature subset.
6. Step 5 - Model testing and evaluation: The model was tested with the test dataset. The test consisted in decoding the most likely state sequence given a previously unseen sequence of observations. This path prediction was evaluated based on the evaluation criterion presented in
7. Step 6 - Iteration: Step 3 through step 5 were repeated while varying the feature subset size until the maximum size, which is 18 in this case. In each iteration, the prediction accuracy was stored in an array for visualisation.
8. The different accuracies were finally plotted as a function of the observation dimensionality. Moreover, the observations were grouped based on the corresponding hidden state sequence and scattered in a 2-dimensional principal component space. This is to compare the decoded states against the ground-truth.

4.1.4 Experiment results

The results of the experiments are presented in figure 4.1, 4.2, 4.3, 4.4, 4.5, 4.6.

4.1 and 4.2 show how the hidden state decoding performance and the log-likelihood estimated by the CHMM model, train with just 539 observations, varies as the observation dimensionality increases.

4.3 and 4.4, 4.5 and 4.6 illustrate how the estimated state sequences compare to reality for observation sequences of 5-dimensions and 18-dimensions. 5 and 18 dimensions were presented because they are the two extremes in terms of accuracy. Other dimensions may be found in the appendices,

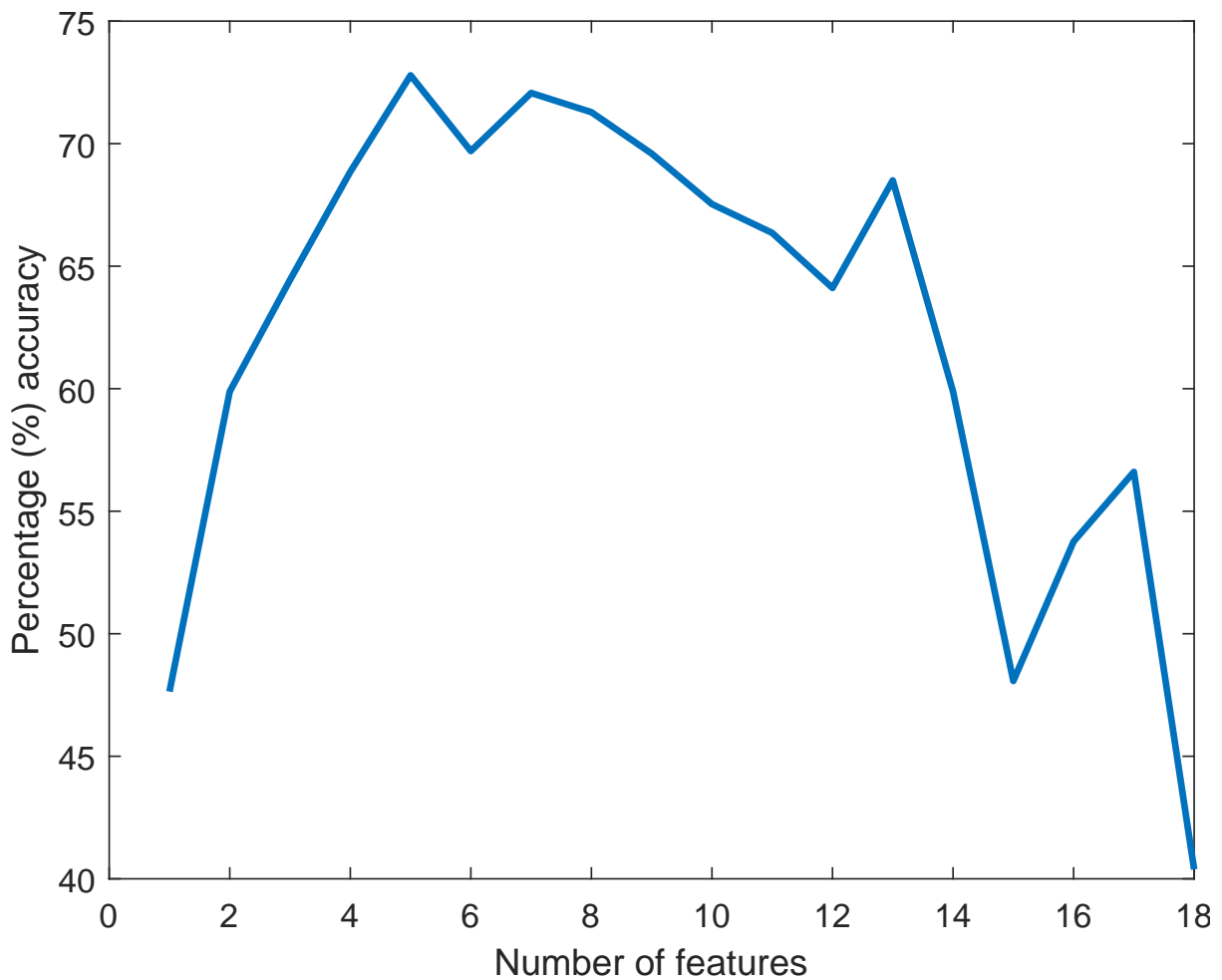


Figure 4.1: The effect of CHMM's observation dimensionality the state sequence decoding accuracy

4.1. EXPERIMENT 1: THE EFFECT OF A CHMM' OBSERVATION DIMENSIONALITY ON ITS PERFORMANCE

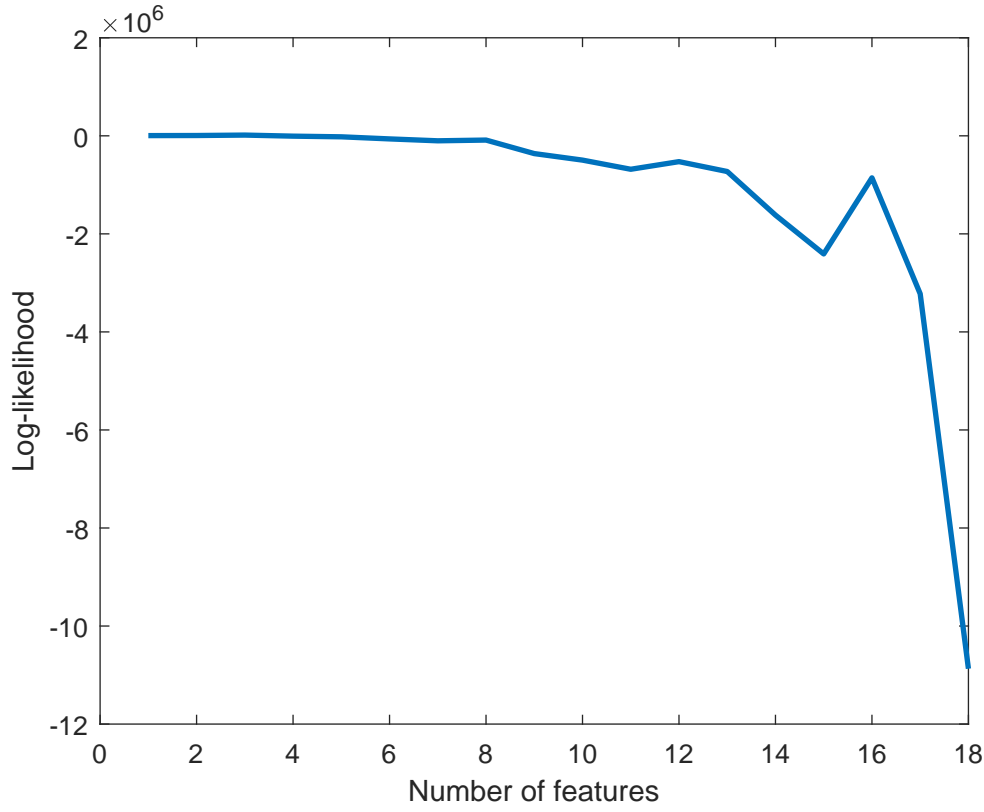


Figure 4.2: The effect of CHMM's observation dimensionality the log-likelihood

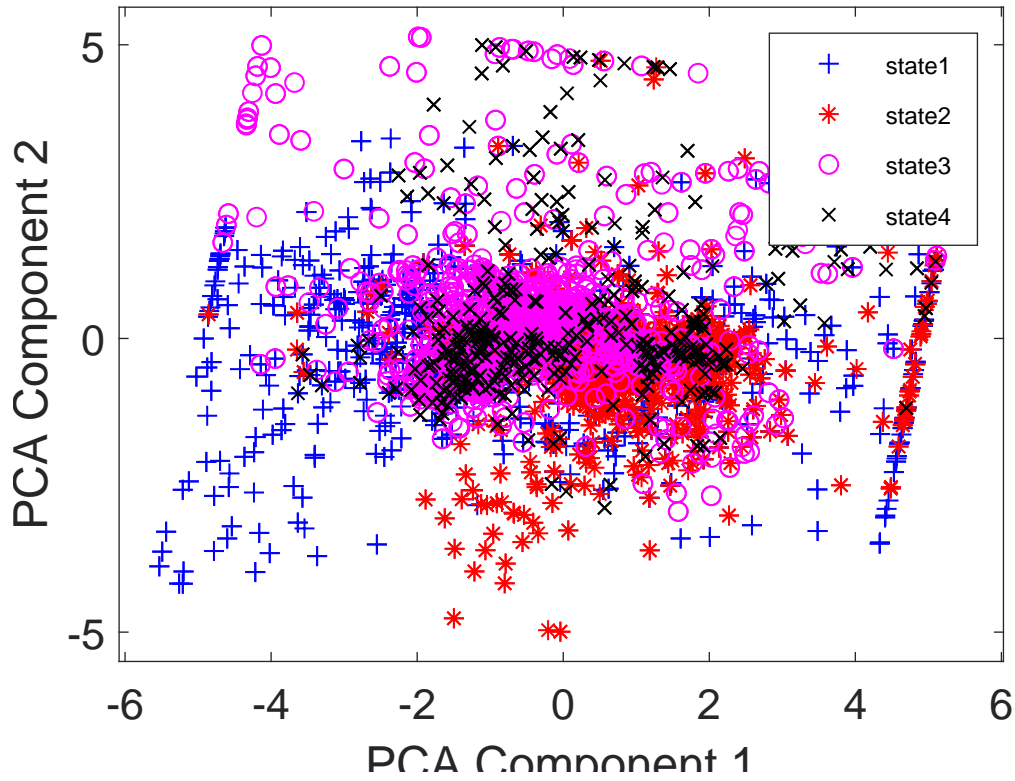


Figure 4.3: Scatter plot of 5-dimensional observations grouped per state based on ground-truth state sequence

4.1. EXPERIMENT 1: THE EFFECT OF A CHMM' OBSERVATION DIMENSIONALITY ON ITS PERFORMANCE

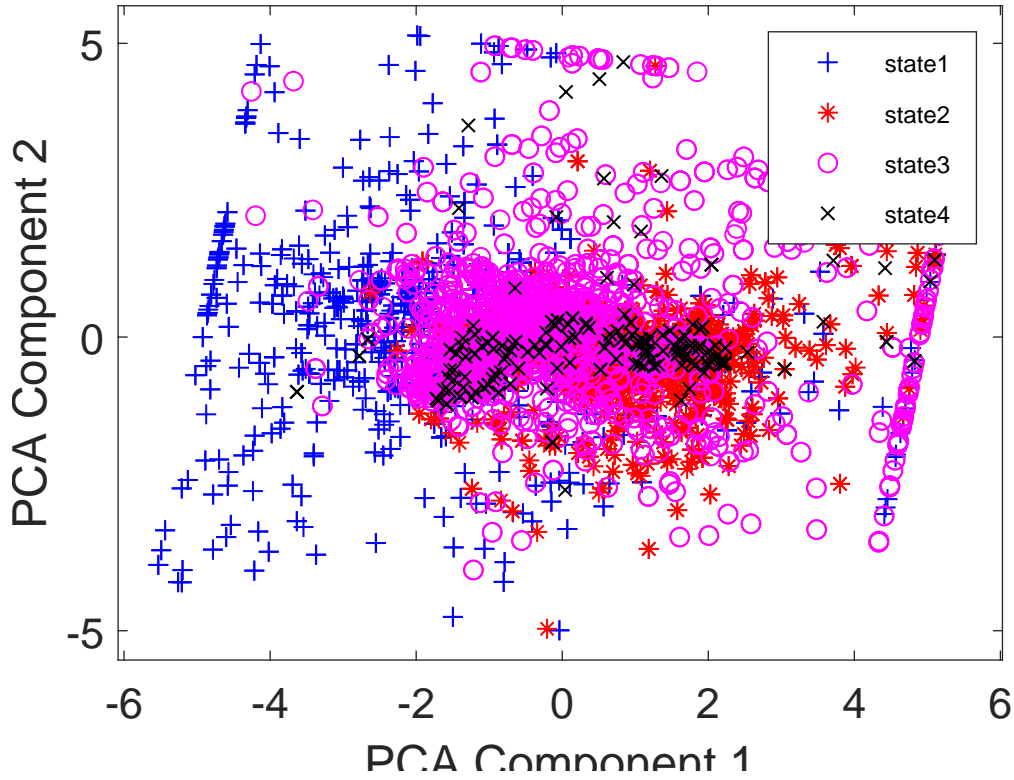


Figure 4.4: Scatter plot of 5-dimensional observations grouped per state based on estimated state sequence

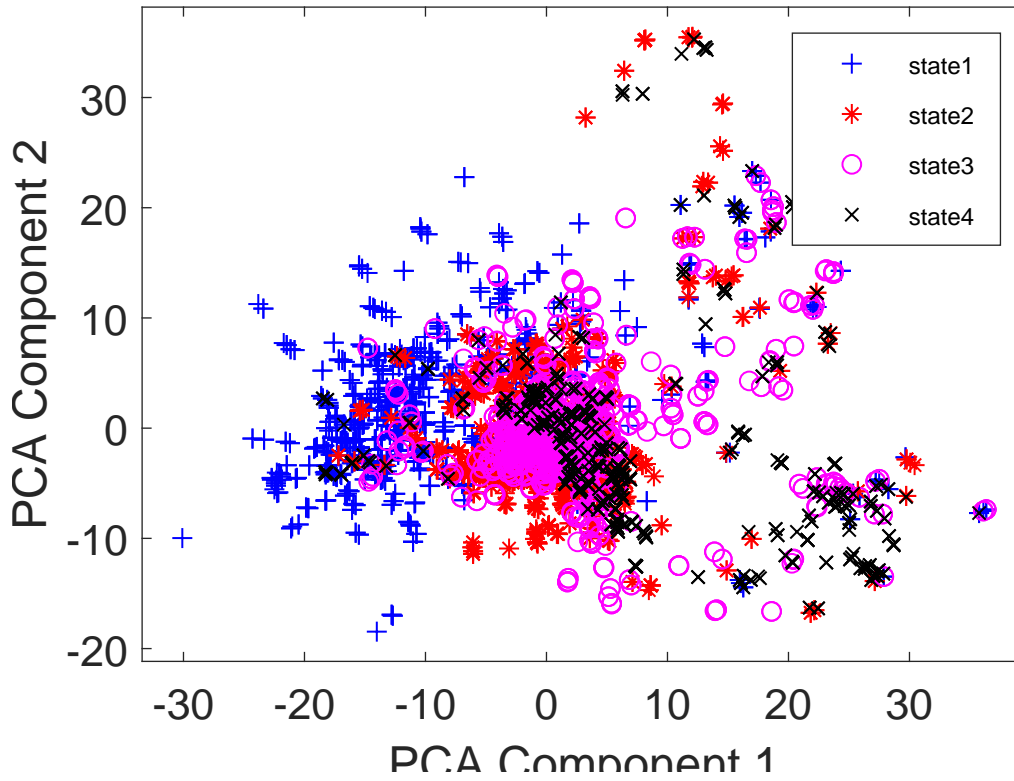


Figure 4.5: Scatter plot of 18-dimensional observations grouped per state based on ground-truth state sequence

4.2. EXPERIMENT 2: THE EFFECT OF DIMENSIONALITY REDUCTION ON CHMM'S PERFORMANCE

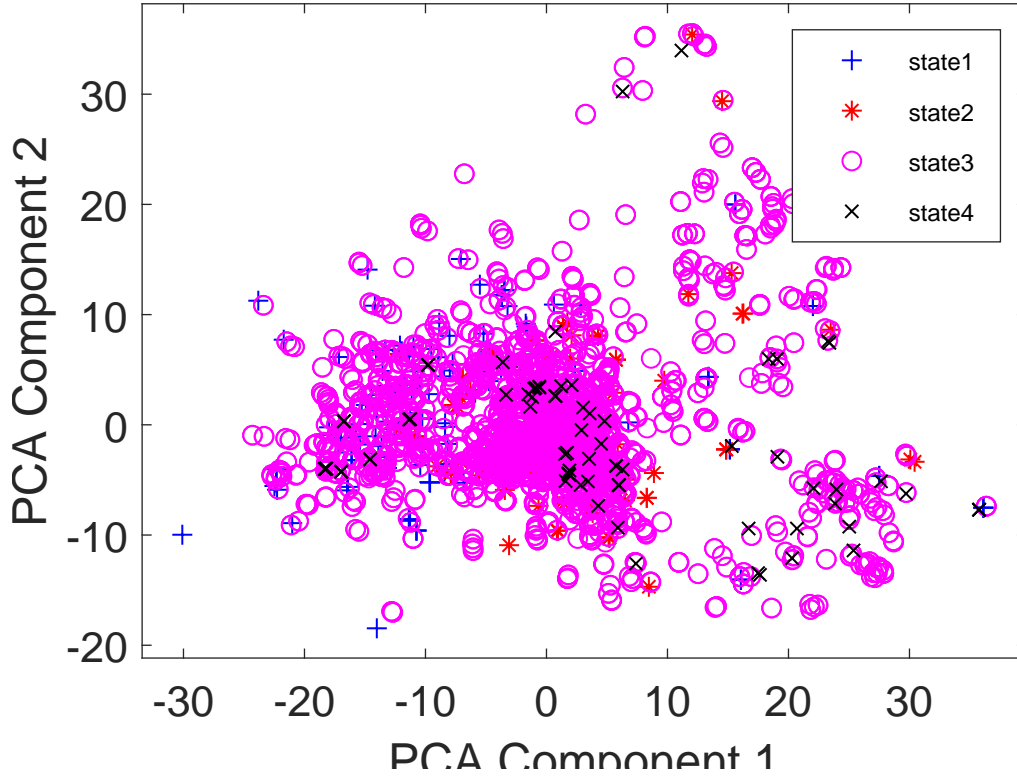


Figure 4.6: Scatter plot of 18-dimensional observations grouped per state based on estimated state sequence

4.1.5 Analysis of results

4.1.6 Conclusions and recommendations of the experiment

4.2 Experiment 2: The effect of dimensionality reduction on CHMM's performance

4.2.1 Aim of the experiment

The aim of this experiment is to investigate the effect of dimensionality reduction on the performance of a continuous Hidden Markov Model (CHMM). The hypothesis under investigation is therefore the following: *Dimension reduction can cause an increase in a CHMM model performance when there is not enough training data.* Thus, the performance of the CHMM without and with various dimensionality reduction are compared to test the hypothesis.

4.2. EXPERIMENT 2: THE EFFECT OF DIMENSIONALITY REDUCTION ON CHMM'S PERFORMANCE

4.2.2 Experiment apparatus

The assets needed to perform the experiment are listed below.

- λ , a continuous Hidden Markov Model specified by $\lambda = (A, \beta_{jm}, \mu_{jm}, \Sigma_{jm}, \pi)$.
- Dimensionality reduction methods. Two wrapper and two filter methods were considered. The wrapper methods were Principle Component Analysis (PCA), Linear Discriminant Analysis (LDA). The two filters methods were feature ranking with similarity index and a combination of forward feature selection and similarity index.
- A Performance metric. The metric used were the hidden state decoding accuracy and the log-likelihood of the EM algorithm.

4.2.3 Experiment procedure

The experiment was performed as follows. First, the dataset was partition into two different set for training and testing using random sampling. Using the same training dataset five different models were built and trained, $\Lambda = (\lambda_{NoReduction}, \lambda_{PCA}, \lambda_{LDA}, \lambda_{SI}, \lambda_{SI-forward})$. Then the different models were tested with the same test dataset and the prediction accuracy as well as the EM algorithm loglikelihood were recorded. The training and the testing were repeated while varying the proportion of training set used from 10% to 90% of the total dataset. The prediction accuracies and the EM algorithm likelihoods were finally plotted as a function of the training data size for each model. These findings are presented in the figures 4.7 and 4.8.

4.2.4 Experiment results

Firstly, figure 4.7 how the performances of the five CHMMs compare against each other as the training data size increases. Secondly, the loglikelihoods presented in 4.8 show how effectively can each model recognise an observation sequence generated by the underlying mechanism.

4.2. EXPERIMENT 2: THE EFFECT OF DIMENSIONALITY REDUCTION ON CHMM'S PERFORMANCE

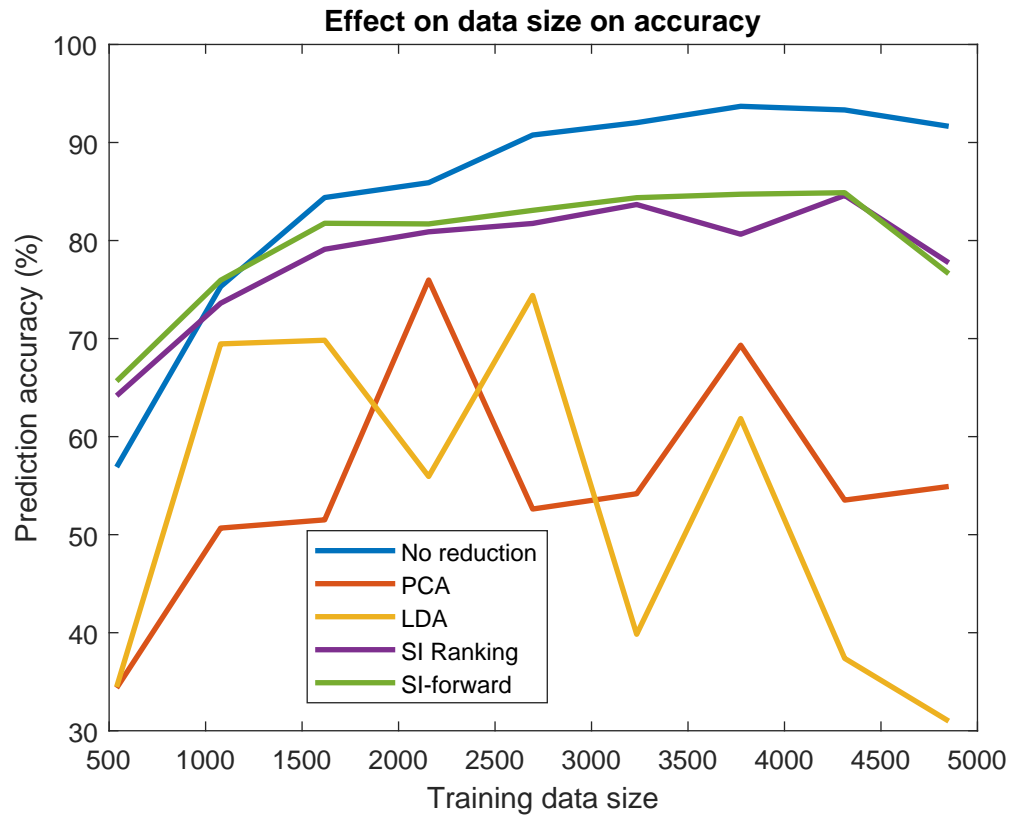


Figure 4.7: The effect of training datasize on the prediction accuracy

4.2.5 Analysis of results

4.2.6 Conclusions and recommendations of the experiment

4.2. EXPERIMENT 2: THE EFFECT OF DIMENSIONALITY REDUCTION ON CHMM'S PERFORMANCE

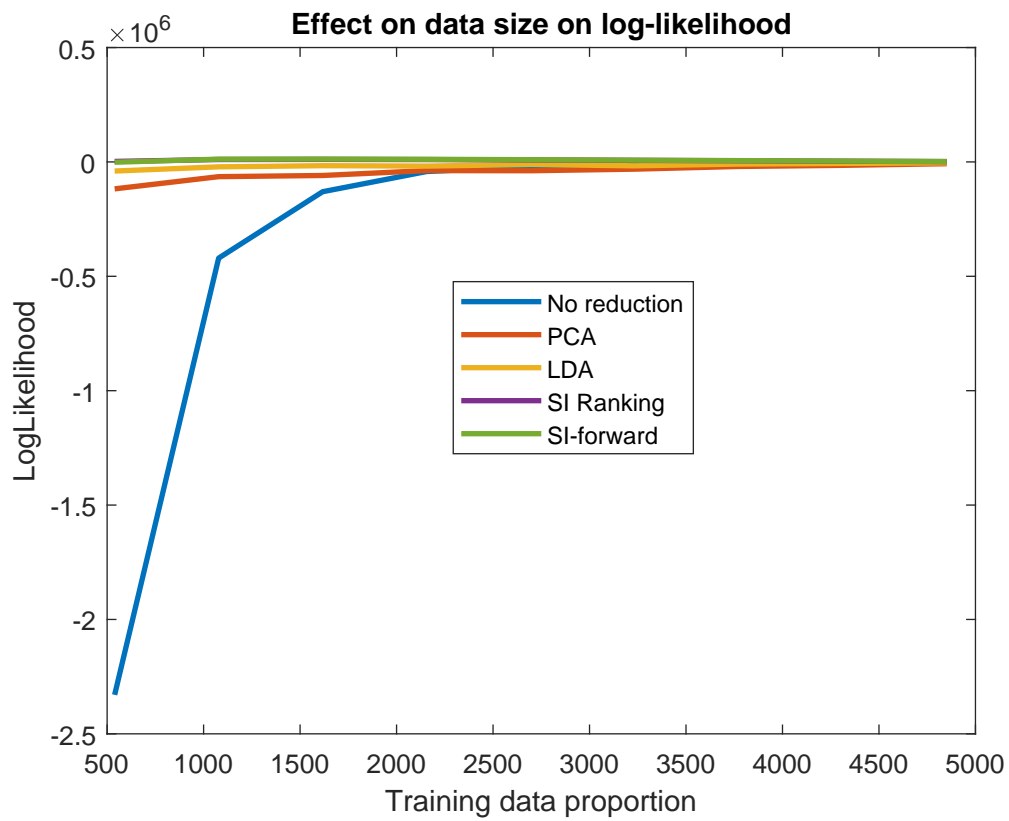


Figure 4.8: The effect of training datasize on the log likelihood

4.2. EXPERIMENT 2: THE EFFECT OF DIMENSIONALITY REDUCTION ON CHMM'S PERFORMANCE

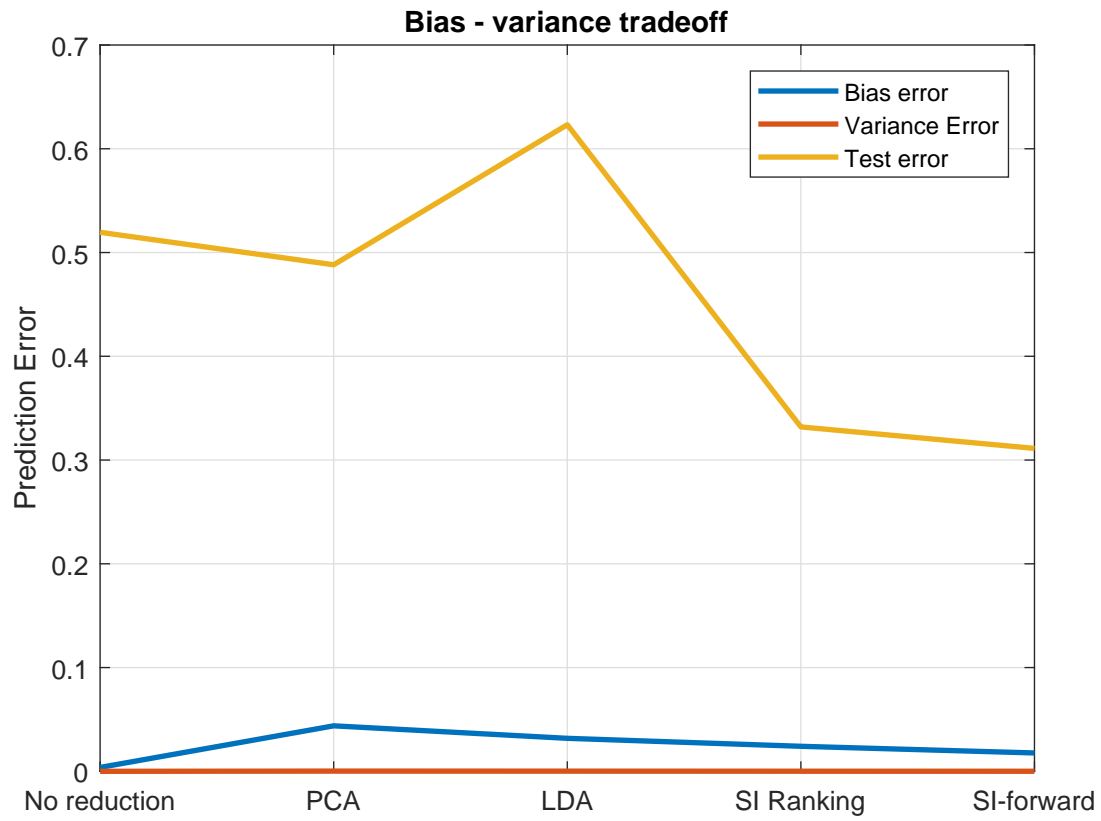


Figure 4.9: Bias-Variance tradeoff analysis

Chapter 5

Discussion

Here is what the results mean and how they tie to existing literature...

Discuss the relevance of your results and how they fit into the theoretical work you described in your literature review.

Chapter 6

Conclusions

These are the conclusions from the investigation and how the investigation changes things in this field or contributes to current knowledge...

Draw suitable and intelligent conclusions from your results and subsequent discussion.

Chapter 7

Recommendations

Make sensible recommendations for further work.

Use the IEEE numbered reference style for referencing your work as shown in your thesis guidelines. Please remember that the majority of your referenced work should be from journal articles, technical reports and books not online sources such as Wikipedia.

Bibliography

- [1] M. S. Tsoeu and M. Braae, “Control Systems,” *IEEE*, **vol. 34(3)**, pp. 123-129, 2011.
- [2] J. C. Tapson, *Instrumentation*, UCT Press, Cape Town, 2010.

Appendix A

Additional Files and Schematics

Add any information here that you would like to have in your project but is not necessary in the main text. Remember to refer to it in the main text. Separate your appendices based on what they are for example. Equation derivations in Appendix A and code in Appendix B etc.

Appendix B

Addenda

B.1 Ethics Forms