# Gait sequence modelling and estimation

## using Hidden Markov Models

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# Declaration

1. I know that plagiarism is wrong. Plagiarism is to use another's work and pretend that it is one's own.
2. I have used the IEEE convention for citation and referencing. Each contribution to, and quotation in, this report from the work(s) of other people has been attributed, and has been cited and referenced.
3. This report is my own work.
4. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as their own work or part thereof.

Signature:...........................
Kouame H. Kouassi

Date:...............................

# Acknowledgments

Abstract

- Open the **Project Report Template.tex** file and carefully follow the comments (starting with %).
- Process the file with **pdflatex**, using other processors may need you to change some features such as graphics types.
- Note the files included in the **Project Report Template.tex** (with the .tex extension excluded). You can open these files separately and modify their contents or create new ones.
- Contact the latex namual for more features in your document such as equations, subfigures, footnotes, subscripts & superscripts, special characters etc.
- I recommend using the **kile** latex IDE, as it is simple to use.

# Contents

# Chapter 1

# Introduction

## 1.1 Background to the study

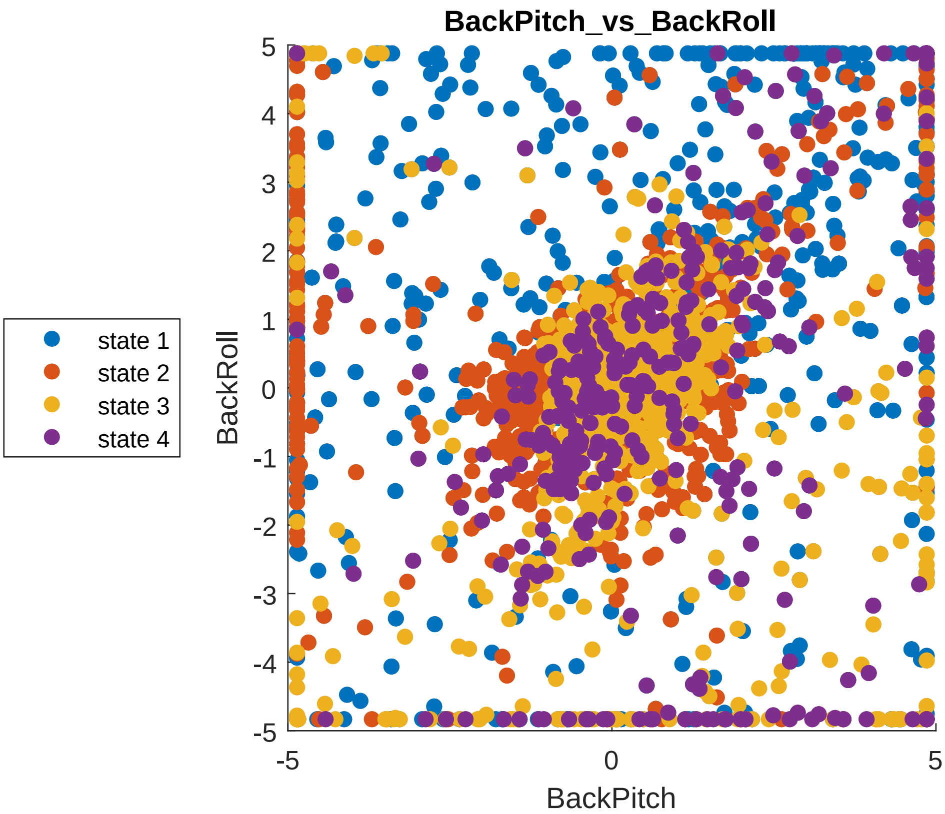
A very brief background to your area of research. Start off with a general introduction to the area and then narrow it down to your focus area. Used to set the scene [1].

Bio-inspired robotics uses nature to inform real-world engineering systems. Research has been conducted at UCT to investigate the manner in which a cheetah uses its tail for stability during high acceleration, quick turns and sudden braking, with an aim to incorporating identified mechanisms into sophisticated robot designs. One way to acquire useful data is to strap an inertial measurement unit (IMU) to an animal, and log the sensor data while certain actions are being performed. We currently have such a dataset of a dog moving, along with corresponding video data.

## 1.2 Objectives of this study

The objective of this project is to design, implement, and test Hidden Markov Models (HMM) for estimating gait sequence from Inertia Measurement Unit (IMU) data.

so that specific models can be formulated and their parameters estimated and interrogated. The project can be extended to include any other useful analysis of gait patterns from similar sensor measurements




1 - formulate model 2 - estimate its parameters 3 - Interrogate its parameters 4 - Useful analysis of gait patterns from IMU measurements

### 1.2.1 Problems to be investigated

Description of the main questions to be investigated in this study.

The main questions to be answered are the following:

1. How well can HMM model gait sequence dynamics using IMU data, in the abscence of enough training samples?
2. Can dimensionality reduction cause an increase in performance of HMM models when there is not enough training data?

### 1.2.2 Purpose of the study

Give the significance of investigating these problems. It must be obvious why you are doing this study and why it is relevant.

## 1.3 Scope and Limitations

Scope indicates to the reader what has and has not been included in the study. Limitations tell the reader what factors influenced the study such as sample size, time etc. It is not a section for excuses as to why your project may or may not have worked.

1 - Does not include data collection 2 - Focus on design of HMM only 3 - Focus on analysis of the model 4 - Focus on impact of dimensionality reduction

## 1.4 Plan of development

Here you tell the reader how your report has been organised and what is included in each chapter.

**I recommend that you write this section last. You can then tailor it to your report.**

# Chapter 2

# Literature Review

Once upon a time engineers and researchers believed... In this area of research, they used the following methods... [2]

Write this section first as it will take you the longest. I suggest you start writing this as soon as you have done your initial research at the beginning of your project. You can then return to it once you have completed your work to edit and adjust it.

A literature review forms the theoretical basis of your project. You need to read a large number of journal papers, sections in books, technical reports etc. relevant to your work at the start of project. This will give you a good idea of the field of research.

When writing your review start of with the general concepts and move to the more specific aspects explaining the necessary theory as you go. This section is NOT a copy and paste from others work or a rewrite-but-change-one-word section. I suggest you read all your material, and then put it down and write this section, referring back to the work only when you need to check something.

See your PCS textbook for more details on how to write a literature review.

If you include a figure or a table in your text please see the example in Fig. 3.1 as to how to caption it. Please make sure that all text in your figures is readable and that you reference your figures if they are from another source.

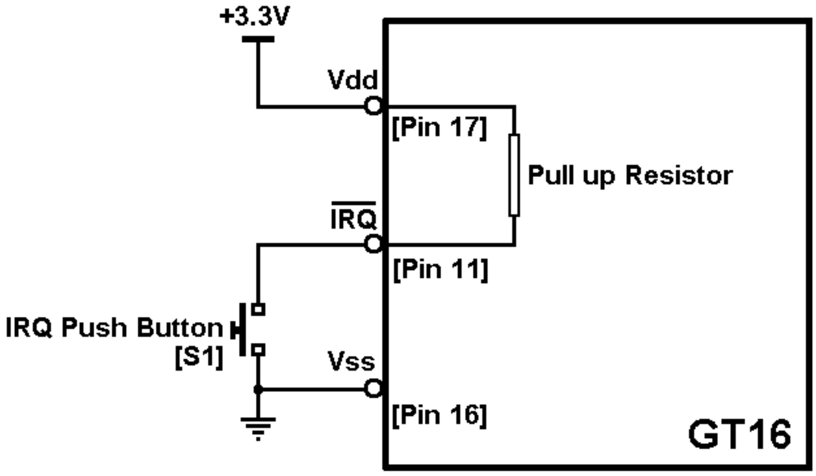



Figure 2.1: A block diagram illustrating the connections to the IRQ pin on the MCS08GT16A microcontroller (Please note that your headings should be short descriptions of what is in the diagram not simply the figure title)

## 2.1 Gait sequence modelling and estimation

### 2.1.1 Quadrupede gait modelling

**Periodicity**

### 2.1.2 Quadrupede gait estimation

## 2.2 Case study: Inertia Measurement Unit

## 2.3 Hidden Markov Models

Hidden Markov Models (HMMs) are doubly embedded stochastic processes with a rich underlying statistical structure. Introduced at the end of the 1960s by Baum and colleagues, they have become one of the prefered techniques in speech recognition after the implementation of Baker and Jelinek in 1970s. HMMs have been successfully applied to various other engineering problems in pattern recognition for classification and fraud detection purposes, amongst others.

### 2.3.1 HMM parameters specification

An HMM is fully specified the following parameters

1. N, the number of distinct states of the model. Together they form the set of individual states $S = \{S_1, S_2, ..., S_N\}$.
2. T, the number of observations. A sample observation sequence is denoted as $O = \{O_1, O_2, ..., O_T\}$.
3. $Q = q_t$, the set of states with $q_t$ denoting the current state at time instance, t such that $q_t \epsilon S$ and $t = 1, 2, ..., T$.
4. K, the number of distinct observation symbols per state.
5. $V = \{v_1, v_2, ..., v_K\}$, the feature set of K dimensions.
6. $A = \{a_{ij}\}$, the state transition probabilities. It $a_{ij}$ denotes probability of transitioning from state $S_i$ to state $S_j$.
7. $\Phi = \{\phi_j(k\}$, the probability distribution of observation symbols in state j.
8. The initial state distribution, $\pi = \pi_i$

For continuous HMM (CHMM), i.e, HMM with continuous-valued observations, $\Phi$ consists in a probability distribution function. Many applications have continous distributions with mixtures of Gaussian distribution. As such, $\phi$ is approximated by a weighted sum of M multivariate Gaussian distributions $\eta$,

$$\phi(O_t) = \sum_{m=1}^{M} \beta_{jm}\eta(\mu_{jm}, \Sigma_{jm}, O_t), \tag{2.1}$$

$$\eta(\mu, \Sigma, O) = \frac{1}{\sqrt{(2\pi)^K|\Sigma|}} exp(-\frac{1}{2}(O-\mu)'\Sigma^{-1}(O-\mu) \tag{2.2}$$

$$1 \leq j \leq N; 1 \leq m \leq M; \beta_{jm} \geq 0; \sum_{m=1}^{M} \beta_{jm} = 1$$

where $\beta_{jm}$ is the mixture composition coefficient; $\mu_{jm}$, $\Sigma_{jm}$, respectively the mean vector and covariance matrix of state j; M is the number of mixture components and K is the dimensionality of O.

Thus, the compact specification of a continous valued observation HMM defined in 2.3 and that of a discrete HMM in 2.4.

$$\alpha = (A, \beta_{jm}, \mu_{jm}, \Sigma_{jm}, \pi) \tag{2.3}$$

$$\alpha = (A, b_j(k), \pi) \tag{2.4}$$

**Basic assumptions of HMMs theory**

HMM theory is built on three basic assumptions listed below. They not only, make it a relatively simple graphic modelling tool but, also simplify its implementation. This naturally comes with some limitations in modelling more complex problems, which however, may be modelled with higher order HMMs.

1. *The Markov assumption*: HMM assumes that the probability of being in the current at any instance of time t, is uniquely dependent on the previous state, at time, t + 1. More specifically, $a_{ij} = P[q_t = S_j | q_{t+1} = S_i]$. This assumption makes it unsuitable for long-range correlation capturing applications.
2. *The stationary assumption*: Furthermore, HMM state transition probabilities are assumed to be time-independent. Thus, the transition probabilities of two distinct time, $t_1$ and $t_2$ are identical, $P[q_{t_1} = S_j | q_{t_1 - 1} = S_i] = P[qt_2 = S_i | q_{t_2 - 1} = S_i]$. HMMs can therefore effectively model mechanisms with stationary observations.
3. *The output/observation independence assumption*: The current observation also known as emission symbol is statistically independent of the previous observations. It is "emitted" only by the current state, $P[O | q_1, q_2, ..., q_T, \alpha] = \prod_{t=1}^{T} P[O_t | q_t, \alpha]$.

The above three assumptions are similar to those of a Markov chain. This is because the stochastic process of an HMM pertaining to the hidden states can be reduced to a Markov chain. Thus, the essential difference between an HMM and a Markov process is that with the former there is not a one-to-one correspondence between the states and the symbols.

### 2.3.2 Three fundamental problems for HMM design

In , Lawrence stipulates an HMM design needs to answer three fondamental problems. The three problems and their solutions are discussed in the next sub-sections.

**The training problem**

**The evaluation problem**

**The decoding problem**

### 2.3.3 Assumptions

Assumption of statistical model: Signal can be parametrised as a parametric random process and that the parametrers of the stochastic process can be determined/estimated in a precise, well-defined manner.
Observation is a probabilitistic function of the state - doubly embedded stochastic process. Each state characterised by the probability distribution of observations, and transitions between states are characterised by a state transition matrix.

**First order Markov model - current and predecessor only considered**

### 2.3.4 Transition Probability Matrix

### 2.3.5 Emission Probability Matrix

### 2.3.6 Initial distribution

### 2.3.7 Elements of an HMM

1. Number of states, N
2. Number of distinct observation symbols per state, M
3. The initial state distribution, pi

### 2.3.8 Types of HMM

#### Ergodic model

#### Left-Right model or Bakis model

#### Evaluation of the probability of a sequence of observations

#### The determination of a best sequence states

#### The adjustment of model parameters to account for observed signal

## 2.4 k-Nearest Neighbour

## 2.5 Dimension reduction

### 2.5.1 Feature selection

## 2.6 Sufficiency of Training Data

## 2.7 Techniques to increase Training Data

### 2.7.1 Mirroring

# Chapter 3

# Hidden Markov Model design

This section focuses on the design of the HMM used to test the hyphotheses postulated above.

## 3.1 Description of available dataset

The available dataset was acquired from a moving dog using Inertia Measurement Units. Two inertial measurements units (IMU) were straped to the front and back of a dog. Each unit has an accelarometer, a gyroscope and a magnetometer. The dataset contains calibrated measurements of a dog running, walking, and trotting then walking; together with the footfalls. The footfall is represented by a binary value that indicates the state of the dog's leg: if it is on or above the ground, at a particular instant in its gait sequence. More specifically, the value 0 means leg up and the value 1 means leg down. The four variables representing the footfalls effectively constitute the ground truth, informing us about the state in which the dog is, at a given time in its movement.

The dataset can be retrieved from nine different matlab files. Each file contains twenty four matlab variables. The variables of interest are listed in the table 3.1.

The observations are continous and the statistical property are assumed to be stationary, i.e, the do not vary over time. In this sta

| Observations | | | |
|---|---|---|---|
| Body part | Accelerometer | Gyroscope | Magnetometer |
| Front | accFrontX<br>accFrontY<br>accFrontZ | FrontPitch<br>FrontRoll<br>FrontYaw | magFront_cal<br>magFront_cal2<br>magFront_cal3 |
| —<br>Back | accBackX<br>accBackY<br>accBackZ | BackPitch<br>BackRoll<br>BackYaw | magBack_cal<br>magBack_cal2<br>magBack_cal3 |

Table 3.1: IMU measurements and footfall variables in dataset

**stationary: statistical property do not vary over time or non-stationary: properties vary over time**

**pure or corrupted?**

### 3.1.1 Quadrupede Gait sequence modelling

One of the objectives of this project is to effectively model the gait sequence dynamic of the dog from IMU measurements using HMM. Based on the fact that quadrupedes achieve inverted pendulum-like movements like humans, their gait dynamic can be modelled as a succession of latent states observed by measurements such as IMU data. The states representing the footfalls and the observations, the outputs of the accelerometer, gyroscope and the magnetometer. Similar to human gait mechanism, it is sound to assume that the current state of a quadrupede is conditionally dependent on its previous state. This inference combined with the statistical robustness of HMM makes it the best model candidate when the available dataset is not large enough.

#### HMM model elements: states and observations properties

The problem at hand requires 16 distinct state that make up the state vector S, shown in equation 3.1

$$S = S_i = \{(LF, RF, LB, RB)\} = \{0000, 0001, 0010, ..., 1111\}. \tag{3.1}$$

$$|S| = N = 2^4 = 16$$

$$i = 1, 2, ..., 16$$

The 16 distinct states are derived from the combination of the four binary footfalls. In practice, the dataset may not reveal all the 16 states.

The stream of IMU measurement form the observation sequence. An observation instance is a row vector of K dimensions. The initial K value before any dimensionality reduction is 18, from the 18 IMU measurements. Thus, an observation sequence O is a TxK matrix of continuous values as presented in 3.2. T is the total number of the successive measurements.

$$O = \{Ok_t\} = O1_t, O2_t, ..., O18_t. \tag{3.2}$$

$$k = 1, 2, ..., 18. \tag{3.3}$$

$$t = 1, 2, ..., T. \tag{3.4}$$

**Splitting the 16-states HMM in two 4-states HHMs**

In order to simplify the problem, it was decided to split the the four legs in two sub-parts: two front legs and two back legs. This approach for spliting it was based on the two inverted-like movements for a quadrupede such as a dog has investigated in As a result, the initial 16-states HMM becomes, two distinct 4-states HMMs. These two models may be combined to get back the holistic 16-states model. This design decision was motivated by the fact that it is a simpler task to dsicriminate between 4 distinct classses than 16 classes. From here onward, attention will be given to the 4-states HMM model.

**Transition between states**

This design assumes that a dog can transition from one state to any other possible state. So, for any transtion from S_i to S_j both S_i and S_j may be any of the element of the state space

$$S = \{S1, S2, S3, S4\}$$

.

For instance, if a dog has its left leg above ground and its right leg on ground, at time instance t, it may move to any of the 3 other possible positions or remain in the same state, in the next time instance, t + 1. This consideration yielded in an ergodic HMM where, all the transitions are possible. The graphical model of the simplified HMM is illustrated by figure 3.1

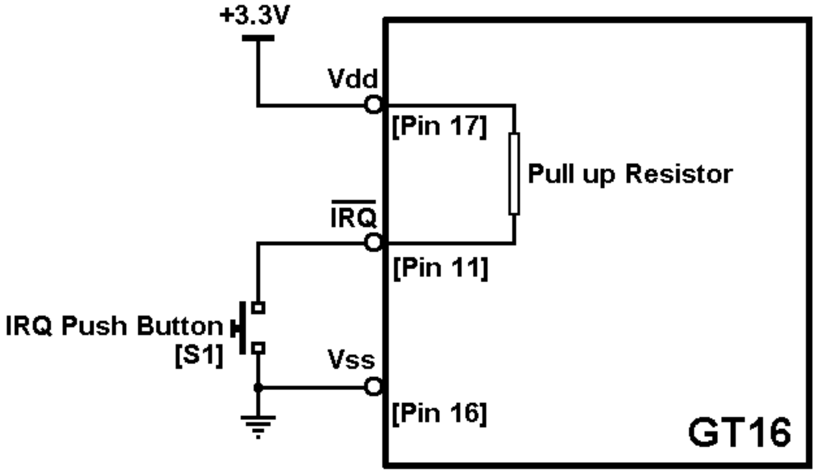



Figure 3.1: Ergodic HMM graphical model showing the hidden states, observation, and transitions between states

### 3.1.2 Data pre-processing

### 3.1.3 Model parameters estimation

As a reminder, a continuous HMM model is completely specified by its initial state distribution: $\pi$ transition matrix: A; the mean covariance matrices: $\mu$, $\Sigma$ which can be combined into $\Phi$. If the observations are modelled with gaussian mixture distributions, one addition parameter is required for the initial mixture distribution: $\beta$. The next sub-sections discuss how each parameters was estimated in this project.

#### Transition matrix: A

For each of the front and back 4-state HMM, the state transition matrix A, is a 4-by-4 matrix. Two different approaches were considered in the estimation of A. The two methods make use of the expectation maximisation algorithm but differ in the input arguments considered.

1. Approach 1: Exploiting the available ground truth
   This approach takes advantage of the ground truth for a labelled dataset to reduce

the HMM model to a Markov Chain. This is done by making the hypothetical observation sequence identical to the state sequence. Then, using approach used in discrete HMM, the transition matrix can be estimated using maximum likelihood algorithm. For each state, the pseudocount was set to the number of occurences of that state in the training data plus a constant value: $PseudoA = |S_i| + C$. The additional constant C is to avoid having 0 in the transition matrix for states and transitions not reflected in the dataset. It can be chosen empirically. This method is very simple however, it has two limitations. It not only assumes that the number of states is known but also requires the training data to be labelled. Although, the dataset at hand satisfies the two constraints, the second approach which eliminates these possible setbacks, was also considered.

2. Approach 2: This method is the standard approach found in literature using expectation algorithm such as Welch-Baum algorithm. and described in the literature review.

**Mean and covariance matrices: $\mu$, $\Sigma$ and $\beta$**

The observations were modelled using mixture Gaussian distributions, characterised by a KxMxN mean matrix: $\mu$ and a KxKxMxN co-variance matrix: $\Sigma$. where: $K = number\ \ of\ \ features,\ \ M = mixture\ \ number,\ \ and\ \ N = number\ \ of\ \ states.$ The observations were grouped K classes following based on the ground-truth represented by the state sequence. The each group of observation was used as input to the EM algorithm to estimate the $\mu$ and $\Sigma$. In general, the optimal number of mixture, M is chosen empirically, in this project, it was estimated using akaike information criterion (AIC). So, gaussian mixture models were built using EM algorithm while varying the M from 1 to K, the feature number. Since AIC is a measure of information loss, the model with the minimum AIC best represents the dataset. The number of mixture M, is therefore set to the mixture number of this model. This algorithm is outlined below.

```
function M = best_M(data, K)
  data = training_set
  AIC = zeros(1, K);
  models = cell(1, K);
  for m = 1:K
    model = gauss_mixture(data, m);
    AIC(m) = model.AIC;
  end
```

```
  [minAIC, minAIC_Idx] = min(AIC);
  M = minAIC_Idx;
end
```

The mixture components were considered evenly distributed initially. In order to avoid zero values in the covariance matrix, it was regularized with $10^-10$. The maximum number of iteration of the EM algorithm was empirically set to 1000.

**Inital state distribution:** $\pi$

The initial state distribution was estimated using the probability of occurence of each state in the training data sample. Thus, $\pi = \{\pi_i\} = \frac{|S_i|}{T}$

### 3.1.4 Dimension reduction

**From 18 to 4 features using classification with knn**

### 3.1.5 Continuous observations/features to Discrete observations/features

## 3.2 HMM implementation

# Chapter 4

# Results

These are the results I found from my investigation.

Present your results in a suitable format using tables and graphs where necessary. Remember to refer to them in text and caption them properly.

## 4.1 Aim

The objetive of this project is to design, implement and evaluate an algorithm, a model or a machine to predict the gait sequence of an animal (quadrupede or bipede) using Markov models. Thus, for a given state S, the model should be able to predict the next state S + 1, with a certain degree of confidence.

## 4.2 Apparatus

## 4.3 Methods

## 4.4 Results

## 4.5 Analysis

### 4.5.1 Discrete Probability density function duration d in state i

### 4.5.2 Expected number of observations (duration) in a state

## 4.6 Experimental Results

# Chapter 5

# Discussion

Here is what the results mean and how they tie to existing literature...

Discuss the relevance of your results and how they fit into the theoretical work you described in your literature review.

# Chapter 6

# Conclusions

These are the conclusions from the investivation and how the investigation changes things in this field or contributes to current knowledge...

Draw suitable and intelligent conclusions from your results and subsequent discussion.

# Chapter 7

# Recommendations

Make sensible recommendations for further work.

Use the IEEE numbered reference style for referencing your work as shown in your thesis guidelines. Please remember that the majority of your referenced work should be from journal articles, technical reports and books not online sources such as Wikipedia.

# Bibliography


[1] M. S. Tsoeu and M. Braae, "Control Systems," *IEEE*, **vol. 34(3)**, pp. 123-129, 2011.

[2] J. C. Tapson, *Instrumentation*, UCT Press, Cape Town, 2010.

# Appendix A

# Additional Files and Schematics

Add any information here that you would like to have in your project but is not necessary in the main text. Remember to refer to it in the main text. Separate your appendices based on what they are for example. Equation derivations in Appendix A and code in Appendix B etc.

# Appendix B

# Addenda

## B.1 Ethics Forms