Models of ongoing and evoked activity

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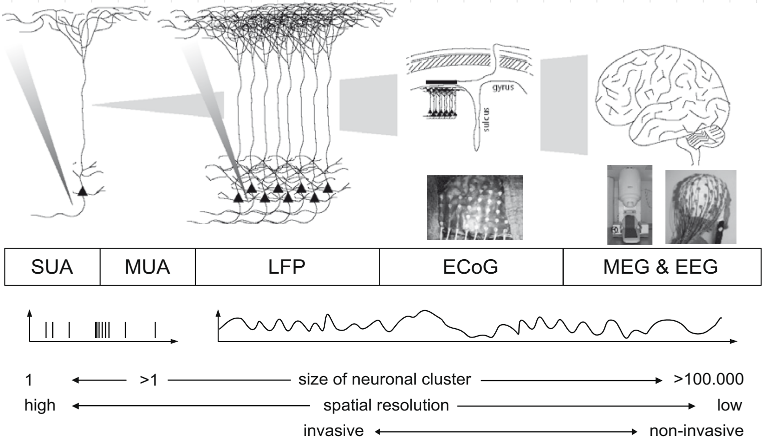
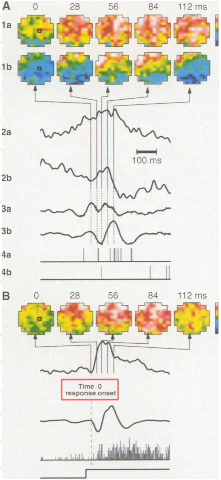
Multiscale organization of the brain

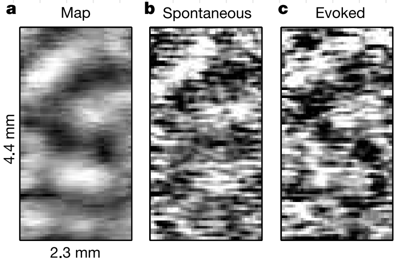


Waldert et al. 2009

Outline

Variability of cortical activity is as large as the mean

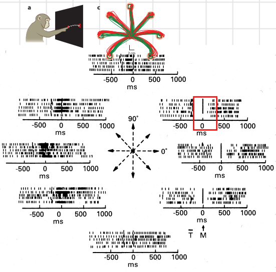




Arieli et al. 1996 || Kenet et al. 2003

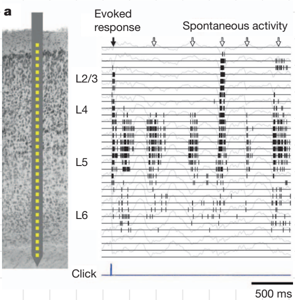
Task related firing rate modulation

Motor cortex



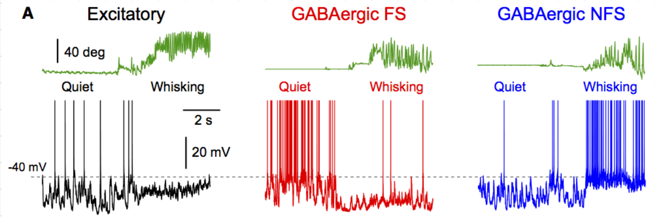
Georgeopoulous et al. 1982

Auditory cortex



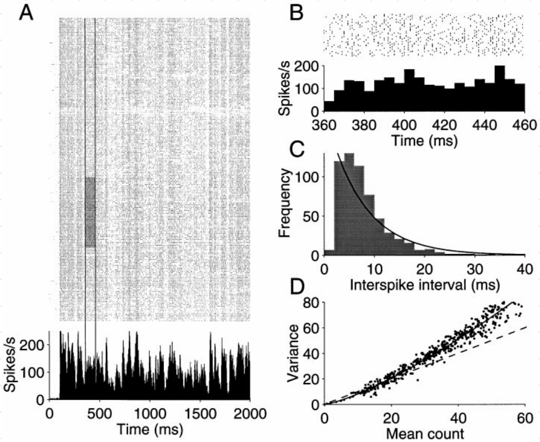
Sakata & Harris 2009

Somatosensory cortex



Gentet et al. 2010

Statistics of stimulus evoked spiking activity

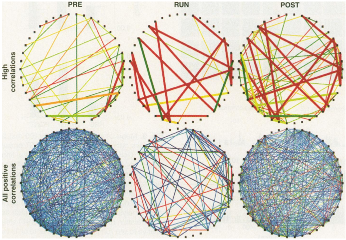


⇒ Spike response is approximately Poisson point process

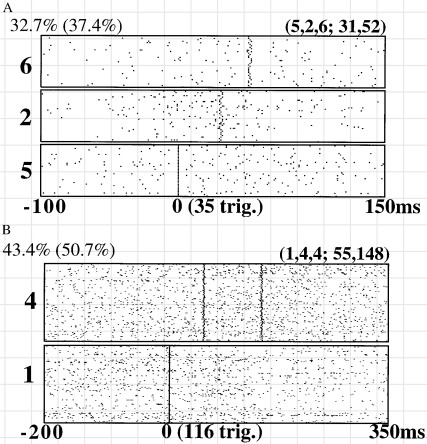
- Mean of spike count is proportional to the variance
- Spike intervals are exponentially distributed

Shadlen and Newsome 1998

Task related neural activity: Correlations

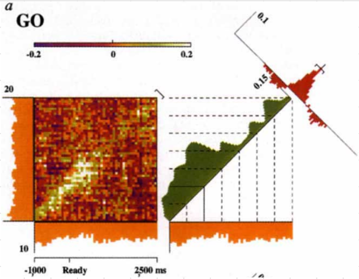


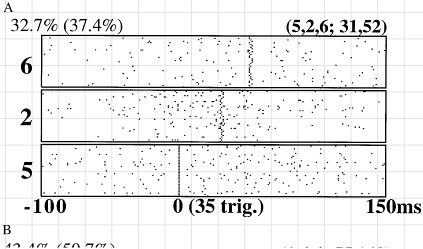
Wilson and McNaughton 1994

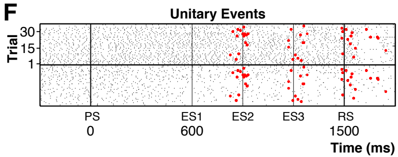


Prut et al. 1998

Spike correlations are modulated by stimulus/task



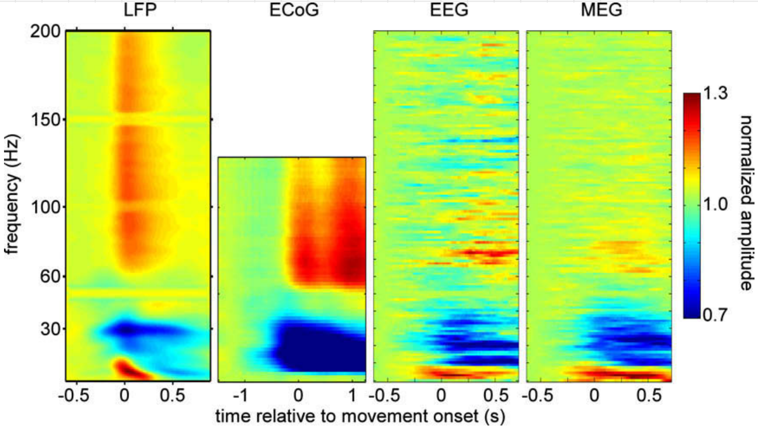




- Spike activity correlations exist
- Correlations are modulated in a highly coordinated manner
- Correlations can affect the statistics of the network activity
- and can be propagated with high fiedility

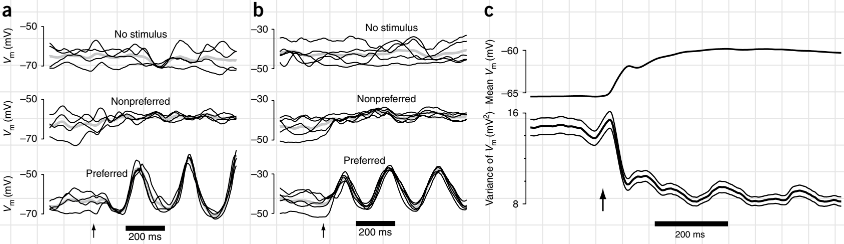
Vaadia et al. 1995, Riehle et al. 1997, Prut et al. 1998 and many since

Task related neural activity: Oscillations



Waldert et al. 2009

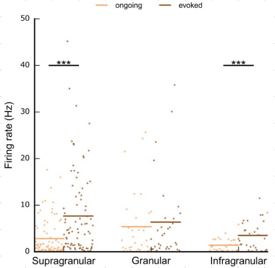
Evoked activity dynamics



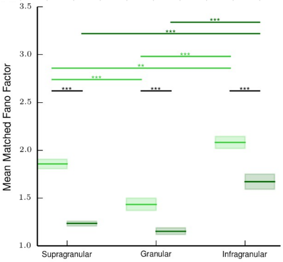
Churchland et al. 2010

Evoked activity dynamics: Correlation and variability reduction

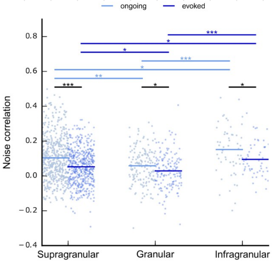
Firing rate



Variability



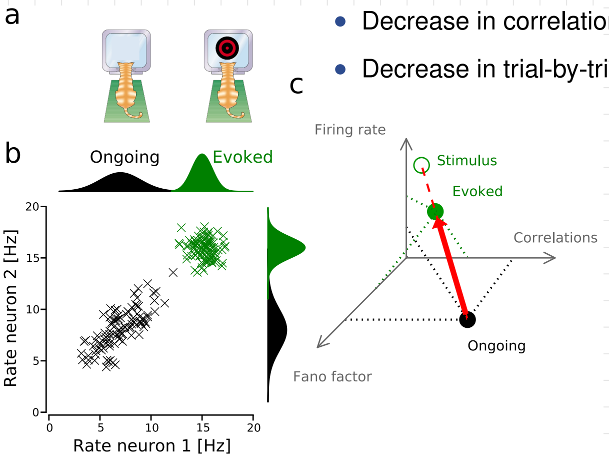
Noise correlations



Jasper et al. in prep.

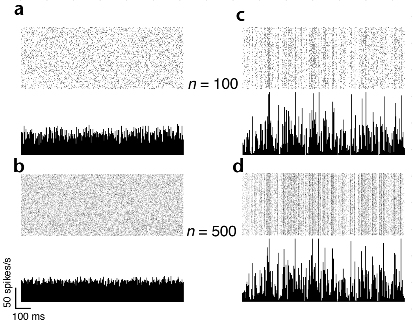
Features of the evoked activity

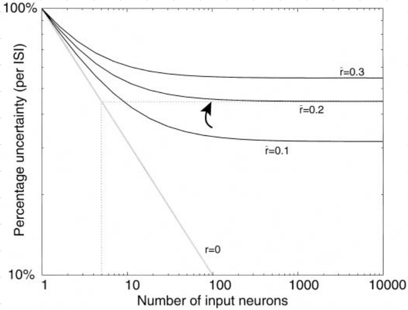
- Increase in the firing rate
- Decrease in correlations
- Decrease in trial-by-trial variability



Bujan et al. 2015, Churchland et al. 2010, Oram 2012

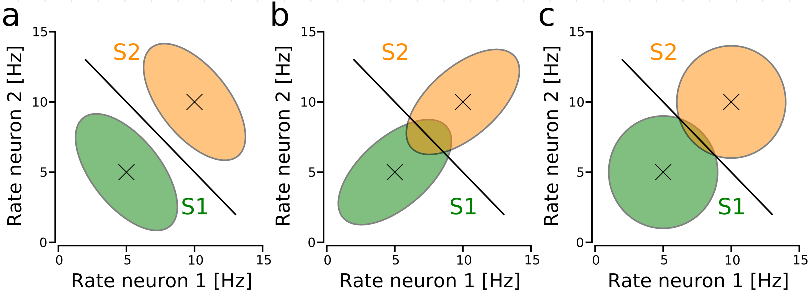
Correlations: signal encoding





Shadlen and Newsome 1998, Mazurek et al. 2002

Correlations: Signal decoding



Averbeck et al. 2006

Properties of ongoing and evoked activity

- Ongoing activity
  - Low firing rates on average
  - Irregular spike pattern
  - High trial-by-trial variability
  - Weak pair-wise correlations
- Evoked activity
  - Moderate increase in firing rates
  - Reduction in correlations
  - Reduction in trial-by-trial variability
  - Emergence of task-related oscillations
- Brain hadware
  - On average there are 80% excitatory 20% inhibitory neurons
  - $1mm^3$ of brain tissue contains 100,000 neurons
  - Each neuron makes sparse connections with a probability of 10%
  - Each neuron receives 10^4 synapses from its neighbors in mouse and upto 10^6 in humans
  - Neurons and synapses are stochastic
  - Synapses can be changed depending on the properties of the synapse and neural activity

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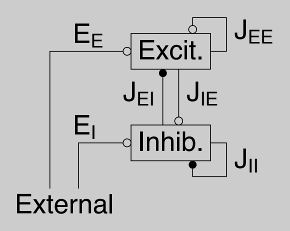
Outline

1. Ongoing and evoked activity dynamics
2. **Modelling of ongoing activity dynamics**
3. Modelling of evoked activity dynamics

Firing rate dynamics of random recurrent networks

When the variance of the input can be ignored (e.g. large sparse networks) average firing rate of the neuronal populations can be described as

$$\tau_e \frac{dr_e}{dt} = -r_e + J_{ee}K_{ee}r_e - J_{ei}K_{ei}r_i$$
$$\tau_i \frac{dr_i}{dt} = -r_i + J_{ie}K_{ie}r_e - J_{ii}K_{ii}r_i$$



or in more general form

$$\tau_e \frac{dr_e}{dt} = -r_e + \phi_e(r_e, r_i)$$
$$\tau_i \frac{dr_i}{dt} = -r_i + \phi_i(r_e, r_i)$$

$\tau_{e,i}$	effective time constant of the population
$r_{e,i}$	average firing rate of the population
$K_{1,2}$	No. of connections from population '2' to '1'
$J_{1,2}$	Connection strength from population '2' to '1'
$\phi_{e,i}$	effective transfer function of the population

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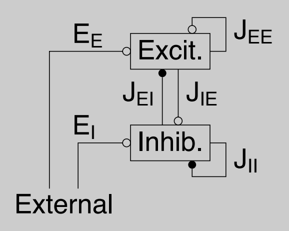

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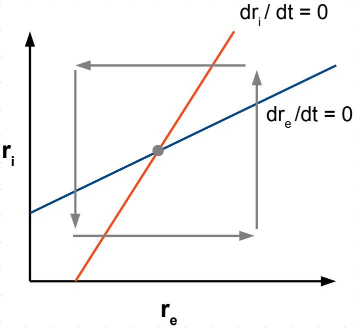
Stability analysis of a linear system: Geometrical approach

Lets assume a linear dynamics:

$$\begin{aligned} 0 &= -r_e + J_{ee}r_e - J_{ei}r_i + U_e \\ 0 &= -r_i + J_{ie}r_e - J_{ii}r_i + U_i \end{aligned}$$



- For stability Inhibitory nullcline must intersect the Excitatory nullcline from below.
- Slope at the intersection point determines whether it is stable or metastable

Firing rate models

- It is not possible to relate the rate model parameters to the neuron/synapse parameters
- We get only a qualitative description of the dynamics

Firing rates interact multiplicatively

$$\tau \dot{V}_i(t) = -V_i(t) + RI^{syn}(t)$$
$$\tau \dot{V}_i(t) = -V_i(t) + \sum_j \alpha_{ij} S_j(t)$$

S_j is the spike train of the jth neuron and α_{ij} is the synaptic weight

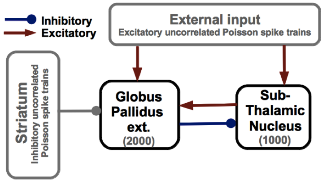
$$r_i(t) = f(V_i(t))$$
$$r_i(t) = exp(V_i(t))$$

If we now formulate the rate equation

$$\dot{r}_i(t) = r_i(t) \sum_j \alpha_{ij} r_j(t)$$

Cardanobile and Rotter 2011

Oscillations in EI networks



Reduced analytical model:

$$\dot{r}_G = a r_S r_G - I_G r_G + \eta(t)$$

$$\dot{r}_S = -b r_G r_S + I_S r_S + \zeta(t)$$

Fixed Point I:

$$r_G = 0 \quad r_S = 0$$

Eigen values:

$$r_1 = -I_G \,; \quad r_2 = I_S$$

Fixed Point II:

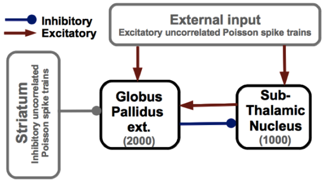
$$r_S = \frac{I_G}{a} \quad r_S = \frac{I_S}{b}$$

Eigen values:

$$r_1 = j\sqrt{I_S I_G} \,; \quad r_2 = -j\sqrt{I_S I_G}$$

Complex eigenvalues imply oscillations

Oscillations in EI networks



Reduced analytical model:

$$\dot{r}_G = ar_S r_G - I_G r_G + \eta(t)$$

$$\dot{r}_S = -br_G r_S + I_S r_S + \zeta(t)$$

Fixed Point I:

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Eigen values:

$$r_1 = -I_G \; ; \quad r_2 = I_S$$

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Eigen values:

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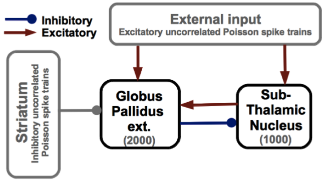
Complex eigenvalues imply oscillations

Oscillations sign rule



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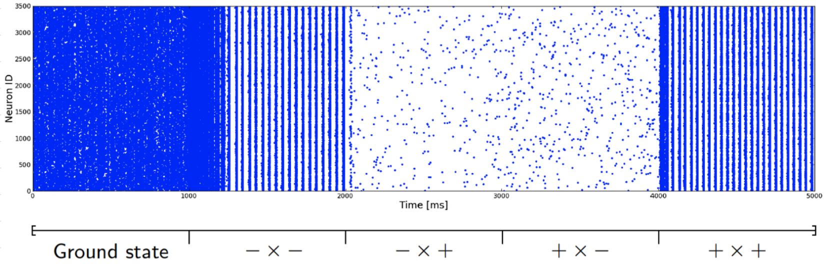
$$r_S = \frac{I_G}{a} \qquad r_S = \frac{I_S}{b}$$

Eigen values:

$$r_1 = j\sqrt{I_S I_G}\ ; \qquad r_2 = -j\sqrt{I_S I_G}$$

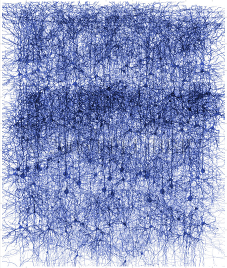
Effect on oscillations = sign (input) X sign (receiver)

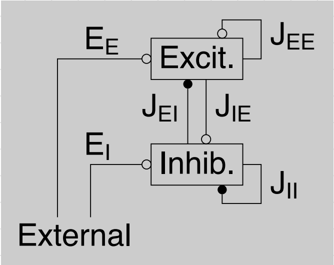


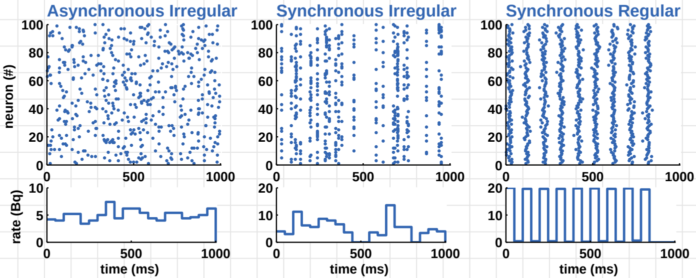
Limits of the rate based models

- Provides a qualitative description of the firing rate and population oscillations
- Not possible to relate the model parameters to the neuron/synapse properties
- Effect of fluctuations is ignored
- It is not possible to understand correlations in the activity

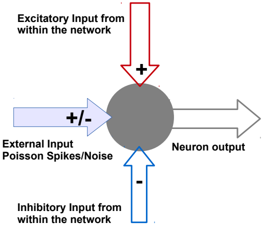
Dynamics of random recurrent neural networks

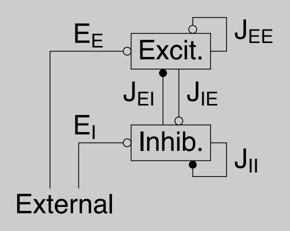


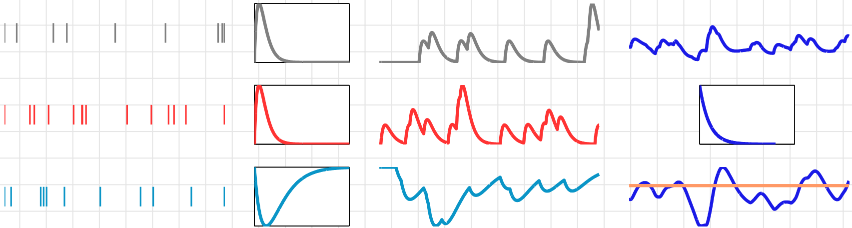




Single neuron approximation







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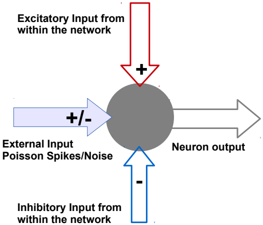
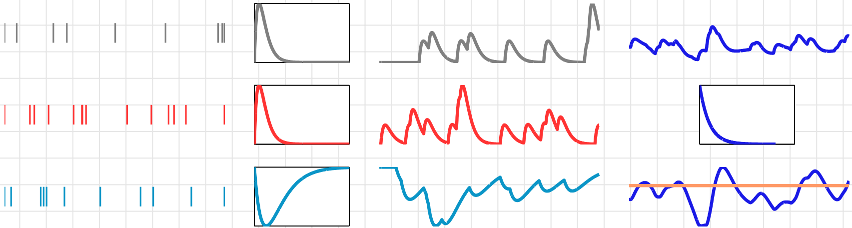


$$
\begin{aligned}
\tau \dot{V}_i(t) &= -V(t) + R(I_i^{syn}) \\
I_i^{syn} &= I_i^{ext}(t) + I_i^{exc}(t) - I_i^{inh}(t) \\
r_i(t) &= f(V_i(t))
\end{aligned}
$$



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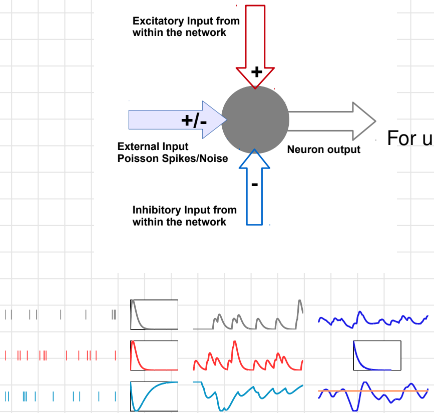
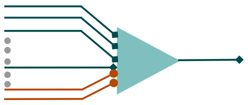


$$\begin{aligned} \tau \dot{V}_i(t) &= -V(t) + R(I_i^{syn}) \\ I_i^{syn} &= I_i^{ext}(t) + I_i^{exc}(t) - I_i^{inh}(t) \\ r_i(t) &= f(V_i(t)) \end{aligned}$$

For uncorrelated Poisson type spike trains:

$$\begin{aligned} \mu_I &= r_x K_x \int J_x \times PSC(t) dt \\ \sigma_I^2 &= r_x K_x \int (J_e \times PSC(t))^2 dt \end{aligned}$$

Balanced excitatory and inhibitory inputs



- *in vivo* a neuron receives both excitatory and inhibitory inputs
- Using the Cambell's theorem we can calculate the mean and variance of the membrane potential.

$$\mu_V = r_{exc} J_e \int EPSP(t)dt - r_{inh} J_i \int IPSP(t)dt$$

$$\sigma_V^2 = r_{exc} J_e^2 \int EPSP^2(t)dt + r_{inh} J_i^2 \int IPSP^2(t)dt$$

- because IPSP's and EPSP's have opposite sign, for balanced input the mean will be reduced
- but variance calculation involves sum of square of the IPSP's and EPSP's variance increases for balanced input

Single neuron approximation

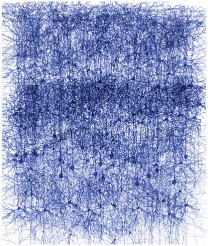
The mean and variance of the synaptic current

$$\begin{aligned}\mu(I^{syn}) &= \mu(I^{exc}) + \mu(I^{exc}) - \mu(I^{inh}) \\ \sigma^2(I^{syn}) &= \sigma^2(I^{ext}) + \sigma^2(I^{exc}) + \sigma^2(I^{inh})\end{aligned}$$

- Inhibitory inputs reduce the mean input current
- Inhibitory inputs increase the variance of the input current
- Increase in weight of the synapses always increases variance unless inputs are correlated
- If total mean synaptic current is constant:

  increases in network size (i.e. connections) will reduce the variance

Firing rate dynamics of random recurrent networks: fluctuations



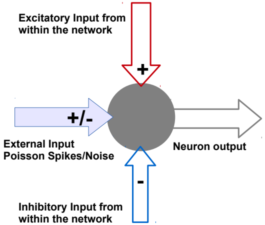
$$\begin{aligned}\tau \dot{V}_i(t) &= -V(t) + RI_i^{syn}(t)\\ r_i(t) &= f(V_i(t))\end{aligned}$$

When input is Poisson type then the synaptic input is shot noise

$$RI_i^{syn}(t) = \mu(t) + \sigma\sqrt{\tau}\eta_i(t)$$

$$\begin{aligned}\mu(t) &= \mu_{net}(t) + \mu_{ext}\\ \mu_{net}(t) &= K_{exc}J_E r(t-D)\tau - K_{inh}J_I r(t-D)\tau\\ \mu_{ext}(t) &= K_{exc}J_E r_{ext}\tau\end{aligned}$$

$$\begin{aligned}\sigma^2(t) &= \sigma^2_{net}(t) + \sigma^2_{ext}\\ \sigma^2_{net}(t) &= J_E^2 K_{exc} r(t-D)\tau + J_I^2 K_{inh} r(t-D)\tau\\ \sigma^2_{ext} &= J_E^2 K_{exc} r_{ext}\tau\end{aligned}$$



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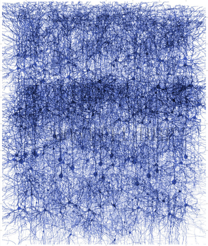

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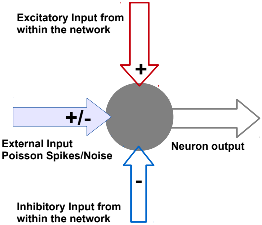
$$RI_i^{syn}(t) = \mu(t) + \sigma\sqrt{\tau}\eta_i(t)$$

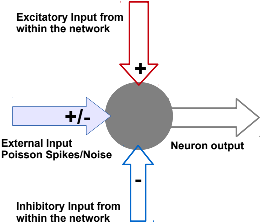
$$\begin{aligned}\mu(t) &= \mu_{net}(t) + \mu_{ext}\\ \mu_{net}(t) &= K_{exc}J_E r(t-D)\tau - K_{inh}J_I r(t-D)\tau\\ \mu_{ext}(t) &= K_{exc}J_E r_{ext}\tau\end{aligned}$$

$$\begin{aligned}\sigma^2(t) &= \sigma^2_{net}(t) + \sigma^2_{ext}\\ \sigma^2_{net}(t) &= J_E^2 K_{exc} r(t-D)\tau + J_I^2 K_{inh} r(t-D)\tau\\ \sigma^2_{ext} &= J_E^2 K_{exc} r_{ext}\tau\end{aligned}$$



Firing rate dynamics of random recurrent networks: fluctuations

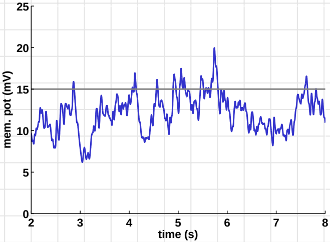
$$\frac{\partial P(V,t)}{\partial t} = \underbrace{\frac{\sigma^2(t)}{2}\frac{\partial^2 P(V,t)}{\partial V^2}}_{\text{diffusion}} + \underbrace{\frac{\partial}{\partial V}[(V-\mu(t))P(V,t)]}_{\text{drift}}$$

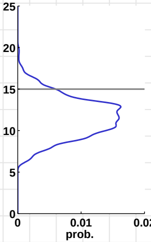


Lets define a probability current $S(V,t)$

$$\frac{\partial P(V,t)}{\partial t} = -\frac{\partial S(V,t)}{\partial V}$$

$$S(V,t) = \frac{\sigma^2(t)}{2}\frac{\partial P(V,t)}{\partial V} - \frac{[(V-\mu(t))}{\tau}P(V,t)]$$





Probability current at V_{th} is the firing rate i.e.

$$r(t) = S(V_{th}, t)$$

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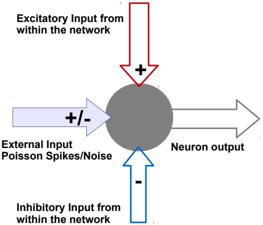

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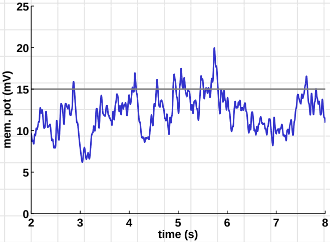

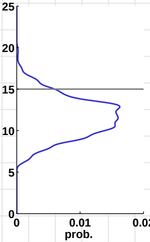

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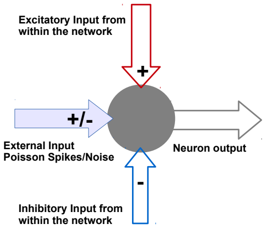


$$\frac{\partial P(V,}{\partial t}$$

Lets define

$S(V$

Boundary conditions

$\Rightarrow$ $P(V_{th},t) = 0$

$\frac{\partial P}{\partial V}(V_{th},t) = -\frac{2r(t)\tau}{\sigma^2(t)}$

$\Rightarrow$ at $V = V_r$

$S(V_r^+,t) - S(V_r^-,t) = r(t-\tau_{ref})$

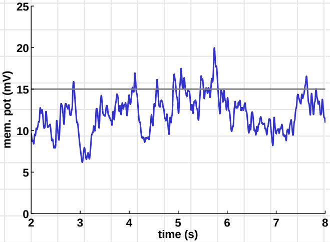
$\frac{\partial P}{\partial V}(V_r^+,t) - \frac{\partial P}{\partial V}(V_r^-,t) = -\frac{2r(t-\tau_{ref})\tau}{\sigma^2(t)}$

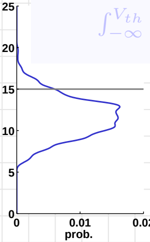
$\Rightarrow$ $V{=}-\infty$

$\lim_{V\to\infty} P(V,t) = 0; \lim_{V\to\infty} VP(V,t) = 0$

$\Rightarrow$ Finally

$\int_{-\infty}^{V_{th}} P(V,t)dV - p_r(t) = 1$





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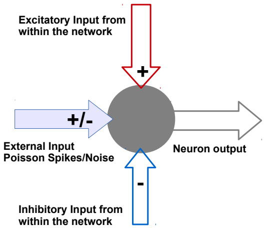

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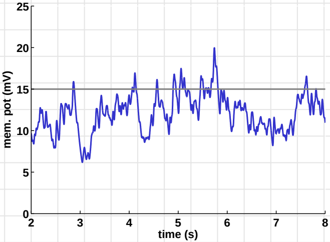

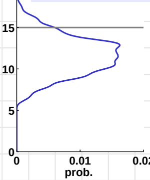

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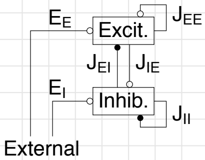
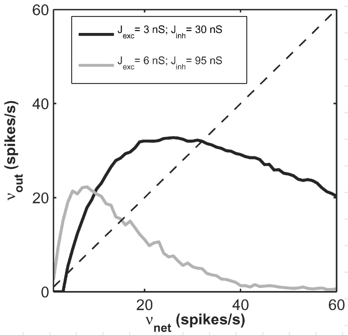


$$\mu_0 = K_{ext} J \tau r_{ext} + K_E J \tau r_{exc} r_0 - K_I J \tau r_{inh} r_0$$
$$\sigma_0 = K_{ext} J^2 \tau r_{ext} + K_E J^2 \tau r_{exc} r_0 + K_I J^2 \tau r_{inh} r_0$$

Find a self-consistent solution for the following:

$$\frac{1}{r_0} = \tau_{ref} + \tau\sqrt{\pi} \int_{\frac{V_r - \mu_0}{\sigma_0}}^{\frac{V_{th} - \mu_0}{\sigma_0}} du\, e^{u^2} (1 + \text{erf}(u))$$



For a generic neocortical network when $K_I = \gamma K_E$ and $J_I = g J_E$

$$\mu_0 = K_E J \tau [r_{ext} + r_0(1 - g\gamma)]$$
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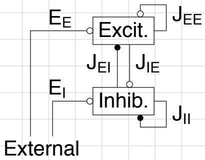
Firing rate dynamics of random recurrent networks: fluctuations



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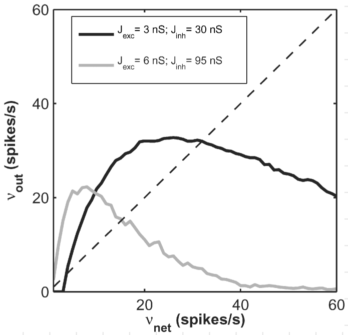

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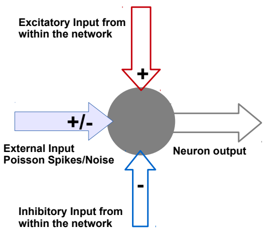


In the so-called mean driven regime when $(V_{th} - \mu_0) \gg \sigma_0$ or $g < 4$

$$r_0\tau \approx \frac{(V_{th} - \mu_0)}{\sigma_0\sqrt{\pi}} exp(-\frac{(V_{th} - \mu_0)^2}{\sigma_0^2})$$

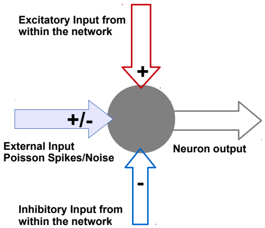
$$r_0 = \frac{1}{\tau_{ref}}[1 - \frac{(V_{th} - V_r)}{K_E J(1 - g\gamma)}]$$

In the so-called fluctuation driven regime when $(V_{th} - \mu_0) \approx \sigma_0$ or $g > 4$

$$r_0 = \frac{(r_{ext} - r_{thr})}{g\gamma - 1}$$

$$r_{thr} = \frac{V_{th}}{K_E J\tau}$$

Firing rate dynamics of random recurrent networks



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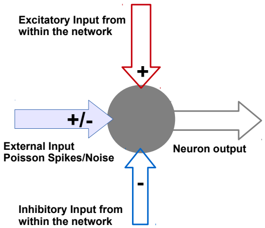

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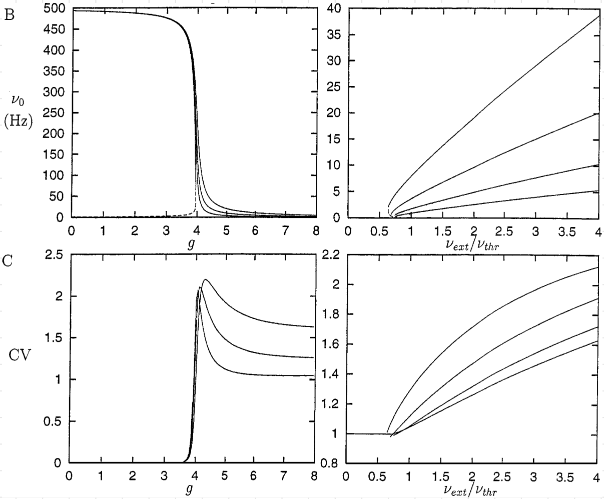

Brunel 2000, Brunel and Hakim 1999

Linear stability analysis of the stationary state

Perturb the steady state with an oscillatory input

$$r = r_0 + \delta r e^{\lambda t}$$

Input rate fluctuation $\rightarrow$ synaptic current $\rightarrow$ membrane pot. response $\rightarrow$ new output rate

When Input rate fluctuation = output rate fluctuation

Fluctuations do not die out and therefore, the asynchronous state is unstable

Alternatively find the eigenvalues λ and their signs will tell about the stability

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Perturb the steady state with an oscillatory input

$$r = r_0 + \delta r_e^{\lambda t}$$

Synapses behave as low pass filters

$$S(\lambda) = \frac{e^{-\lambda D}}{(1+\lambda\tau)^2}$$

so the input synaptic current fluctuation is

$$\delta I(t) = JS(\lambda)\delta r(t)e^{\lambda t}$$

The change in the output firing rate then is

$$\delta r(t)e^{\lambda t)} = R(\lambda)\delta I(t)$$

$$\begin{aligned} \delta r(t)e^{\lambda t)} &= R(\lambda)JS(\lambda)\delta r(t)e^{\lambda t} \\ 1 &= R(\lambda)JS(\lambda) \end{aligned}$$

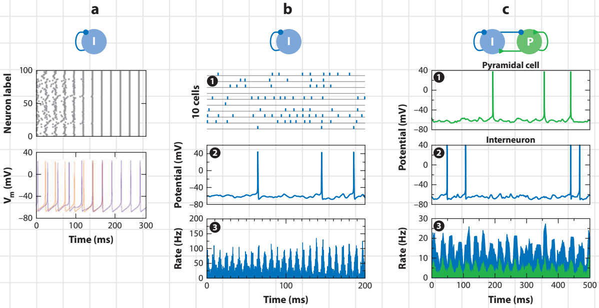
Oscillations ING and PING and all that

Conditions for oscillations

For an EI network

$$1 = A_{EE}(\lambda)(1 + A_{II}(\lambda)) - A_{II}(\lambda) - A_{EI}(\lambda)A_{IE}(\lambda)$$

$$A_{xy}(\lambda) = \underbrace{R_x(\lambda)}_{neuron\ tx\ fun} \times \underbrace{J_{xy}}_{syn.\ weight} \times \underbrace{S_{xy}(\lambda)}_{syn.tx.fun}$$



Buzsaki 2012

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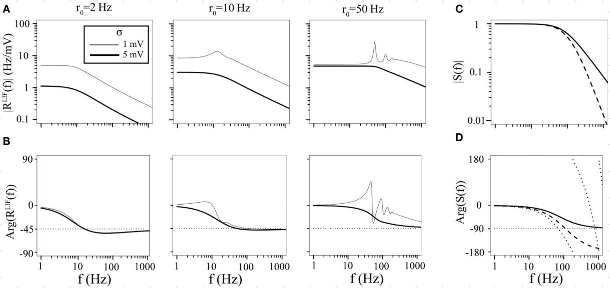


For the rate model neuron tx fun.

$$R^{rm}(\lambda) = \frac{1}{1+\lambda\tau}$$

The LIF neuron response function is

$$R_{LIF}(\lambda) = \frac{\tau_m r_0}{\sigma(1+\lambda\tau)}\left[\frac{\frac{\partial U}{\partial y}(y_t \lambda\tau_m) - \frac{\partial U}{\partial y}(y_r, \lambda\tau_m)}{U(y_t, \lambda\tau_m) - U(y_r, \lambda\tau_m)}\right]$$

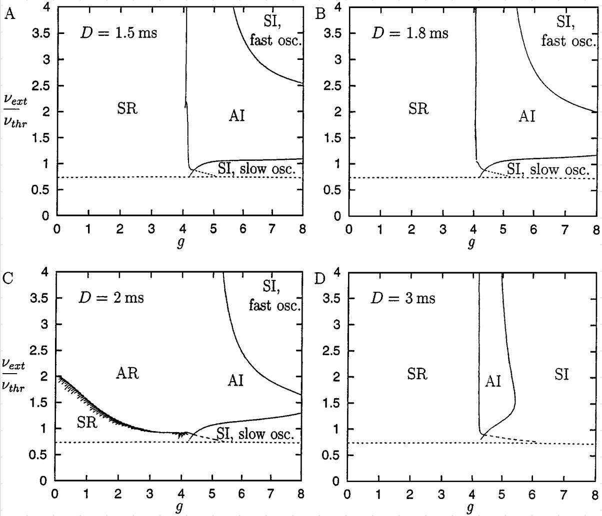
where
$y_t = V_{th} - (V_r + I_0)/\sigma$ and
$y_r = V_r - (V_r + I_0)/\sigma$

$U(y, \lambda)$ is

$$U(y,\lambda) = \frac{e^{y^2}}{\Gamma((1+\lambda)/2)} M\left(\frac{1-\lambda}{2}, \frac{1}{2}, -y^2\right) + \frac{2ye^{y}}{\Gamma(\lambda/2)} M\left(1-\frac{\lambda}{2}, \frac{3}{2}, -y^2\right)$$

Ledoux and Brunel 2011

Linear stability analysis of the stationary state



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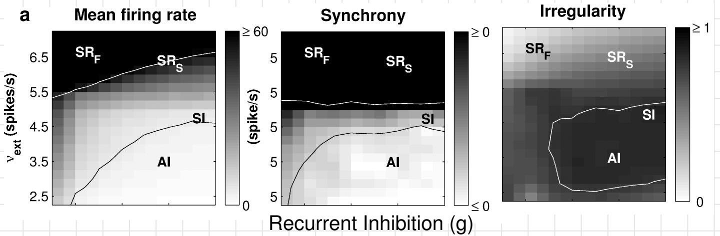
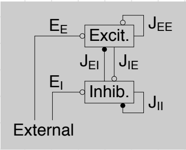
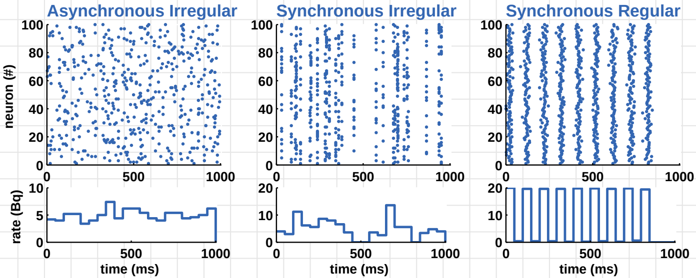
Dynamics of random recurrent neural networks







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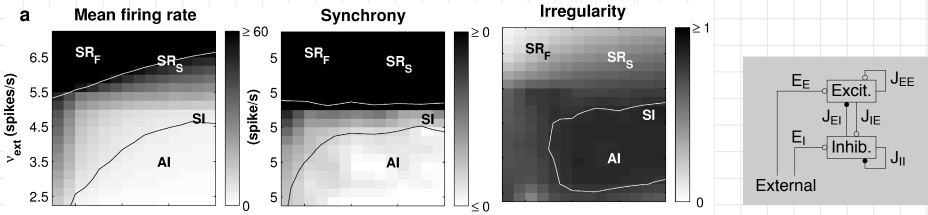
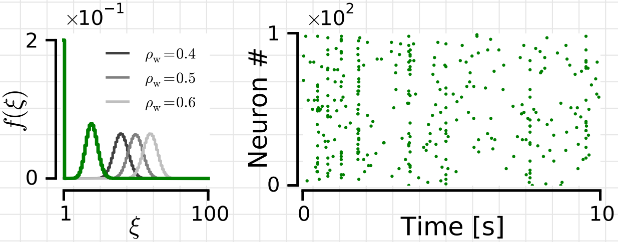


- Basic properties of the ongoing and stimulus induced activity such as the low firing rate, asynchronous-irregular spiking are observed in a fluctuation or inhibition driven regime.
- Oscillations arise due to delays, imbalance of feedforward drive to the exc/inh neurons, or imbalance of exc/inh recurrent weights.

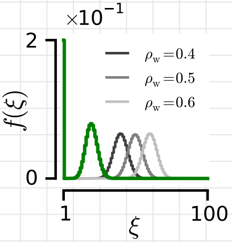
Outline

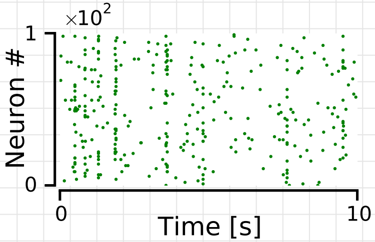
1. Ongoing and evoked activity dynamics
2. Modelling of ongoing activity dynamics
3. **Modelling of evoked activity dynamics**

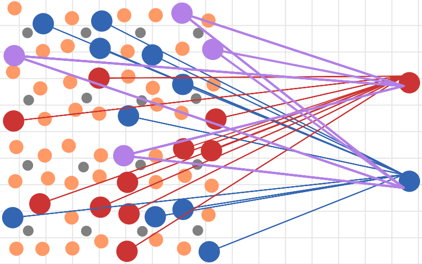
Origin of correlations/synchrony



Origin of correlations





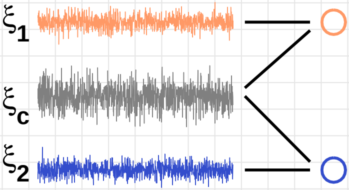


- Common inputs
- Pooling of inputs
- Change in synaptic strengths
- Multiple stochastic synapses
- Daleian synapses

Correlation transfer function

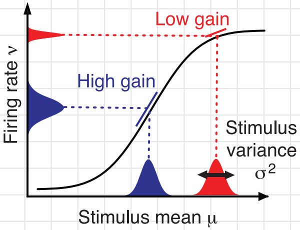
$$\tau_m \frac{dV_1}{dt} = -V_1 + \mu + \sigma\sqrt{\tau_m}[\sqrt{1-c}\xi_1(t) + \sqrt{c}\xi_c(t)]]$$

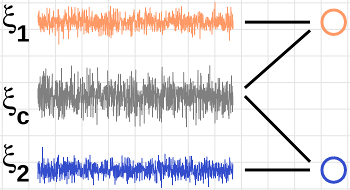
$$\tau_m \frac{dV_2}{dt} = -V_2 + \mu + \sigma\sqrt{\tau_m}[\sqrt{1-c}\xi_2(t) + \sqrt{c}\xi_c(t)]]$$



Membrane potential → Neuron transfer function → Output firing rate → Cross-spectrum (freq. domain) → correlations (time domain)

Correlation transfer function





Hong et al. 2012

The correlation susceptibility can be approximated as:

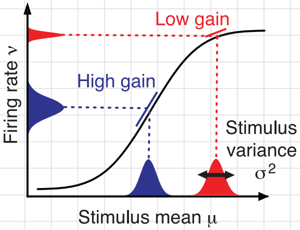
$$\lim_{\omega \to 0} A(\omega) = \frac{dr}{d\mu}$$

essentially the slope of the neuron transfer function

so the correlation coefficient is defined as

$$\rho = \frac{\int_{-\infty}^{+\infty} C_{ij}(\tau)}{\int_{-\infty}^{+\infty} C_{ii}(\tau)} = \frac{\sigma^2 |\frac{dr}{d\mu}|^2}{rCV^2} c$$

Correlation transfer function



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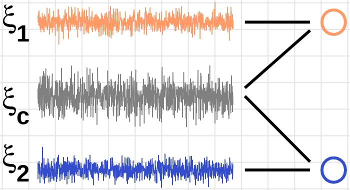

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Firing rate, CV and transfer function

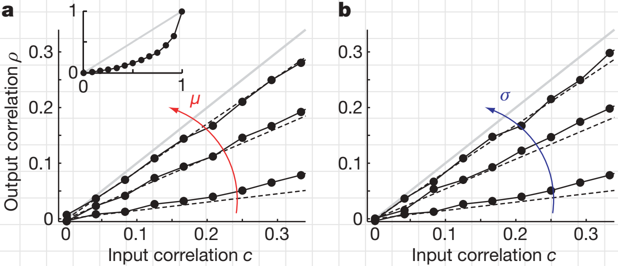
$$r^{-1} = \sqrt{\pi} \int_{(\mu - V_T)/\sigma}^{(\mu - V_R)/\sigma} dx \; e^{x^2} \text{erfc}(x)$$

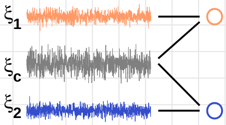
$$CV = 2\pi r \int_{(\mu - V_T)/\sigma}^{\infty} dx \; [e^{x^2} \text{erfc}(x)]^2 \int_{(\mu - V_T)/\sigma}^{x} dy \; e^{y^2} \Theta(\frac{\mu - V_R}{\sigma} - y)$$

$$\frac{dr}{d\mu} = r^2 \frac{\pi}{\sigma} [e^{(\mu - V_T)^2/\sigma} \text{erfc}(\mu - V_T)/\sigma) - e^{(\mu - V_R)^2/\sigma} \text{erfc}(\mu - V_R)/\sigma)]$$

Transfer of correlations

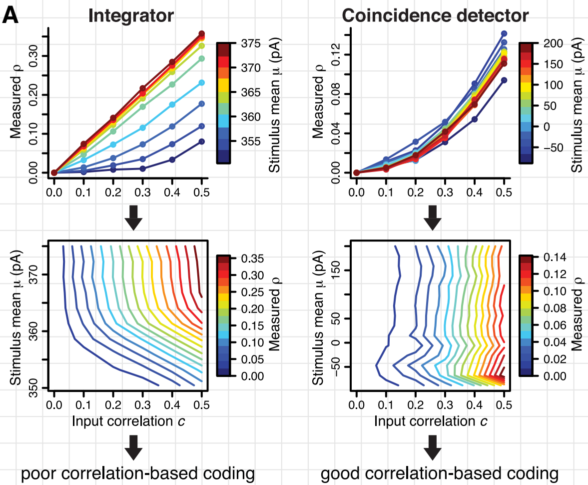
Firing rates change the output correlations





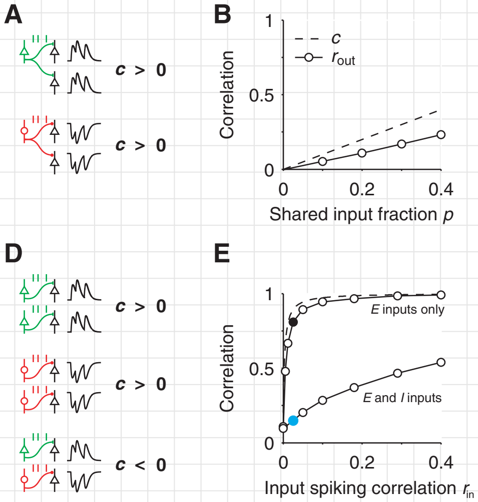
de La Rocha et al. 2007

Transfer of correlations: Coincidence detector vs integrators



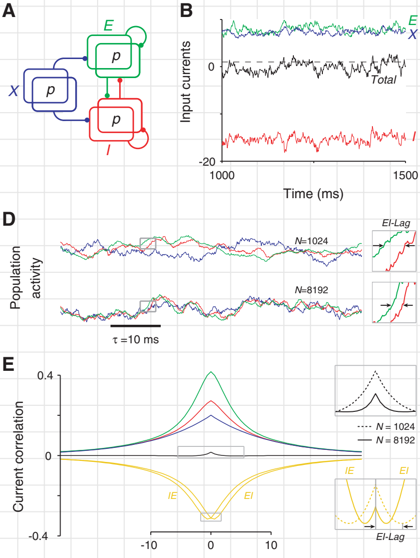
Hong, et al. 2012

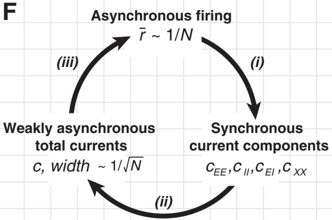
De-correlation by feedback inhibition



Renart et al. 2010, Tetzlaff et al. 2012

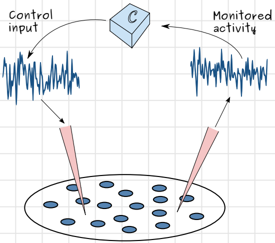
De-correlation by feedback inhibition

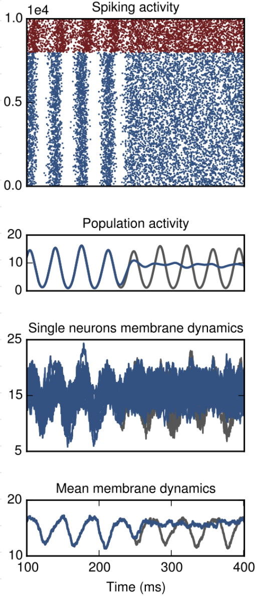




Renart et al. 2010, Tetzlaff et al. 2012

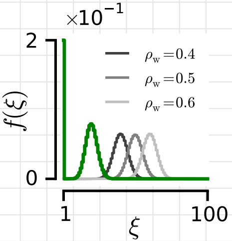
Even shared input can de-correlation: delayed feedback control

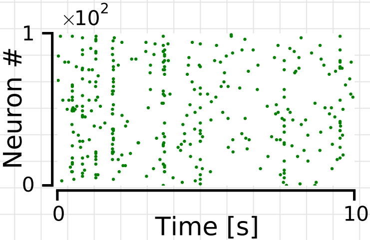




Vlachos et al. 2016

Many mechanisms to de-correlations





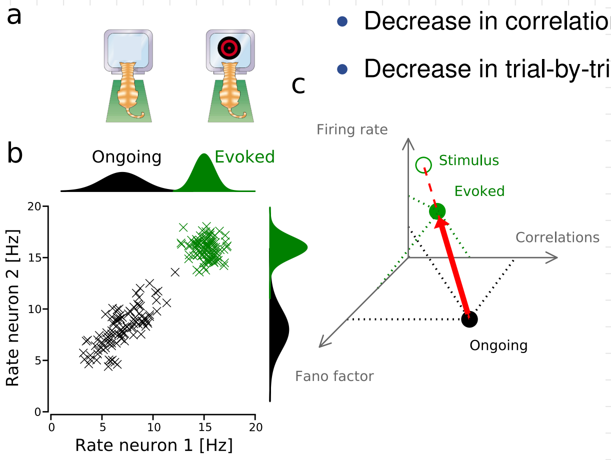
Potential reasons for de-correlations

- Sub-linear gain of correlation transfer
- Low firing rates
- Neuron integration properties
- Balance of excitation and inhibition

Potential reasons for correlations

- Common inputs
- Pooling of inputs
- Change in synaptic strengths
- Multiple stochastic synapses
- Daleian synapses

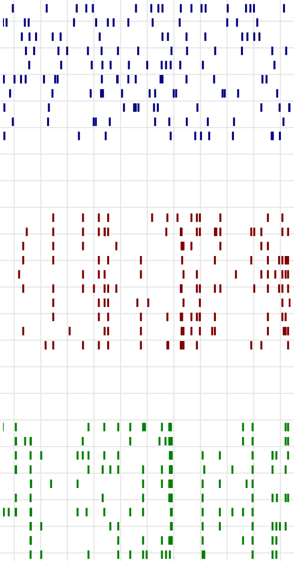
Features of the evoked activity



- Increase in the firing rate
- Decrease in correlations
- Decrease in trial-by-trial variability

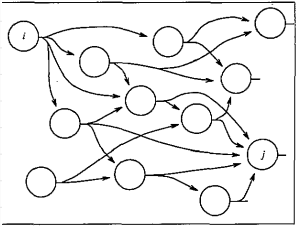
Bujan et al. 2015, Churchland et al. 2010, Oram 2012

Natural inputs have spatio-termporal correlations



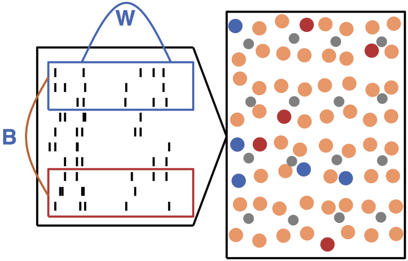
⇒ Natural stimuli imply correlated inputs

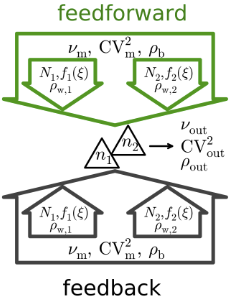
⇒ Convergent-divergent connections



Bienenstock 1995

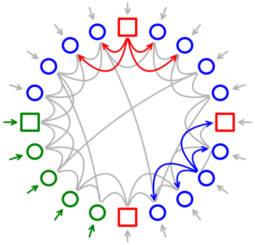
Parameterisation of the input activity





Bujan et al. 2015 J. Neurosci.

Evoked activity in response to correlated input



Bujan et al. 2015 J. Neurosci.

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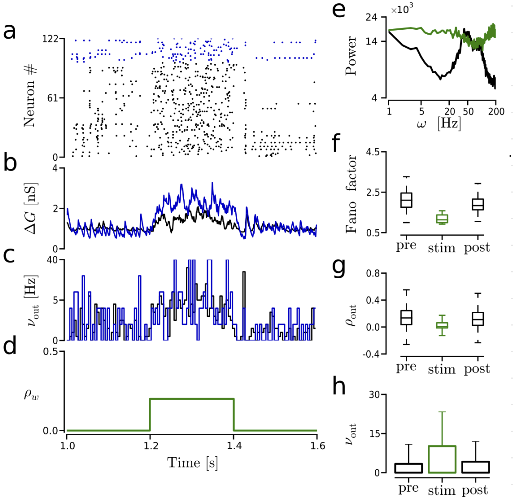

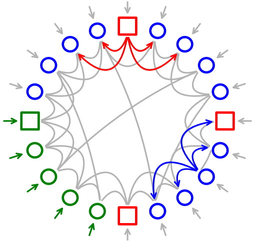

Bujan et al. 2015 J. Neurosci.

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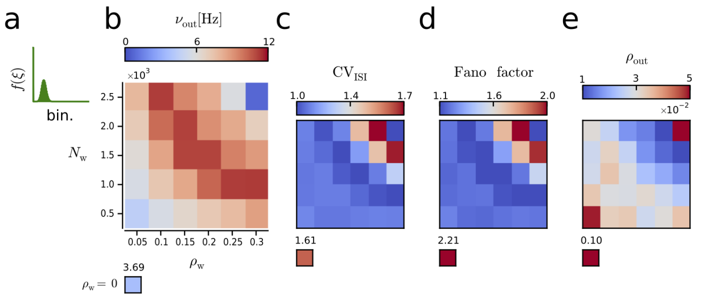
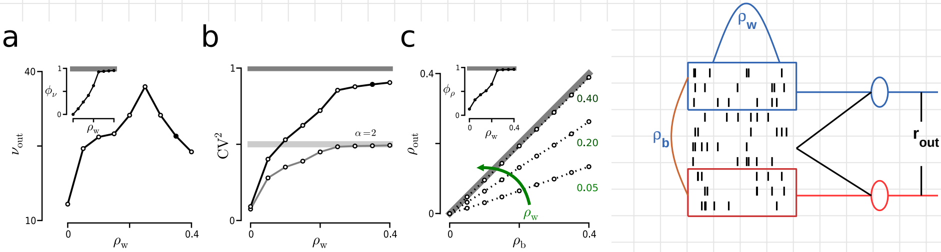


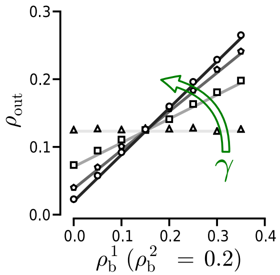
Bujan et al. 2015 J. Neurosci.

Transfer of correlation: Effect of within correlations

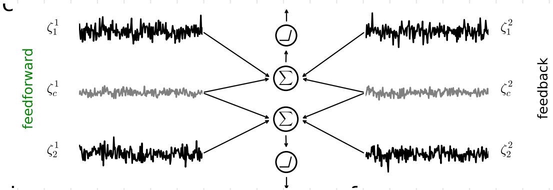


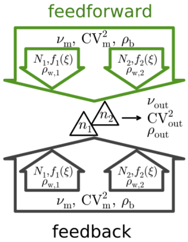
Bujan et al. 2015 J. Neurosci.

Interaction of the feedforward input with ongoing activity



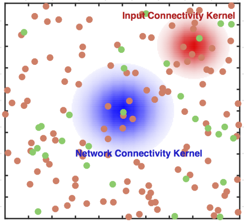
$$\gamma = \frac{\sigma_1}{\sigma_2}$$





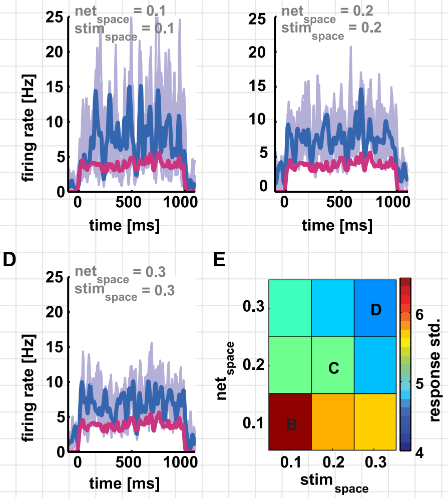
Bujan et al. 2015 J. Neurosci.

Interaction of the feedforward input with ongoing activity: Spatially structured network

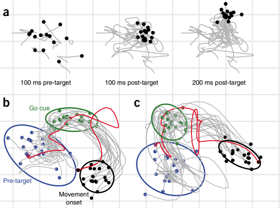


Network and input connectivity extents affect variability and correlations

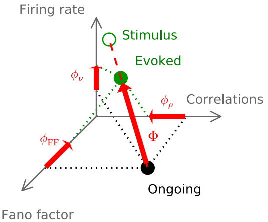
Schnepel et al. 2015



Summary: Input and ongoing activity interactions



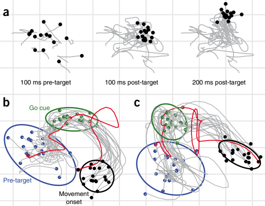
Churchland et al. 2010



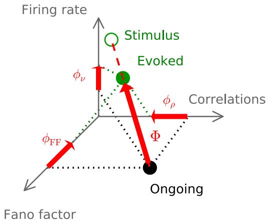
Bujan et al. 2015 J. Neurosci.

- ⇒ Input correlation are sufficient to explain the observed *increase* in firing rate *decrease* in variability and noise correlation
- ⇒ Depending on the state of the network response variability and noise correlation can both increase and decrease
- ⇒ The transfer of correlations or correlation susceptibility depends on various parameters, most importantly, slope of neuron transfer function, CV of the spike input and variance (correlations) of the input to the two neurons.
- ⇒ It is useful to divide correlations in two categories: 'W' and 'B' and it is consistent with clustered connectivity in the network
- ⇒ 'B' and 'W' correlations refer to $convergent$ and $divergent$ projections in the network
- ⇒ Variability and noise correlations depend on the stimulus

Summary: Input and ongoing activity interactions



Churchland et al. 2010



Bujan et al. 2015 J. Neurosci.

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Further reading

- Brunel N (2000) Dynamics of sparsely connected networks of excitatory and inhibitory spiking neurons *J. Comp. Neurosci.* 8:183:208
- Bujan A et al. (2015) Role of input correlations in shaping the variability and noise correlations of evoked activity in the neocortex. *J. Neurosci.* 35(22):8611–8625
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- Tetzlaff et al. (2012) Decorrelation of Neural-Network Activity by Inhibitory Feedback. *PLoS Comput Biol* 8(8): e1002596.