AI1103 Assignement 8

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Paper - Gate-2021-Statistics

Q1. Let X be a non-constant positive random variable such that $\mathbb{E}(X) = 9$. Then which of the following statesments is True?

1) $\mathbb{E}(\frac{1}{X+1}) > 0.1$ and $P(X \geq 10) \leq 0.9$
2) $\mathbb{E}(\frac{1}{X+1}) < 0.1$ and $P(X \geq 10) \leq 0.9$
3) $\mathbb{E}(\frac{1}{X+1}) > 0.1$ and $P(X \geq 10) > 0.9$
4) $\mathbb{E}(\frac{1}{X+1}) < 0.1$ and $P(X \geq 10) > 0.9$

Ans. We need to inequalities to solve the problem. The first one is Jensen's inequality, and the second one is Markov's inequality.
Jensen's inequality states that, for a random variable X and a convex function ϕ the below inequality holds,

$$\mathbb{E}[\phi(X)] \ > \ \phi(\mathbb{E}[X])$$

We need to show, $\phi(X) = \frac{1}{X+1}$ is convex. Now, $\phi'(X) = -\frac{1}{(1+X)^2}$ and $\phi"(X) = \frac{2}{(1+X)^3}$. We observe,

$$\phi"(X) > 0 \ for \ X > 0$$

So, ϕ is a convex function, and we can say

$$\mathbb{E}(\frac{1}{X+1}) > 0.1$$

Markov's inequality states that, for a non-negative random variable X and $a > 0$, the below inequality holds,

$$P(X \geq a) \leq \frac{\mathbb{E}(X)}{a}$$

Putting $a = 10$, we get

$$P(X \geq 10) \leq 0.9$$

So,

$$\mathbb{E}(\frac{1}{X+1}) > 0.1 \ and \ P(X \geq 10) \leq 0.9$$