AI1103 Assignement 3

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Q53. Given the observations $0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62$ from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$, which of the following is a maximum likelihood estimate for θ ?
1) 0.7
2) 0.9
3) 1.1
4) 1.3

Answer.

$$X \sim U(\theta - 0.2, \theta + 0.8)$$

Let the pdf be denoted by f, then

$$f(x|\theta) = \begin{cases} \frac{1}{(\theta+0.8)-(\theta-0.2)} = 1 & (\theta - 0.2 \leq x \leq \theta + 0.8) \\ 0 & \textit{(Otherwise)} \end{cases}$$

Now, the likelihood function for the observations $\underline{x} = \{x_1, x_2, \cdots, x_n\}$ is given by

$$\begin{aligned} L(\underline{x}|\theta) &= P(\underline{x}|\theta) \\ &= f(x_1|\theta) \times f(x_2|\theta) \times \cdots \times f(x_n|\theta) \qquad \textit{(i.i.d assumption)} \end{aligned}$$

So,

$$L(\underline{x}|\theta) = \begin{cases} 1 & when\ all(\underline{x}) \geq \theta - 0.2\ and\ all(\underline{x}) \leq \theta + 0.8 \\ 0 & \textit{(Otherwise)} \end{cases}$$

maximising the likelihood implies satisfying the following condition

$$all(\underline{x}) \geq \theta - 0.2\ and\ all(\underline{x}) \leq \theta + 0.8$$

which is same as satisfying

$$min(\underline{x}) \geq \theta - 0.2\ and\ max(\underline{x}) \leq \theta + 0.8$$

For the given question $min(\underline{x}) = 0.71$ and $max(\underline{x}) = 1.68$
So,

$$0.71 \geq \theta - 0.2 \ and \ 1.68 \leq \theta + 0.8$$

$$\implies 0.91 \geq \theta \ and \ 0.88 \leq \theta$$

$$\implies 0.88 \leq \theta \leq 0.91$$

The only option that satisfies the condition is the second option 0.9 (Answer)