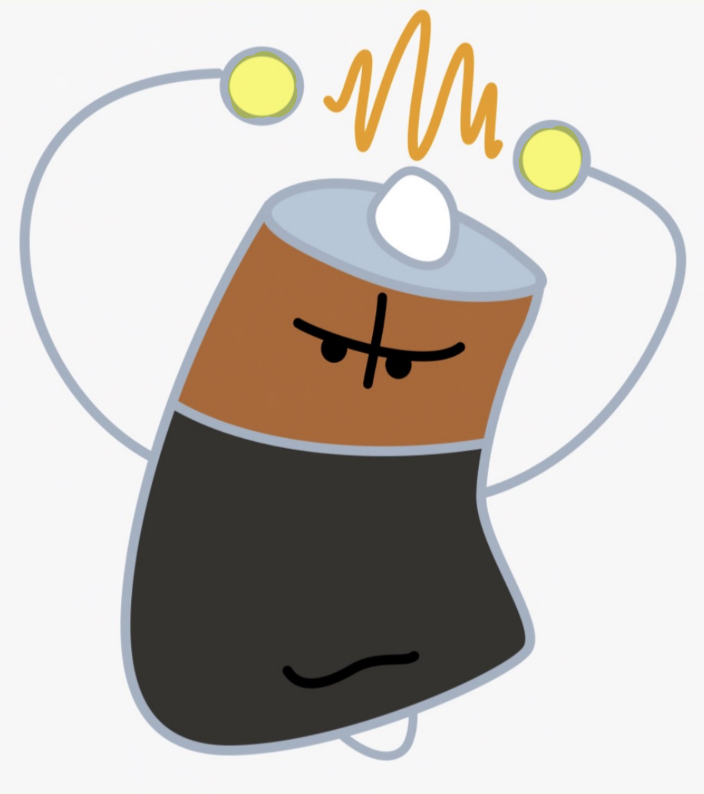
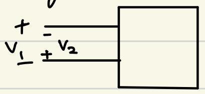
ANALOG ELECTRONIC

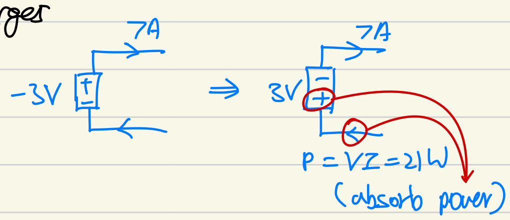


Voltage & Current

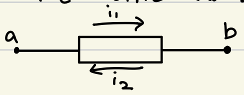
↳ Voltage (Electric force) : separation of charges



☆ $v_2 = -v_1$



↳ Current (Electric flow): motion of charges



☆ $i_2 = -i_1$

☆ – If current goes in from ⊖ve terminal / leaves from ⊕ve terminal

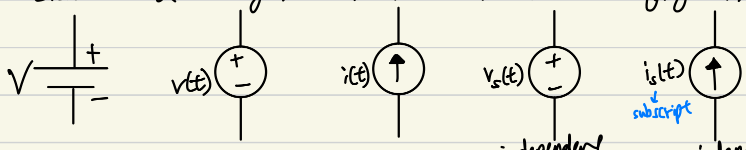
↳ source generating power → power ⊖ve no. (Active element → batt, voltage & current sources)

☆ – If current goes in from ⊕ve terminal / leaves from ⊖ve terminal

↳ source absorbing power → power ⊕ve no. (Passive element → resistor, capacitor & inductor)

Ideal Independent sources

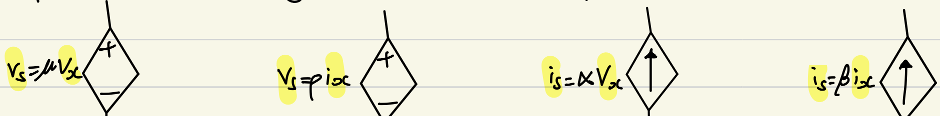
↳ establishes a voltage/current in circuit without relying on voltage/current somewhere else in circuit.



☆ lower case for time varying
upper case for constant (DC)

Ideal Dependent sources

↳ output is determined by voltage/current at a specific location in circuit



Series & Parallel Connections

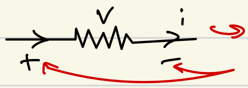
↳ Connections that are in series share the same current

↳ Connections that are in parallel share the same voltage

☆ resistor can also be neither series/parallel

Ohm's law

$v = iR$ → refers only to voltage & current across resistor ☆



↳ for resistor, current must flow from high to low potential

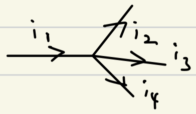
↳ Conductance ($G = \frac{1}{R}$)

$i = \frac{v}{R} = Gv$ Unit is Siemens (Ω^{-1}/℧)

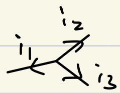
Power

1. $P = vi$
2. $P = i^2R$
3. $P = \frac{v^2}{R}$

Kirchhoff's Current Law (KCL)

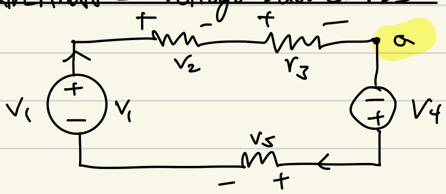


$i_1 = i_2 + i_3 + i_4$



$i_1 + i_2 + i_3 = 0 \rightarrow$ also can write this

Kirchhoff's Voltage Law (KVL)



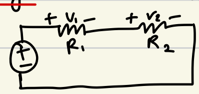
Starting from a: $+V_4 - V_5 + V_1 - V_2 - V_3 = 0$

Resistor

↳ in series → $R_{eff} = R_1 + R_2$

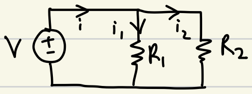
↳ parallel → $R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$

→ Voltage Division for resistors in series



$V_1 = \frac{R_1}{R_1 + R_2} V$ & $V_2 = \frac{R_2}{R_1 + R_2} V$

→ Current Division for resistors in parallel

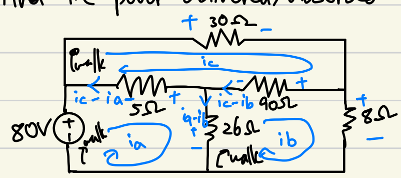


$i_1 = \frac{R_2}{R_1 + R_2} i$ & $i_2 = \frac{R_1}{R_1 + R_2} V$

Mesh Analysis

↳ Assume mesh current that flows around a mesh in a chosen direction

E.g. Find the power delivered/absorbed by the 80V source & power dissipated by resistors.



* If have a dependent voltage source → include it in KVL.

* If have independent current sources, need assume it has a voltage & add in the voltage to KVL

Mesh current equations:

☆ KVL →

Mesh i_a: $80 + 5(i_c - i_a) - 26(i_a - i_b) = 0$

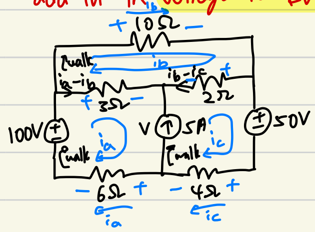
$80 + 5i_c - 5i_a - 26i_a + 26i_b = 0$

Mesh i_b: $26(i_a - i_b) + 90(i_c - i_b) - 8i_b = 0$

$26i_a - 26i_b + 90i_c - 90i_b - 8i_b = 0$

Mesh i_c: $-30 - 90(i_c - i_b) - 5(i_c - i_a) = 0$

$-30 - 90i_c + 90i_b - 5i_c + 5i_a = 0$



☆ Need 4 independent equations ↳ 4 unknowns

$+100 - 3(i_a - i_b) - V - 6i_a = 0$

$-10i_b - 2(i_b - i_c) + 3(i_a - i_b) = 0$

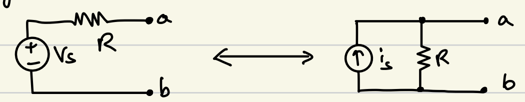
$+V + 2(i_b - i_c) - 50 - 4i_c = 0$

☆ $i_c = i_a + 5 \Rightarrow 5 = i_c - i_a$ (sort like net force)

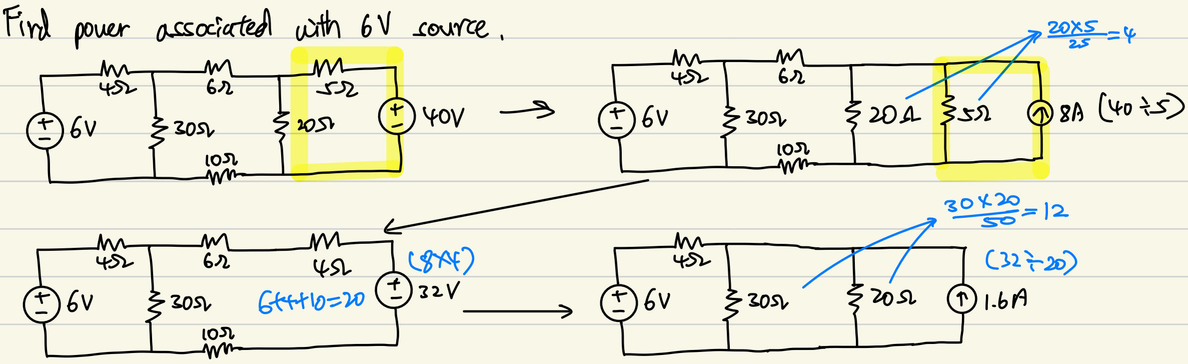
Norton & Thevenin's Theorem

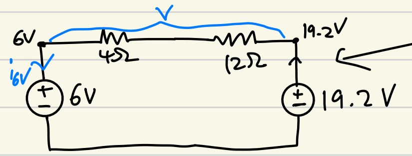
★ Source Transformation: $V_s = i_s R$ ★

voltage source in series with resistor = current source in parallel with resistor



E.g. Find power associated with 6V source.



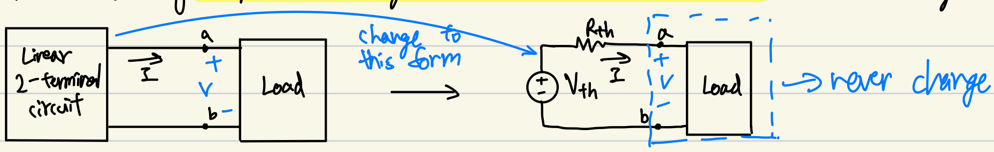


$19.2V - 6V =$ voltage across it.

$i_{6V} = \frac{19.2-6}{4+12} = 0.825A$ $\qquad P_{6V} = 6 \times 0.825 = 4.95W$ (absorbed)

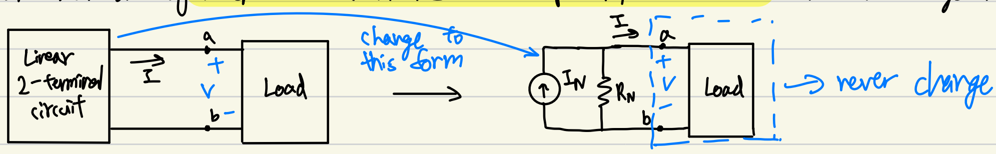
Thevenin's Theorem

↳ Replace any linear network by independent voltage source in series with resistor → current-voltage relationship unchanged.



Norton's Theorem

↳ Replace any linear network by independent current source in parallel with resistor → current-voltage relationship unchanged.



★★ $R_N = R_{th}$ $\qquad I_N = \frac{V_{th}}{R_{th}}$

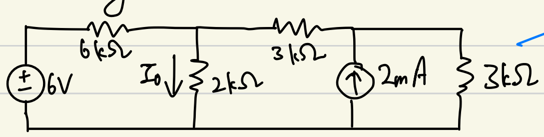
Method A for Determining Equivalent Circuit (No dependent sources) ★★

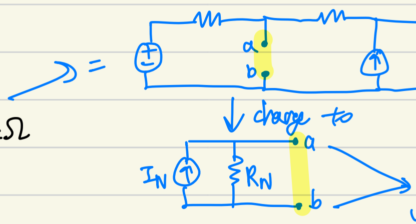
To solve for R_{th} → ignore independent sources

↳ turning off independent voltage source, $V_s = 0$ → replace with wire → short circuit

↳ turning off independent current source, $I_s = 0$ → replace with open-circuit

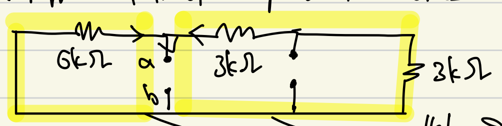
E.g. Find I_o using Norton's theorem





a & b defined as where you want to get current

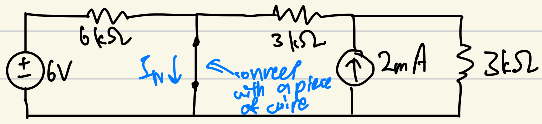
1. Find R_N → kill all independent sources



$$R_N = \frac{(3+3)\times 6}{(3+3)+6} = 3k\Omega$$

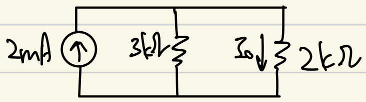
parallel → use current analysis to confirm

2. Use mesh analysis to find I_N / other methods



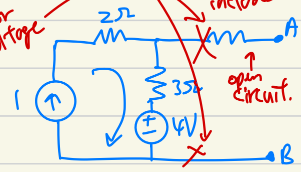
$$I_N = \frac{6}{6} + \frac{3}{3+3} \times 2$$
$$= 2mA$$

3. Convert back to 2kΩ.



$$I_o = \frac{3}{3+2} \times 2$$
$$= 1.2mA$$

☆ But for voltage



do not include these so just one circuit

open circuit.

voltage across 1 is V_{AB}.

☆ Steps same for Thevenin Theorem, but just find V_{th} → which is across a, b → can use mesh analysis also

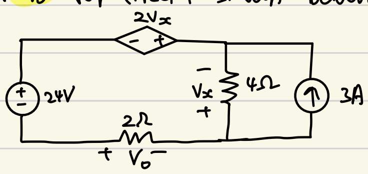
Method B for Determining Equivalent Circuit (have dependent sources)

↳ Terminal open-circuit voltage V_{oc} & short-circuit current i_{sc}.

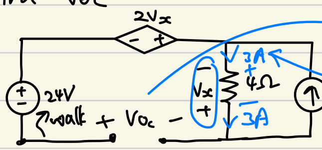
↳ $R_{th} = \frac{V_{oc}}{i_{sc}}$ (Use Thevenin Theorem)

↳ similar to method A.

E.g. Find V_o for circuit shown below.



1. Find V_{oc} (V_{th})



cannot use mesh analysis → no current (open circuit)

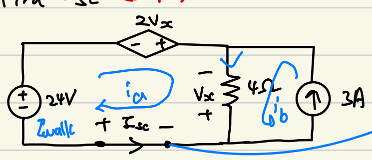
but still have voltage

→ no current does not mean won't kill you

$V_x = -3 \times 4 = -12V$ $\quad 2V_x = -24V$

KVL : $24 - 24 - 12 + V_{oc} = 0$ → $V_{oc} = 12V$

2. Find I_{sc} (I_N)



replace with wire → use mesh analysis

$24 + 2V_x + V_x = 0$ → $3V_x = -24$ → $V_x = -8V$

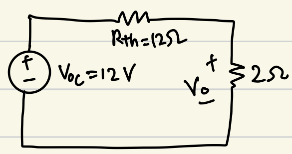
$V_x = 4 \times -(i_b + i_a)$

$2 = 3 + i_a$ → $i_a = -1A$

$I_{sc} = 1A$

3. Find R_{th} & combine Thevenin equivalent circuit to load & solve V_o.

$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{12}{1} = 12\Omega$ $\quad V_{th} = V_{oc} = 12V$



$$V_o = \frac{2}{2+12} \times 12$$
$$= \frac{12}{7}V$$

Maximum Power Transfer

↳ Given a DC circuit → maximum power transfer takes place when the load resistance $R_L = R_{th}$.

Proof:

$$P_L = i^2 R_L = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \times R_L$$

P_L is max when:

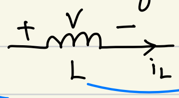
$$\frac{dP_L}{dR_L} = V_{th}^2\left(\frac{(R_{th}+R_L)^2 - R_L \times 2(R_{th}+R_L)}{(R_{th}+R_L)^4}\right)^2 = 0$$

$$(R_{th}+R_L)^2 = 2R_L(R_{th}+R_L)$$

$$R_L = R_{th}$$

maximum power: $P_{L,max} = \left(\frac{V_{th}}{2R_L}\right)^2 \quad R_L = \frac{V_{th}^2}{4R_L}$

Inductors (current flowing through)



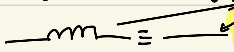
→ inductor (unit: H)

voltage across inductor ← $V = L\frac{di}{dt}$

1) DC condition

i_L = constant (current)

$\frac{di_L}{dt} = 0 \rightarrow V = 0$ → act as a piece of wire

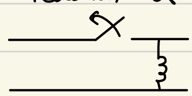


2) Continuity

$V \neq \infty \longrightarrow \frac{di_L}{dt} \neq \infty$

⇒ i_L cannot change instantaneously

↳ reaction of inductor slow



$i(0^-) = i(0) = i(0^+)$

before switch triggered — after switch open

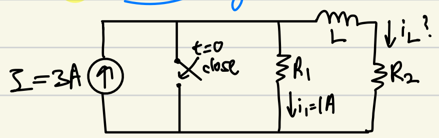
− Power & Energy

$p = vi = Li\frac{di}{dt} \qquad p = \frac{dw}{dt}$

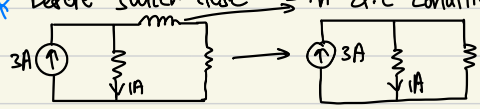
work done → $w_L(t_0) = \int_{-\infty}^{t_0} p\,dt = 0.5Li^2(t_0)$ as $i(-\infty) = 0$

E.g. Given that $i_1 = 1A$ at $t<0$. Find current i_L through the R_2 resistive load when switch is closed at $t = 0^+$.

→ very short time after switch is closed



Before switch close → in d.c condition (3A condition),



can be replaced be wire

Before switch closed at $t=0$,

$I = 3A \qquad i_L(0^-) = 3 - i_1 = 2A$

At $t=0^+$, current shorted, $i_1(0^+) = 0A$,

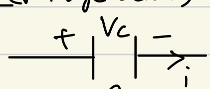
$i_L(0^+) = i_L(0^-) = 2A$ → inductor cannot react immediately

− Inductor in series (same $\frac{di}{dt}$) ⟶ $L_{eq} = L_1 + L_2 + \ldots + L_n$

− Inductor in parallel ⟶ $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}$

→ like resistors

Capacitors (voltage across)



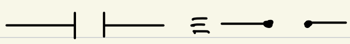
→ if want high current flowing through → need Δ voltage very fast

→ Capacitor (unit F)

$i = C\frac{dV_c}{dt}$

1) DC condition

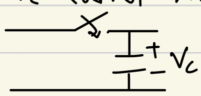
V_c = constant → $\frac{dV_c}{dt} = 0$ → $i = 0$ → act as a open circuit



2) Continuity

$i \neq \infty \rightarrow \frac{dV_c}{dt} \neq \infty$

⇒ V_c cannot change instantaneously



$V_c(0^-) = V_c(0) = V_c(0^+)$

↳ capacitor react slow to Δ

- Capacitor in series → $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$
- Capacitor in parallel → $C_{eq} = C_1 + C_2 + \dots + C_n$

→ opposite of resistor

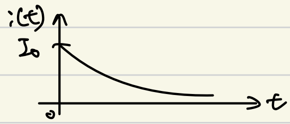
Natural Response of an RL Circuit

RL → inductor

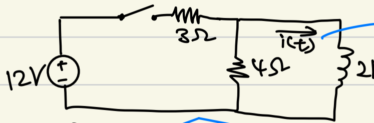
↳ Natural response ⇒ response of system when circuit has reached its constant voltage & currents & is disconnected from any power source

current through inductor ← ★ $i(t) = i(0)e^{-\frac{t}{\tau}}$, $\tau = \frac{L}{R}$ → R that L provide energy to

$i(0)$ → initial current through inductor at $t=0$

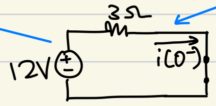


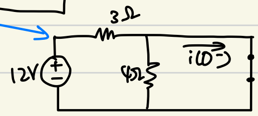
E.g. The switch in the circuit shown has been closed for a long time. At $t=0$, the switch is opened. Calculate $i(t)$ for $t>0$.



→ find how current will die

if can recognise that 4Ω resistor is short-circuited ↳ current choose less resistance route





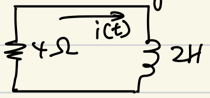
→ replace with wire → DC source

→ mesh analysis

$i(0^-) = \frac{12}{3} = 4A$

Since current through inductor cannot change abruptly, $I_0 = i(0^+) = i(0^-) = 4A$.

- When $t>0$ → voltage source & 3Ω resistor cut off



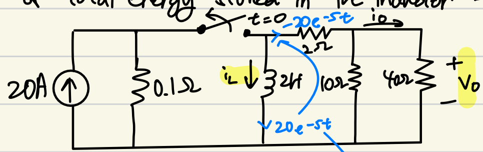
$i(t) = i(0^+)e^{-\frac{t}{\tau}}$ $\tau = \frac{2}{4}$

$= 4e^{-2t} A, \ t>0$

E.g. The switch in the circuit has been closed for a long time before open at $t=0$. Find

a) $i_L(t)$ for $t \geq 0$ b) $i_o(t)$ for $t \geq 0^+$ c) $v_o(t)$ for $t \geq 0^+$

d) Percentage of total energy stored in the inductor that is dissipated in the 10Ω resistor.



a) At $t=0^-$, inductor acts as short circuit causing all of the resistors to be bypassed.

$i_L(0^-) = 20A$ At $t=0^+$, $i_L(0^+) = i_L(0^-) = 20A$

$R_{eq} = 2 + (10 // 40) = 10\Omega$ $\tau = \frac{L}{R_{eq}} = \frac{2}{10} = 0.2s$

$i_L(t) = 20e^{-5t} A, \ t \geq 0$

b) Using current division,

$i_o = -i_L \frac{10}{10+40}$ → 10: opposite of 40Ω

$i_o(t) = -4e^{-5t} A$ → $\frac{10}{50} \times (-20e^{-5t})$

c) $V_o = 40i_o$ → $V_o(t) = -160e^{-5t} V, \ t \geq 0^+$

d) Initial energy stored in 2H inductor,

$W_{2H}(0) = \frac{1}{2}Li^2(0)$ → $(20e^{-0})^2$

$= \frac{1}{2} \times 2 \times 20^2 = 400J$

Power dissipated by 10Ω resistor, $P_{10\Omega} = \frac{V_o^2}{10} = 2560e^{-10t} W, \ t \geq 0^+$

Total energy dissipated in 10 Ω resistor,

$$W_{10\Omega} = \int_0^\infty 2560e^{-10t}\,dt = 256\text{ J}$$

Percentage of energy dissipated in the 10Ω resistor,

$$W_{10\Omega} = \frac{250}{400} \times 100 = 62.5\%$$

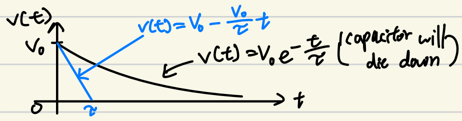
Natural Response of an RC Circuit

capacitor

voltage across capacitor

$$V = V_0 e^{-\frac{t}{\tau}}, \quad \tau = RC, \quad t \geq 0$$

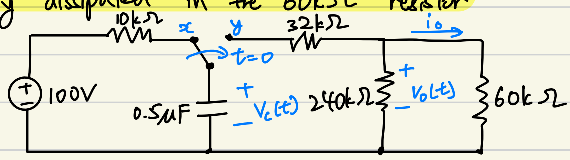
initial voltage across capacitor



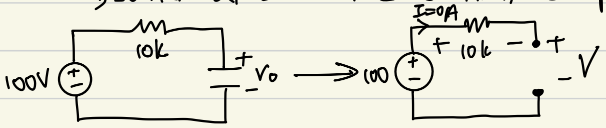
E.g. The switch in the circuit shown has been in position x for a long time. At t=0, the switch moves instantaneously to position y. Find

a) $v_c(t)$ for $t \geq 0$ b) $v_o(t)$ for $t \geq 0^+$ c) $i_o(t)$ for $t \geq 0^+$

d) the total energy dissipated in the 60kΩ resistor



a) At $t = 0^-$, switch at x ⇒ D.C condition → capacitor becomes open circuit



$$R_{eq} = 32k + (240k // 60k) = 80k\Omega$$

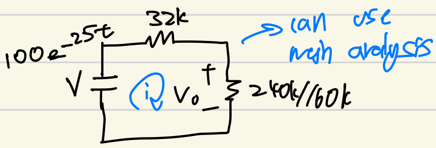
$$\tau = R_{eq}C = 80k \times 0.5\mu = 40ms$$

$$v_c(t) = v_c(0^+)e^{-\frac{t}{\tau}} = 100e^{-25t}\,V, \quad t \geq 0$$

b) Find $v_o(t)$ for $t \geq 0^+$, we employ voltage division/mesh

$$V_0 = \frac{240k//60k}{32k + (240k//60k)} V_c = \frac{48}{80} V_c$$

$$= 60e^{-25t}\,V, \quad t \geq 0^+$$



c) Take voltage across 60kΩ,

$$i_0 = \frac{V_0}{60k} = e^{-25t}\,mA, \quad t \geq 0^+.$$

d) Power dissipated in 60kΩ resistor

$$P_{60k\Omega}(t) = i_o^2 \times 60k\Omega = 60e^{-50t}\,mW, \quad t \geq 0^+$$

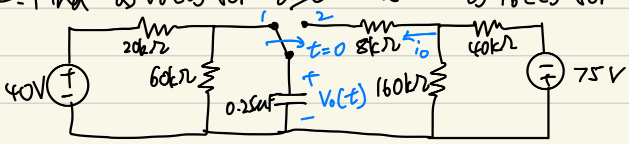
work done (energy) → $$w_{60k\Omega} = \int_0^\infty 60e^{-50t}\,dt = 1.2mJ$$

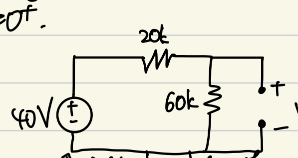
Step Response

- when smt has changed in circuit (remove a resistor) → see how current/voltage through conductor/capacitor react
- when source (voltage/current) is suddenly applied to a storage device (capacitor/inductor)
- anything that is not a natural response

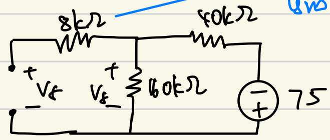
memorise ★ (unknown variable as a function of time) = (final value of the variable) + [(initial value of variable) − (final value of variable)] × $e^{-\frac{[t - (\text{switching time})]}{\tau}}$

E.g. The switch in the circuit has been in position 1 for a long time. At time $t=0$, it moves to position 2. Find a) $V_o(t)$ for $t \geq 0$ & b) $i_o(t)$ for $t \geq 0^+$.





↳ initial value of variable



→ no current & no p.d.

a) Initial value of V_o is given by

$$V_o = \frac{60k}{20k+60k} \times 40 = 30V$$

After switch moves to position 2, at $t \to \infty$,

$$V_\infty = \frac{160k}{40k+160k} \times -75 = -60V$$

$$V_o(t) = V_\infty + [V_o - V_\infty]\, e^{-\frac{(t-0)}{\tau}} \qquad \tau = RC$$

$$= -60 + [30-(-60)]e^{-\frac{t}{0.01}}$$

$$= -60 + 90e^{-100t}\ V, \ t \geq 0$$

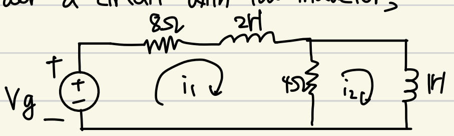
b) $i_o(t) = C\frac{dv_o}{dt}$

$$= 0.25\mu \times -100 \times 90e^{-100t}$$

$$= -2.25e^{-100t}\ mA, \ t \geq 0^+$$

Circuits with Two Energy Storage Elements

Consider a circuit with two inductor → step response



- Mesh-current equations:

$$V_g = 8i_1 + 2\frac{di_1}{dt} + 4(i_1 - i_2) \quad \text{—— ①}$$

$$4(i_2 - i_1) + 1\frac{di_2}{dt} = 0 \longrightarrow i_1 = 0.25\left(\frac{di_2}{dt} + 4i_2\right) \quad \text{—— ②}$$

sub ② into ①

$$V_g = \frac{8}{4}\left(\frac{di_2}{dt} + 4i_2\right) + \frac{2}{4}\frac{d}{dt}\left(\frac{di_2}{dt} + 4i_2\right) + \frac{4}{4}\left(\frac{di_2}{dt} + 4i_2\right) - 4i_2$$

$$2V_g = \frac{d^2 i_2}{dt^2} + 10\frac{di_2}{dt} + 16i_2 \rightarrow \text{use Laplace transform tool}$$

Laplace Transforms

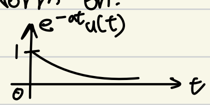
↳ Given time-domain function $f(t)$, its Laplace transform defined as $F(s)$.

$$\mathcal{L}\{f(t)\} = F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

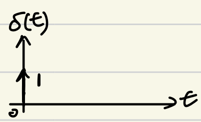
→ formula to perform Laplace Transform on

E.g. Find Laplace transform on:

a) $e^{at}u(t)$, $a \geq 0$



b) $\delta(t)$



$$\mathcal{L}\{e^{-at}u(t)\} = \int_{0^-}^{\infty} e^{-at} \cdot e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} e^{-(a+s)t}\, dt$$

$$= \left[-\frac{1}{a+s}e^{-(a+s)t}\right]_{0^-}^{\infty}$$

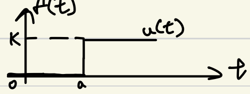
$$= -\frac{1}{a+s}(0 - (1)) = \frac{1}{s+a}$$

$$\mathcal{L}\{\delta(t)\} = \int_{0^-}^{\infty} \delta(t) e^{-st}\, dt$$

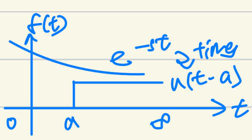
$$= e^{-st}\big|_{t=0}$$

$$= 1$$

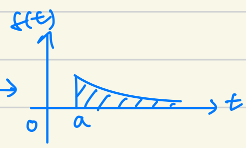
E.g. Laplace transform where $f(t) = \begin{cases} 0 & \text{for } t < a \\ K & \text{for } t > a \end{cases}$



$$f(t) = Ku(t-a)$$



→



$$\mathcal{L}\{K\cdot u(t-a)\} = K\int_0^\infty u(t-a)e^{-st}\,dt$$
$$= K\int_a^\infty 1\cdot e^{-st}\,dt$$
$$= -\frac{K}{s}\left[e^{-st}\right]_a^\infty$$
$$= \frac{K}{s}\left[e^{-\infty} - e^{-as}\right] = \frac{Ke^{-as}}{s}$$

Useful Laplace Transform Pairs

	$f(t)$ transform	$F(s)$
Unit impulse	δ	1
Unit step	$u(t)$	$\frac{1}{s}$
Unit ramp	$r(t) = t$	$\frac{1}{s^2}$
Polynomial	$t^n,\ n \in \mathbb{Z}^+$	$\frac{n}{s^{n+1}}$
Exponential	e^{-at}	$\frac{1}{s+a}$
t-Multiplication exp.	te^{-at}	$\frac{1}{(s+a)^2}$

	$f(t)$	$F(s)$
Sine	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
Damped Sine	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
Damped Cosine	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Laplace Transform Properties

1. Linearity

$$\mathcal{L}\{a_1 f_1(t) + a_2 f_2(t)\} = a_1 F_1(s) + a_2 F_2(s)$$

E.g. Find Laplace transform of the function $\cos(\omega t)$ using linear property & Euler's formula

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \qquad \left[e^{j\omega t} = \cos\omega t + j\sin\omega t \ \ \& \ \ e^{-j\omega t} = \cos\omega t - j\sin\omega t\right]$$

$$\mathcal{L}\{\cos\omega t\} = 0.5\,\mathcal{L}\{e^{j\omega t} + e^{-j\omega t}\}$$
$$= 0.5\left\{\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right\} = \frac{s}{s^2+\omega^2}$$

2. Differentiation

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0^-)$$ [lapase transform of a formula differentiated = s(lapase transform) − initial condition]

$$\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\} = s\,\mathcal{L}\{f'(t)\} - f'(0^-) = s^2F(s) - sf(0^-) - f'(0^-)$$

E.g. Find Laplace transform of $\sin(\omega t)$ $\qquad \frac{d\cos\omega t}{dt} = -(\sin(\omega t))\,\omega$

$$\mathcal{L}\{\sin\omega t\} = \mathcal{L}\left\{-\frac{1}{\omega}\frac{d\cos(\omega t)}{dt}\right\} = -\frac{1}{\omega}\mathcal{L}\left\{\frac{d\cos(\omega t)}{dt}\right\}$$
$$= -\frac{1}{\omega}\left[sF(s) - f(0^-)\right]$$
$$= -\frac{1}{\omega}\left[s\frac{s}{s^2+\omega^2} - \cos(0^-)\right] \qquad \cos(0^-) = 1$$
$$= \frac{\omega}{s^2+\omega^2}$$

E.g. Assuming that $y(0^-) = 3$, $y'(0^-) = 2$, find $Y(s)$ from $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 12$ $\rightarrow 12\times\delta$

$$s^2Y(s) - sy(0^-) - y'(0^-) + 3[sY(s) + y(0^-)] + 2Y(s) = \frac{12}{s}$$
$$Y(s)[s^2+3s+2] = \frac{3s^2+11s+12}{s}$$
$$Y(s) = \frac{3s^2+11s+12}{s(s+1)(s+2)}$$

3. Integration

$$\mathcal{L}\left\{\int_{0^-}^{t} f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$ → use lapase response & then divide by s

E.g. Use Laplace transform to find $V(s)$ if

$$v(t) + 1.6\int_0^t v(x)\,dx + 0.025\frac{dv(t)}{dt} = 5\cos 10t, \quad \text{assume } v(0^-) = 0$$

$$V(s) + 1.6\frac{V(s)}{s} + 0.025\left[sV(s) - v(0^-)\right] = \frac{5s}{s^2+100}$$

$$V(s)\left[0.025s^2 + s + 1.6\right] = \frac{5s^2}{s^2+100}$$

$$V(s) = \frac{200s^2}{(s^2+40s+64)(s^2+100)}$$

Common Laplace Transform Properties

	f(t)	F(s)
Linearity	$c_1 f_1(t) + c_2 f_2(t)$	$c_1 F_1(s) + c_2 F_2(s)$
Differentiation	$\frac{d}{dt} f(t)$	$sF(s) - f(0^-)$
n-fold Differentiation	$\frac{d^n}{dt^n} f(t)$	$s^n F(s) - s^{n-1} f(0^-) - \ldots - f^{n-1}(0^-)$
Integration	$\int_{0^-}^{t} f(\tau) d\tau$	$\frac{F(s)}{s}$
t-Multiplication	$tf(t)$	$-\frac{d}{ds} F(s)$
n-fold t-Multiplication	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
Time shift	$f(t-t_0)u(t-t_0),\ t_0 > 0$	$e^{-st_0} F(s)$
Frequency shift	$e^{-s_0 t} f(t)$	$F(s+s_0)$
Time-frequency scaling	$f(ct),\ c > 0$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
Periodic Function	$f(t) = f_1(t)u(t) + f_1(t-T)u(t-T) + \ldots$	$\frac{1}{1-e^{-sT}} F_1(s)$

Inverse Laplace Transform

↳ transform from $F(s)$ to $f(t)$ → first decompose $F(s)$ into simple terms using Partial Fraction Expansions

1. Simple Poles

$$F(s) = \frac{N(s)}{(s+p_1)(s+p_2)\ldots(s+p_n)} \longrightarrow F(s) = \frac{K_1}{s+p_1} + \frac{K_2}{s+p_2} + \ldots + \frac{K_n}{s+p_n} \longrightarrow \text{convert back to } f(t)$$

$$K_i = \left[(s+p_i) \times F(s)\right]\Big|_{s=p_i}$$

E.g. $F(s) = \frac{96(s+5)(s+12)}{s(s+8)(s+6)}$, find laplace transform of $F(s)$.

$$F(s) = \frac{K_1}{s} + \frac{K_2}{s+8} + \frac{K_3}{s+6}$$

Use cover-up rule,

$$K_1 = \left[s \times F(s)\right]\Big|_{s=0} = \left[\cancel{s} \times \frac{96(s+5)(s+12)}{\cancel{s}(s+8)(s+6)}\right]\Big|_{s=0}$$

$$= \frac{96 \times (0+5) \times (0+12)}{(0+8) \times (0+6)} = 120$$

$$K_2 = \left[(s+8) \times \frac{96(s+5)(s+12)}{s(s+8)(s+6)}\right]\Big|_{s=-8} = -72$$

$$K_3 = \left[(s+6) \times \frac{96(s+5)(s+12)}{s(s+8)(s+6)}\right]\Big|_{s=-6} = 48$$

$$F(s) = \frac{120}{s} - \frac{72}{s+8} + \frac{48}{s+6} \quad \text{giving } f(t) = \mathcal{L}\{F(s)\} = \left[120 - 72e^{-8t} + 48e^{-6t}\right]u(t)$$

2. Order of Numerator ≥ Order of Denominator

Given $F(s) = \frac{x^2+x+7}{x+8} \longrightarrow F(s) = Q(s) + \frac{R(s)}{D(s)}$

E.g. Find $f(t)$ given $F(s) = \frac{5s^2+29s+32}{(s+2)(s+4)}$

$$F(s) = 5 - \frac{s+8}{(s+2)(s+4)}$$

$$= F_1(s) + F_2(s)$$

$$F_2(s) = \frac{K_1}{s+2} + \frac{K_2}{s+4}$$ [do it like above]

$$\begin{array}{r} 5 \\ s^2+6s+8 \overline{)\ 5s^2+29s+32} \\ -\ \ 5s^2+30s+40 \\ \hline -s-8 \end{array}$$

$$f(t) = \mathcal{L}^{-1}\{F_1(s) + F_2(s)\} = f_1(t) + f_2(t)$$
$$= 5\delta(t) - 3e^{-2t}u(t) + 2e^{-4t}u(t)$$

3. Repeated Poles

$$F(s) = \frac{N(s)}{(s+p)^n} \rightarrow F(s) = \frac{K_a}{(s+p)^n} + \frac{K_{a+1}}{(s+p)^{n-1}} + \ldots + \frac{K_{a+n-1}}{(s+p)^1}$$

$$\left.\begin{aligned} K_a &= (s+p)^n F(s)|_{s=-p} \\ K_{a+1} &= \frac{1}{1!}\frac{d}{ds}\{(s+p)^n F(s)\}|_{s=-p} \\ K_{a+2} &= \frac{1}{2!}\frac{d^2}{ds^2}\{(s+p)^n F(s)\}|_{s=-p} \end{aligned}\right\} \text{if } n=3$$

E.g. Find $f(t)$ given that $F(s) = \frac{100(s+25)}{s(s+5)^3}$

$$F(s) = \frac{K_1}{s} + \frac{K_2}{(s+5)^3} + \frac{K_3}{(s+5)^2} + \frac{K_4}{(s+5)^1} \rightarrow \text{change to partial fractions}$$

$K_1 = s \cdot F(s)|_{s=0} = 20$

$K_2 = (s+5)^3 \times F(s)|_{s=-5} = -400$

$K_3 = \frac{1}{1!}\frac{d}{ds}\{(s+5)^3 \times F(s)\}|_{s=-5}$

$= \frac{d}{ds}\left\{\frac{100(s+25)}{s}\right\}|_{s=-5}$

$= 100\left\{\frac{s(1) - (s+25)(1)}{s^2}\right\}|_{s=-5} = -100$

$K_4 = \frac{1}{2!}\frac{d^2}{ds^2}[(s+5)^3 \times F(s)]|_{s=-5}$

$= \frac{1}{2!}\frac{d^2}{ds^2}\frac{100(s+25)}{s}|_{s=-5}$

$= 50\frac{d}{ds}\left[\frac{-25}{s^2}\right]|_{s=-5} = -20$

$F(s) = \frac{20}{s} + \frac{-400}{(s+5)^3} + \frac{-100}{(s+5)^2} - \frac{20}{(s+5)^1}$

$\therefore f(t) = \left[20 - 400\frac{t^2}{2!}e^{-5t} - 100\frac{t}{1!}e^{-5t} - 20e^{-5t}\right]u(t)$

$= [20 - 200t^2e^{-5t} - 100te^{-5t} - 20e^{-5t}]u(t)$

※ to power $3 \rightarrow t^2$

4. Distinct Complex Poles

$F(s) = \frac{N(s)}{(s+p_i)(s+p_i^*)}$, $p_i = -(\alpha + j\beta)$ & $p_i^* = -(\alpha - j\beta)$

$F(s) = \frac{K}{s+p_i} + \frac{K^*}{s+p_i^*} \rightarrow K = (s+p_i)F(s)|_{s=-p_i} = |K|\angle\theta$

$\sqrt{a^2+(bi)^2}$ of numerator

argument of numerator

$\{\mathcal{L}\}^{-1}\{F(s)\} = 2|K|e^{\alpha t}\cos(\beta t + \theta)u(t)$ → ⊖ve of real part $(3 \rightarrow -3)$

E.g. Find the inverse Laplace transform of $F(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)}$

$s = -6$ & $\frac{-6 \pm \sqrt{36-100}}{2}$

$= -6$ & $-3 \pm j4$ → like i

$p_1 = 6$, $p_2 = 3 - j4$, $p_2^* = 3 + j4$, ⊖ve $\alpha = -3$ & $\beta = 4$

$F(s) = \frac{K_1}{s+6} + \frac{K_2}{s+p_2} + \frac{K_2^*}{s+p_2^*}$

$K_1 = (s+6)F(s)|_{s=-6} = -12$

$K_2 = (s+p_2)F(s)||_{s=-p_2}$

$= \left[(s+3-j4) \times \frac{100(s+3)}{(s+6)(s+3-j4)(s+3+j4)}\right]|_{s=-p_2=-3+j4}$

$= \left[\frac{100(-p_2+3)}{(-p_2+6)(-p_2+p_2^*)}\right]|_{-p_2=-3+j4}$

$= \frac{100(j4)}{(3+j4)(j8)} = 10\angle -53.13°$

$\therefore f(t) = [K_1e^{-6t} + 2|K_2|e^{\alpha t}\cos(\beta t + \theta)u(t)$

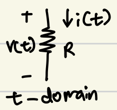
$= [-12e^{-6t} + 20e^{-3t}\cos(4t - 53.13°)]u(t)$

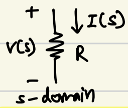
$-3t \rightarrow \alpha = -\alpha$

$\beta = 4$ (don't care about ⊕ve/⊖ve)

Application of Laplace Transform for Circuit Analysis

1. Resistor (R) in s-Domain



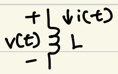


Laplace transform: $v(t) = i(t) \times R \longrightarrow V(s) = I(s) \times R$

Question given for final
a) find initial condition
b) convert circuit to Laplace domain
c) Find $V(s)$ or $I(s)$ mesh/KVL/KCL
d) Find $v(t)$ or $i(t)$ [12 mark]

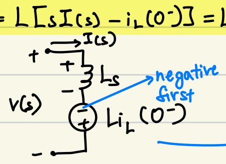
2. Inductor (L) in the s-Domain

In time domain $\longrightarrow v(t) = L\frac{di(t)}{dt}$



Taking Laplace transform on both sides, $V(s) = L[sI(s) - i_L(0^-)] = LsI(s) - Li_L(0^-)$

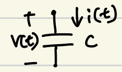
↳ after converting to laplace
↳ treat storage element (inductor/capacitor) like resistors



→ no initial condition → 0V like wire

3. Capacitor (C) in the s-Domain

In time domain ⟶ $i(t) = C\frac{dv(t)}{dt}$

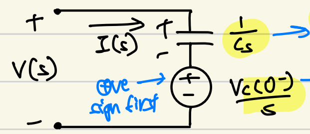


Taking Laplace transform, $I(s) = C[sV(s) - v_c(0^-)]$

$$= CsV(s) - Cv_c(0^-)$$

$$V(s) = \frac{1}{Cs}I(s) + \frac{v_c(0^-)}{s}$$

$$V(s) = \frac{I(s) + Cv_c(0^-)}{Cs}$$



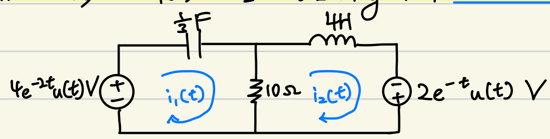
$\frac{1}{Cs}$ → can be treated as resistor

$\frac{v_c(0^-)}{s}$ → initial voltage

Circuit Solutions Without Initial Conditions

E.g. For circuit shown below, find $i_1(t)$ & $i_2(t)$ assuming that no initial energy is stored in the circuit.

↳ both storage element not connected to power source at first



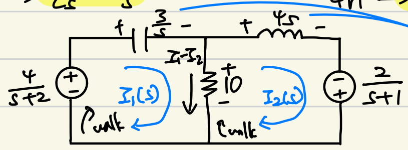
Taking Laplace transform of variables,

$4e^{-2t}u(t) \longrightarrow \frac{4}{s+2}$ $\qquad 2e^{-t}u(t) \longrightarrow \frac{2}{s+1}$

$\frac{1}{3}F \rightarrow \frac{1}{Cs} = \frac{3}{s}$ $\qquad 4H \rightarrow Ls = 4s$

→ treat them as resistors

s-domain circuit,



$$+\frac{4}{s+2} - I_1\left(\frac{3}{s}\right) - 10(I_1 - I_2) = 0 \quad \text{—— ①}$$

$$+10(I_1 - I_2) - 4sI_2 + \frac{2}{s+1} = 0 \quad \text{—— ②}$$

substitution

$$I_1 = \frac{2s(4s^2+19s+20)}{20s^4+66s^3+73s^2+57s+30} \text{ A}$$

$$I_2 = \frac{30s^2+43s+6}{(s+2)(s+1)(20s^2+6s+15)} \text{ A}$$

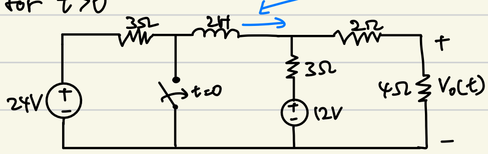
→ use partial fraction & solve this as above

$$i_2(t) = [-0.4819e^{-2t} - 0.2414e^{-t} + 0.864e^{-0.15t}\cos(0.8529t - 33.16)]u(t) \text{ A}$$

$$i_1(t) = [-0.0964e^{-2t} - 0.3448e^{-t} + 0.841e^{-0.15t}\cos(0.8529t) + 0.1977e^{-0.15t}\sin 0.8529t]u(t) \text{ A}$$

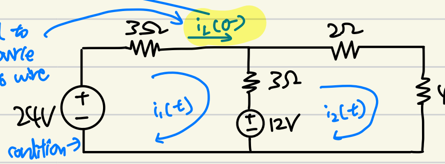
Solving Networks with Non-Zero Initial Conditions

E.g. The switch in the circuit is in the open position for a long time. At $t=0$, it is closed instantaneously. Find $v_o(t)$ for $t>0$



same

inductor connected to d.c source
↳ act as wire



capacitor will out as open circuit

initial condition

With mesh eqn, $24 = 3i_1 + 3(i_1 - i_2) + 12 \longrightarrow 12 = 6i_1 - 3i_2$ —— ①

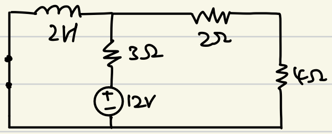
$6i_2 - 12 + 3(i_2 - i_1) = 0 \longrightarrow i_2 = \frac{12+3i_1}{9}$ —— ②

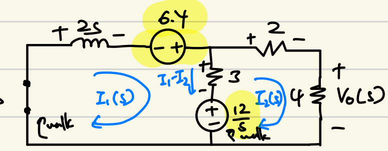
Sub ② into ①, $12 = 6i_1 - 3\left[\frac{12+3i_1}{9}\right] \longrightarrow i_1 = 3.2$ A

$i_L(0^-) = 3.2$ A

time-domain circuit for $t>0$ (switch closed),

if unsure can add the rest of the circuit but current take least resistance path





$$2sI_1(s) - 6.4 + 3[I_1(s) - I_2(s)] + \frac{12}{s} = 0 \quad \text{--- (1)}$$

$$2sI_2(s) + 4I_2(s) - \frac{12}{s} + 3I_2(s) - 3I_1(s) = 0 \quad \text{--- (2)}$$

sub (1) into (2), $I_1(s) = \dfrac{-\frac{12}{s} + 9I_2(s)}{3} = -\frac{4}{s} + 3I_2(s)$ --- (3)

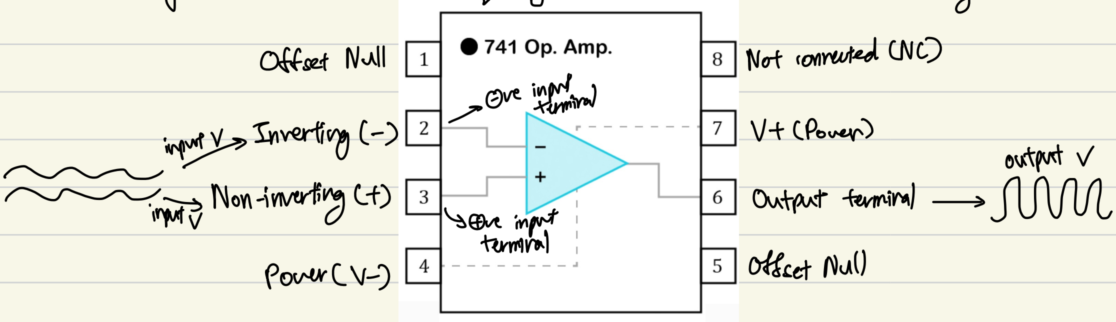
sub (3) into (2), $(2s+3)\left[-\frac{4}{s} + 3I_2(s)\right] - 3I_2(s) = 6.4 - \frac{12}{s}$

$$I_2(s) = \frac{2.4}{s+1}$$

$$V_o(s) = 4 \times I_2(s) = \frac{9.6}{s+1} \xrightarrow[\text{transform}]{\text{inverse laplace}} V_o(t) = 9.6e^{-t}u(t), \text{ for } t>0$$

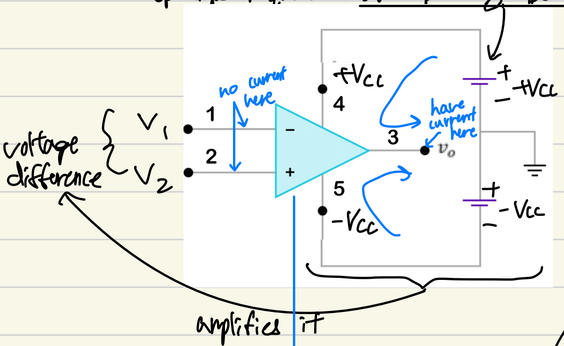
Operational Amplifier / Op-Amp

↳ an integrated circuit that produces an output that is an amplified replica of the difference b/w 2 input voltages
↳ may include terminals for frequency compensation & for offset nulling.

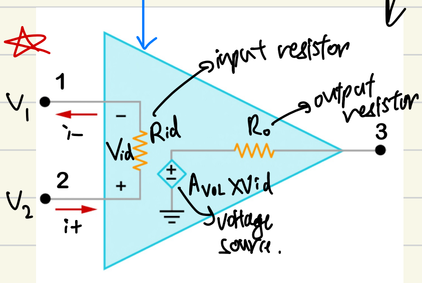


A Typical OP-AMP Device

↳ Most IC op-amps require a dual polarity power supply (All op-amp circuit require power for their operation)



- Rid → input resistance of the op-amp
- Ro → output resistance
- i+ & i− are current flowing in the (+ve) & (−ve) input terminals
- $V_{id} = V_2 - V_1$ → input differential voltage



Op-Amp

An op-amp is suppose to:
↳ sense the difference b/w the voltage signals $V_2 - V_1$
↳ multiply the difference by constant A_{VOL}
↳ Give output as $A_{VOL}(V_2 - V_1)$

A_{VOL} → open-loop voltage gain of amplifier → measure how much gain op-amp can provide if op-amps output is not connected back to input

Conditions of the Ideal Op-Amp

↳ performance of a typical op-amp is very close to that of an ideal op-amp

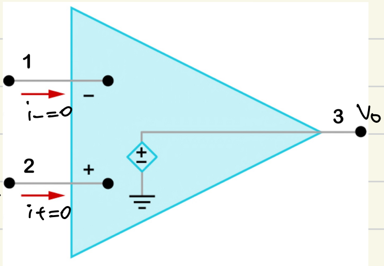
1. Input resistance is infinite ($R_{id} = \infty$)
   ↳ input connection becomes an open-circuit

2. Output resistance is zero ($R_o = 0$)
   ↳ no voltage drop across R_o → becomes wire

3. Open-loop voltage is infinite ($A_{VOL} = \infty$)
   ↳ in order to have finite output voltage → differential input should be 0.

4. Bandwidth is infinite ($BW = \infty$)
   ↳ no phase shift b/w input & output signals

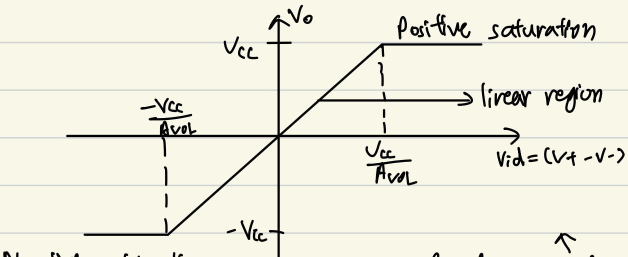
5. Zero input offset voltage
↳ $V_0 = 0$ if $V_1 = V_2$ / $V_{id} = 0$



← Circuit of an ideal op-amp

$$V_0 = \begin{cases} +V_{cc} & \text{if } A_{VOL}V_{id} > V_{cc} \\ A_{vol}(V_+ - V_-) & \text{if } -V_{cc} \le A_{VOL}V_{id} \le V_{cc} \\ -V_{cc} & \text{if } A_{VOL}V_{id} < -V_{cc} \end{cases}$$

$V_{id} = V_+ - V_-$

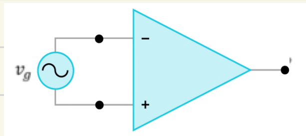


Transfer function of ideal op-amp

$A_{VOL}(V_+ - V_-) > V_{cc}$ → ⊕ve saturation region
$A_{VOL}(V_+ - V_-) < -V_{cc}$ → ⊖ve saturation region
$V_0 = A_{VOL}V_{id}$ → linear region

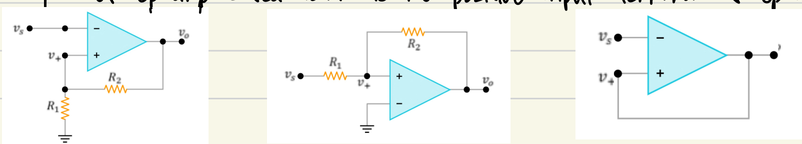
Op-Amp Configurations

× 1. Open-loop configuration (not impt) [Formula: $V_{out} = A_{VOL}(V_+ - V_-)$]
↳ output of op-amp is not fed back to the input of the op-amp

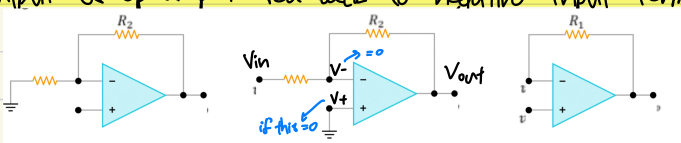


2. Closed-loop configuration

× a) Positive feedback (not impt) → make system unstable (only want if wanna generate digital signal)
↳ output of op-amp is fed back to the positive input terminal of op-amp



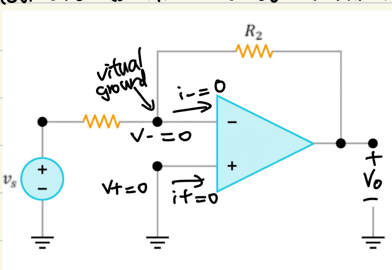
b) Negative feedback (impt)
↳ output of op-amp is fed back to negative input terminal of op-amp



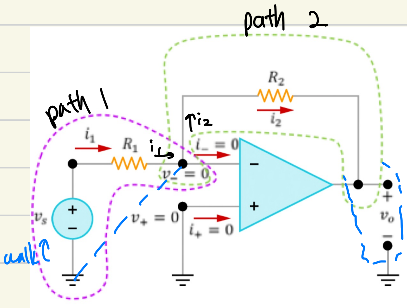
↳ ideal op-amp in negative feedback will operate in linear region: $V_0 = A_{VOL}(V_+ - V_-)$
$A_{VOL} \to \infty$, we have $V_+ - V_- \to 0$, $V_+ = V_-$

Analyse of Inverting Feedback Amplifier Circuits

E.g. You are to find V_0 as a function of input voltage V_s for the amplifier circuit using an ideal op-amp

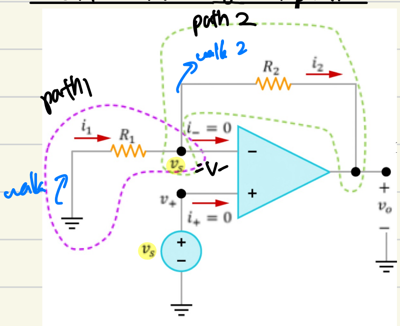


→ Negative-feedback circuit → op-amp is operating as a linear device: $V_+ = V_-$
→ Ideal op-amp: $I_+ = 0$, $I_- = 0$
→ Because V_- is not physically connected to ground → engineers term this a virtual ground



KVL & path 1: $-V_s + i_1 R_1 = 0$
$$i_1 = \frac{V_s}{R_1}$$
KVL & path 2: $i_2 R_2 + V_o = 0$
$$i_2 = \frac{-V_o}{R_2}$$
KCL for $v_- = 0$ node: $i_1 = i_2 + \cancel{i_-}^{\,0}$
$$\frac{V_s}{R_1} = -\frac{V_o}{R_2} \Rightarrow \frac{V_o}{V_s} = -\frac{R_2}{R_1}$$
output, input → inverting amplifier (closed-loop inverting gain)

Non-Inverting Amplifier

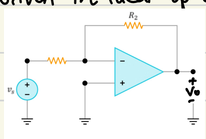


$i_- = i_+$, $V_- = V_+ = V_s$

KVL for path 1: $i_1 R_1 + V_s = 0$
$$i_1 = \frac{V_s}{R_1}$$
KVL for path 2: $-V_s + I_2 R_2 + V_o = 0$
$$i_2 = \frac{V_s - V_o}{R_2}$$
Node v_s: $i_1 = i_2 + \cancel{i}^{\,0}$
$$\frac{V_s}{R_1} = \frac{V_s - V_o}{R_2} \longrightarrow \frac{V_s}{V_o} = 1 + \frac{R_2}{R_1}$$

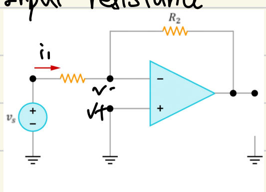
Input & Output resistance

↳ Given the ideal op-amp circuit shown, what is the input & output resistance of the circuit?



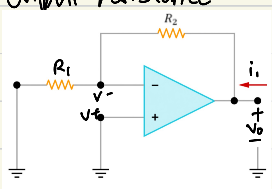
Op-amp ideal → $V_- = V_+ = 0$

a) Input resistance

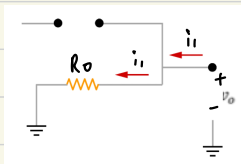


1. Define i_1
2. Input resistance seen by V_s → $R_{in} = \frac{V_s}{i_1}$
3. Take path from V_s to V_-, we have: $0 + V_s - i_1 R_1 = 0 \rightarrow i = \frac{V_s}{R_1}$
$$R_{in} = \frac{V_s}{\frac{V_s}{R_1}} = R_1 \longrightarrow R_{in} = R_1$$

b) Output resistance



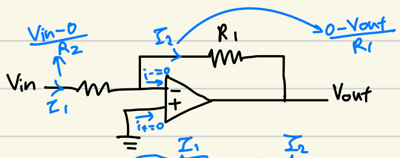
1. We know pratical op-amp has resistance R_o.
2. Output resistance → $R_{out} = \frac{V_o}{i_1} = R_o$
3. Since ideal op-amp → $R_{out} = 0\,\Omega$



easy
Summary

Solve op-amp.

1) Define current directions
2) Use $V_+ = V_-$ & $I_+ = I_- = 0$
3) Figure out the KCL.
4) Equate them



$$\underbrace{\frac{Vin - 0}{R_2}}_{I_1} = \underbrace{\frac{0 - Vout}{R_1}}_{I_2}$$

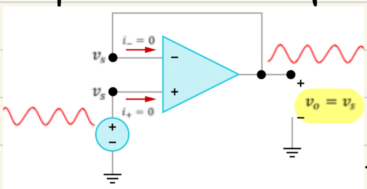
$$Vout = -\frac{R_1}{R_2} Vin$$

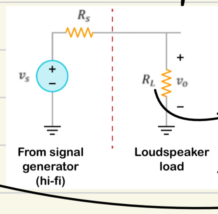
if Vin is connected to ⊖ve terminal & there is a ⊖ve sign → correct

Op-Amp Configurations

1. Voltage Follower

- produces an output signal that is same voltage as input, can be used to lower the output resistance of circuit
- In audio electronics → voltage follower normally used as interface b/w an amplifier circuit having high output impedence (1kΩ) & loudspeaker that has a low input impedance (4–8Ω)

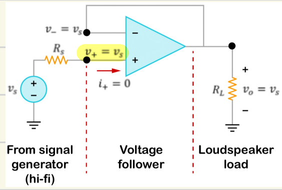




$V_o = \frac{R_L}{R_L + R_s} V_s$

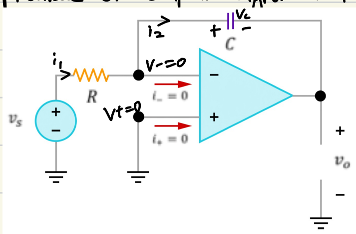
only a fraction of V_s

loudspeaker load resistor



2. Inverting Integrator

- produces an output signal that is proportionate to the integration of the input voltage w.r.t time



current for capacitor: $i = C\frac{dV}{dt} \rightarrow \Delta V$

$i_1 = \frac{V_s - 0}{R}$ $\quad i_2 = C\frac{d(0 - V_o)}{dt}$

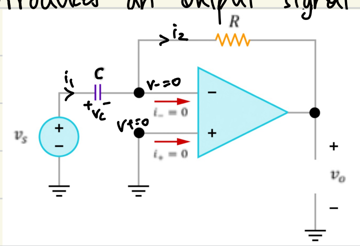
$i_1 = i_2 \rightarrow \frac{V_s}{R} = C\frac{d(-V_o)}{dt}$

$V_s = RC\frac{d(-V_o)}{dt} \rightarrow \int_{-\infty}^{t} V_s(t)dt = -RC(V_o(t))$

$V_o(t) = -\frac{1}{RC}\int_{-\infty}^{t} V_s(t)dt$

3. Inverting Differentiator

- Produces an output signal that is proportionate to the differentiation of the input voltage over time



$i_1 = C\frac{dV}{dt} = C\frac{d(V_s(t) - 0)}{dt} = C\frac{d(V_s(t))}{dt}$

$i_2 = \frac{0 - V_o}{R} = -\frac{V_o}{R}$

$i_1 = i_2 \rightarrow C\frac{dV_s(t)}{dt} = -\frac{V_o}{R}$

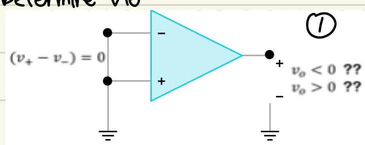
$V_o = -RC\frac{dV_s(t)}{dt}$

Input & Output Offset Voltage

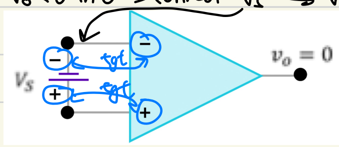
- Use to address the imbalances to force the output voltage to be zero

$V_o = A_{VOL}(V+ - V- + V_{IO})$ → offset voltage

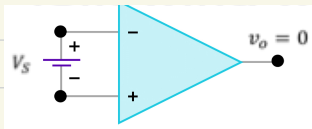
Determine V_{IO}



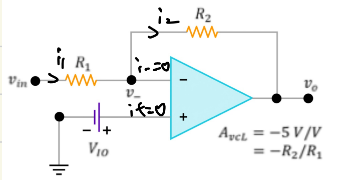
$V_o < 0$ in ① → correct V_s → $V+ - V- = \oplus ve$, $V_{IO} = \ominus ve$



$V_o > 0$ in ① → $V_{IO} > 0$, $V+ - V- < 0$.



E.g. Consider the inverting feedback amplifier with a closed-loop gain $A_{VCL} = -5V/V$. Input signal is $V_{in} = -1V$–DC. Output is $5.12V$–DC. What is the V_{IO} for the op-amp?



$V+ = V- = V_{IO}$

$i_1 = \frac{-1 - V_{IO}}{R_1}$ $\quad i_2 = \frac{V_{IO} - 5.12}{R_2}$

$i_1 = i_2 \rightarrow \frac{-1 - V_{IO}}{R_1} = \frac{V_{IO} - 5.12}{R_2}$

$\frac{1 + V_{IO}}{-R_1} = \frac{V_{IO} - 5.12}{R_2} \rightarrow -\frac{R_2}{R_1} = \frac{V_{IO} - 5.12}{1 + V_{IO}}$

$-5 - 5V_{IO} = V_{IO} - 5.12$

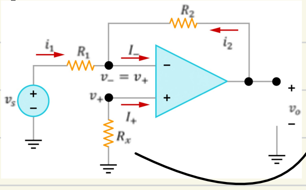
$-6V_{IO} = -0.12$

$V_{IO} = 0.02V$

- Input Bias Current $\rightarrow I_{BIAS} = \frac{1}{2}(I_+ + I_-) \rightarrow$ can be ⊕ve/⊖ve
- Input Offset Current $\rightarrow I_+ \neq I_- \rightarrow I_{IO} = I_+ - I_-$

$\rightarrow$ calculate exactly the same, just $V=IR$, use this.

E.g.



to compensate for effect of biased currents $\rightarrow$ add a resistor

Assume ideal case $\rightarrow V_+ = V_-$, except $I_- \neq 0$, $I_+ \neq 0$

$$V_- = V_+ = 0 - I_+R_x$$

$$i_1 + i_2 = I_- \rightarrow \frac{V_s - V_-}{R_1} + \frac{V_o - V_-}{R_2} = I_-$$

sub $V_- = -I_+R_x$

$$V_o = I_-R_2 - \frac{R_2}{R_1}V_s - I_+R_x\left(\frac{R_2+R_1}{R_1}\right)$$

cancel each other out

$$I_+ = I_- \longrightarrow R_x = \frac{R_1R_2}{R_1+R_2} = R_1 // R_2$$

$$\therefore V_o = -\frac{R_2}{R_1}V_s$$

Slew Rate (not tested)

$\hookrightarrow$ practical op-amp $\rightarrow$ rate of Δ of output limited by value known as slew rate, output of op-amp cannot Δ faster than this

$S_R = \max\left\{\frac{dV_o}{dt}\right\}$ $\rightarrow$ output voltage   Unit: V/μs

E.g. An op-amp with $S_R = 1\,V/\mu s$ is used to build an inverting amplifier with gain $-10V/V$. With a 1.0V-peak sinusoid, output has peak of 10V.

a) At what frequency will output be affected by S_R?

b) If sinusoidal input increased to 1.5V at this frequency, sketch resulting waveform.

a) output voltage of sinusoidal input: $V_o = -V_o \cos\omega t$

$$\frac{dV_o}{dt} = \omega V_o \sin\omega t$$

$$\max\left\{\frac{dV_o}{dt}\right\} = \omega V_o = 2\pi f V_o$$

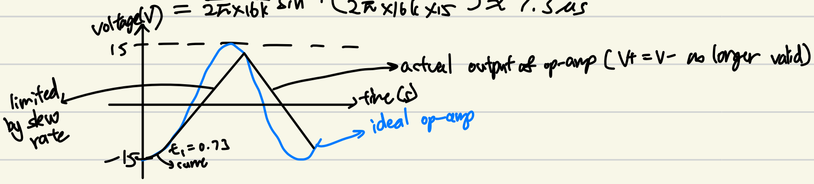
$\omega V_o \leq S_R$ $\rightarrow$ to avoid non-linear distortion

$$2\pi f V_o = S_R \rightarrow f = \frac{S_R}{2\pi V_o} = \frac{(1\times10^6)}{2\pi\times10} \approx 16kHz$$

b) $\omega V_o \sin\omega t \leq S_R \rightarrow \omega V_o \sin\omega t = S_R$

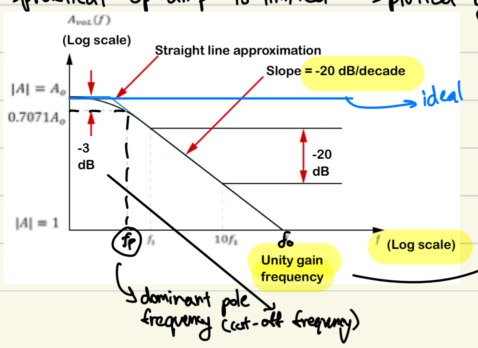
$$t = \frac{1}{\omega}\sin^{-1}\left(\frac{S_R}{\omega V_o}\right)$$

$$= \frac{1}{2\pi\times16k}\sin^{-1}\left(\frac{1\times10^6}{2\pi\times16k\times15}\right) \approx 7.3\mu s$$



Finite Frequency Response (not tested)

$\hookrightarrow$ practical op-amp is limited $\rightarrow$ plotted by Bode plot



Gain of op-amp: $|A(if)| = \frac{A_o}{\sqrt{1+\left(\frac{f}{f_p}\right)^2}}$

When $f = f_p$: $|A(if)| = \frac{A_o}{\sqrt{2}} = 0.7071A_o$

done by setting $|A(if)| = 1$ $\leftarrow$ solve & find f_o [$f_o \approx A_o f_p$ for $A_o >> 1$]

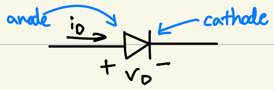
gain bandwidth pdt

The Ideal Diode

— Notations: DC voltage (V_A), DC current (I_A), AC voltage (v_a), AC current (i_b) → both small letters

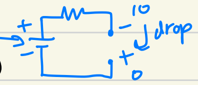
— Sum of DC & AC components: $v_A = V_A + v_a$, $i_A = I_A + i_a$

↳ Diode → 2 terminal device → allow current to flow in a single direction

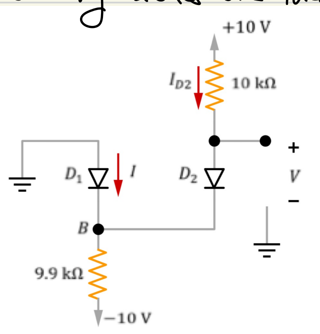


current — 1. Reverse-biased region: $V_D < 0 \rightarrow i_D = 0 \rightarrow$ act as open circuit $(V_D = 0, I_o = \text{small})$

current — 2. Forward-biased region: $V_D > 0 \rightarrow i_D = \infty$ (large) → act as short circuit $\xrightarrow{i_D = \infty}$ $(V_D = 10V, I_o = 0)$



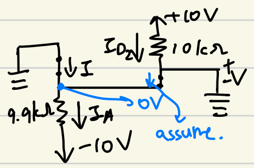
E.g. Assuming diodes are ideal, find I & V in circuit.



1. Make assumptions

D_1	D_2
ON	ON
OFF	OFF
ON	OFF
OFF	ON

ON ON → use this first (both are short circuit)



$V = 0V$

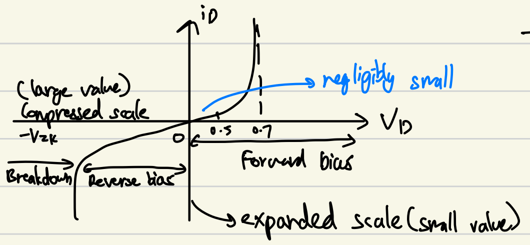
$I_A = I + I_{D2}$

$\frac{0-(-10)}{9.9k\Omega} = I + \frac{10-0}{10k\Omega}$

$I = 0.01mA$ → so the assumption of current flow is correct

need to do again → assuming D_1 is off if negative I → assumption of current wrong

Terminal characteristic of diodes



→ $0 < V_D < 0.5V$, $i_D \approx 0$.

- Forward region → Shockley diode eqn: $i_D = I_S(e^{V_D/(nV_T)} - 1) \approx I_S e^{V_D/nV_T}$

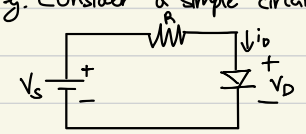
↳ i_D (diode current), I_S (saturation current), $n = 1$ (emission coefficient), $V_T = \frac{kT}{q}$ (thermal voltage), $k = 1.38 \times 10^{-23} J/K$ (Boltzmann's constant), $q = 1.602 \times 10^{-19} C$ (electronic charge), T (absolute temperature, K)

↳ $V_D = nV_T \ln \frac{i_D}{I_S}$

$\frac{\Delta V_D}{\Delta T} \approx -2mV/°C$ → for every 1°C ↑ in T, V_D drops by ≈ 2mV

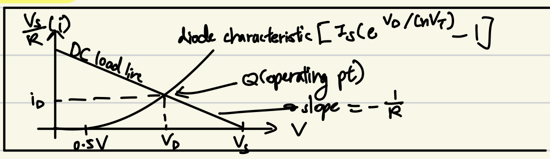
- Reversed region → $i_D = I_S(\underbrace{e^{V_D/(nV_T)} - 1}_{\text{very small}}) \approx I_S$

E.g. Consider a simple circuit as shown, we assume that diode is operating at forward-biased region with $V_S \gg 0.5V$

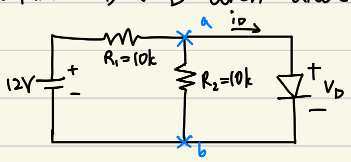


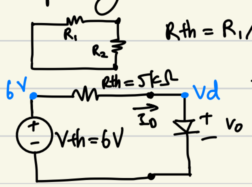
Since $V_S \gg 0.5V$ → forward-biased: $i_D = I_S e^{V_D/(nV_T)}$

Using KVL → $i_D = \frac{V_S - V_D}{R}$



E.g. Find V_D & i_D when diode is operating in forward-biased region $[I_S = 10nA, n = 2, V_T = 26mV]$





$R_{th} = R_1 // R_2 = \frac{10k \times 10k}{10k + 10k} = 5k$

$I_D = i_D = I_S e^{V_D/2V_T}$ → $\ln\left(\frac{i_D}{I_S}\right) = \frac{V_D}{2V_T}$ —— ①

$i_D = \frac{V_{th} - V_D}{R_{th}}$ assume $V_D = 0.7V$ (conducting (forward-biased)) → $i_D \approx 1.06mA$

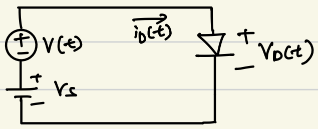
Sub i_D into ①, $V_D \approx 2 \times 0.026 \ln\left(\frac{1.06 \times 10^{-3}}{10 \times 10^{-9}}\right) = 0.602V$

doesn't make sense so input it in & try again until consistent

$I = 1.079mA$, $V_D = 0.603V$

Analysis of Small-Signal Diode Circuits

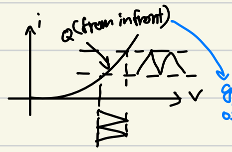
↳ have a DC & AC source → but AC source generating less voltage



$v_D(t) = V_s + v_s(t)$ → sub it into Shockley eqn

$i_D(t) = I_S e^{\frac{v_D(t)}{nV_T}} = I_S e^{\frac{(V_s + v_s(t))}{nV_T}} = I_D e^{\frac{v_s(t)}{nV_T}}$ —①

$\frac{v_s(t)}{nV_T} \ll 1 \longrightarrow e^{\frac{v_s(t)}{nV_T}} \approx 1 + \frac{v_s(t)}{nV_T}$ —②

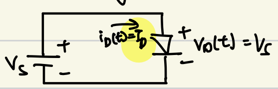


Q to be oscillating

sub ② into ①, $i_D(t) \approx I_D\left(1 + \frac{v_s(t)}{nV_T}\right) = I_D + g_d v_s(t) = I_D + i_d(t)$, $g_d = \frac{I_D}{nV_T}$ → small signal conductance

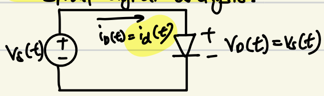
To solve this kind of question:

1. DC Analysis:

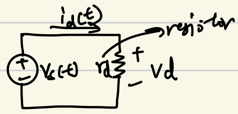


→ solve it like above

2. Small-signal analysis:



→



$v_s(t) = i_d(t) r_d$, $r_d = \frac{1}{g_d} \longrightarrow = \frac{I_D}{nV_T}$

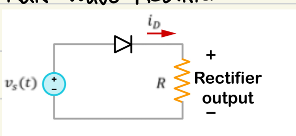
3. Superposition:

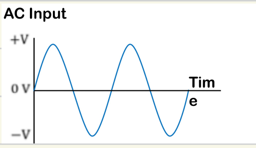
$i_D(t) = I_D + i_d(t)$

$v_D(t) = V_s + v_s(t)$

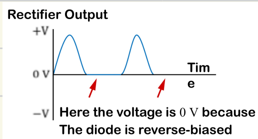
Application of diode

1. Half-wave Rectifier

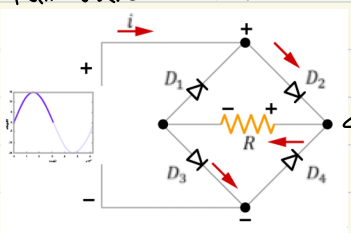




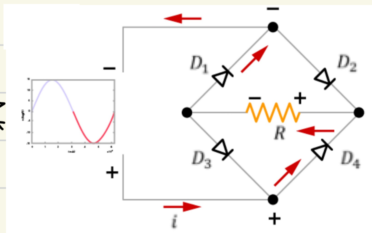
→



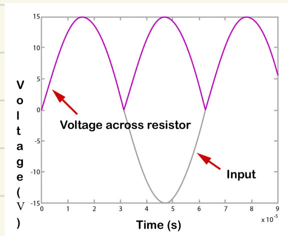
2. Full-wave Rectifier



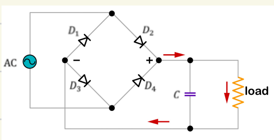
&



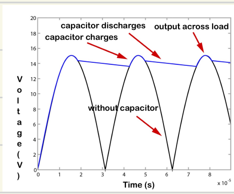
→



3. Elementary DC Supplies



→

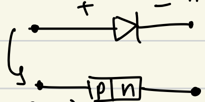


Bipolar Junction Transistors (BJTs)

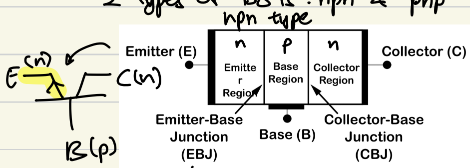
↳ transistor → use of voltage b/w 2 terminal to control current flowing in 3rd terminal

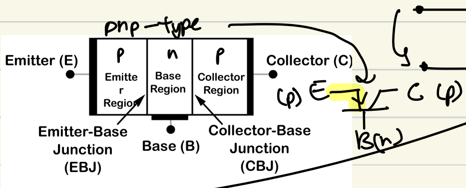
↳ terminals are named: Emitter (E), Base (B) & Collector (C)

↳ 2 types of BJTs: npn & pnp transistor



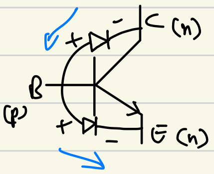
(forward-biased)
$V_p > V_n$ → ON
$V_p < V_n$ → OFF
(reverse-biased)





flow from p to n

↳ like diode



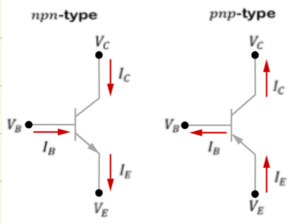
BJT	status of EBJ	status of CBJ
cut-off	reverse	reverse
Active	forward	reverse
saturation (everything conducted)	forward	forward

impl ←

EBJ → emitter base junction

CBJ → collector base junction

Current-voltage relationship in active mode (only)



emit

1) $I_E = I_B + I_C$

2) $I_C = I_S\left(e^{V_{BE}/nV_T} - 1\right) \approx I_S\left(e^{V_{BE}/(nV_T)}\right)$

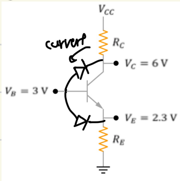
3) $I_C = \beta I_B$ — common emitter current gain ($\beta \gtrsim 100$, $\beta \lesssim 200$)

4) $I_C = \alpha I_E \longrightarrow I_C \approx I_E$ — common-base current gain ($\alpha \approx 1$ (0.99))

5) $\alpha = \frac{\beta}{\beta+1}$; $\beta = \frac{\alpha}{1-\alpha}$

when $\beta \to \infty$, $\alpha \to 1$, $I_C \to I_E$

E.g. Determine the mode of operation for BJT configuration shown below.



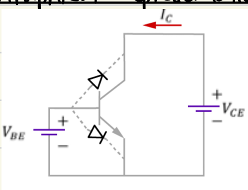
$V_{BE} = V_B - V_E = 0.7V \rightarrow$ ON (Forward-biased) → only bigger than 0.5V then ON

$V_{BC} = 3 - 6 = -3V \rightarrow$ OFF (reverse-biased)

→ BJT is in active mode

— Normally assume $V_{BE} \approx 0.7V$

Graphical Representation of BJT Characteristics



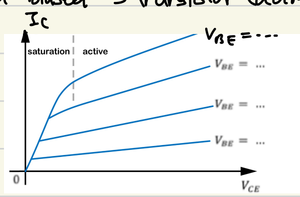
At low V_{CE}: CBJ is forward-biased → transistor is in saturation mode

When $V_{CE}\uparrow$: CBJ becomes increasingly reversed-biased → transistor (active)

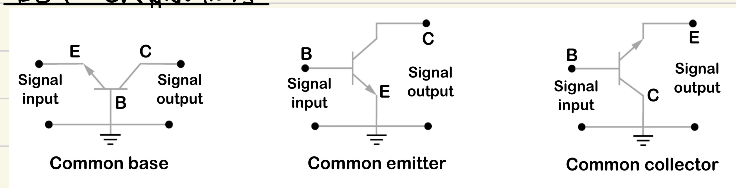
↳ $I_S \uparrow \longrightarrow I_C \uparrow$

↳ width of depletion region ↑

↳ V_T before reaching active region



BJT Configurations



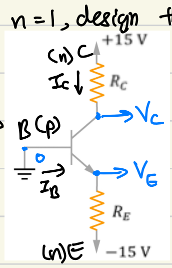
↳ common to both input as well as output

Large Signal (DC) Analysis of BJT Circuits

only

E.g. The npn BJT has shown the following characteristics: $\beta = 100$, $V_{BE} = 0.7V$ when $I_C = 1mA$. Assuming $V_T = 25mV$ & $n = 1$, design the circuit so that $I_C = 2mA$ & $V_C = 5V$.

Common base



$I_C = \frac{15 - V_C}{R_C} \longrightarrow R_C = \frac{15-5}{2\times10^{-3}} = 5k\Omega$

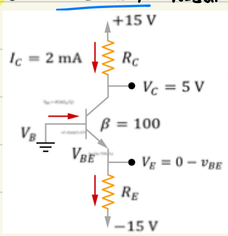
$I_C = I_S e^{V_{BE}/(nV_T)} \longrightarrow V_{BE} = nV_T \ln\left(\frac{I_C}{I_S}\right)$

$V_{BE_1} = nV_T \ln\left(\frac{I_{C_1}}{I_S}\right)$, $V_{BE_2} = nV_T \ln\left(\frac{I_{C_2}}{I_S}\right)$ → 2mA; I_{C_1} = 1mA

$V_{BE_2} = 0.717V$ — figure out I_S & sub in

$V_E = 0 - V_{BE_2} = -0.717V$

$R_E = \frac{V_E - (-15)}{2.02} = 7.07k\Omega$

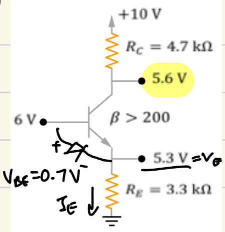


$\alpha = \frac{\beta}{\beta+1} = 0.99$

$I_E = \frac{I_C}{\alpha} = \frac{2\times10^{-3}}{0.99} = 2.02mA$

→ If question say BJT in active mode, but no value of V_{BE} → assume $V_{BE} = 0.7V$

E.g. Analyse given circuit & determine the voltage at all nodes & currents through all branches ($V_B, V_E, V_C = ?$; $I_B, I_E, I_C = ?$)



$I_E = \frac{5.3}{5.3k} = 1mA$

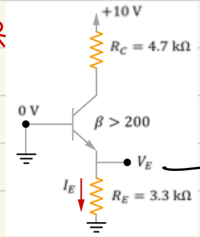
Since β > 200 → $I_E \approx I_C$

$V_C = 2.48V$ → not correct. (both diode forward-biased → saturation mode)

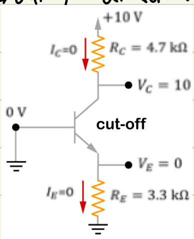
↳ assume in saturation mode, V_{CE} is normally 0.2–0.3V. Assume $V_{CE} = 0.3V$,

$V_C = V_E + V_{CE} = 5.3 + 0.3 = 5.6V$ & solve the rest.

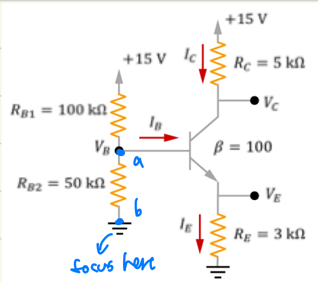
↳ transistor operating at forced β → $\frac{I_C}{I_B} = 1.4$



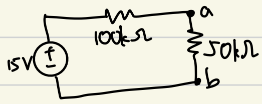
only can = 0, cause if = −0.7V
↳ current cannot flow → 0



E.g.

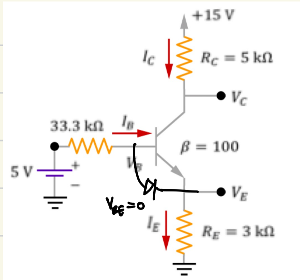


→ convert to Thevenin's equivalent



$R_{th} = 100k\Omega // 50k\Omega$

$V_{th} = 15 \left(\frac{50k}{100k} \right) = 5V$



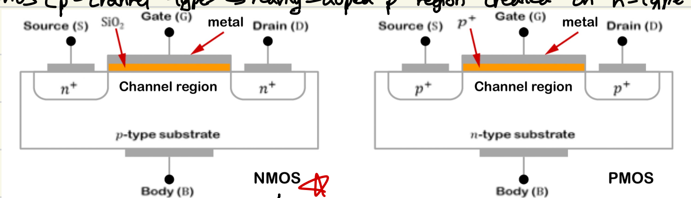
Metal-Oxide Semiconductor Field-Effect Transistor (MOSFET)

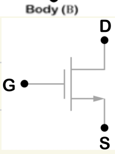
↳ FET uses an electric field to control electrical behaviour

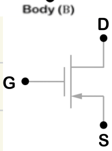
↳ Both BJT & FET can be used as amplifier & switch (logical circuits)

↳ consist of:

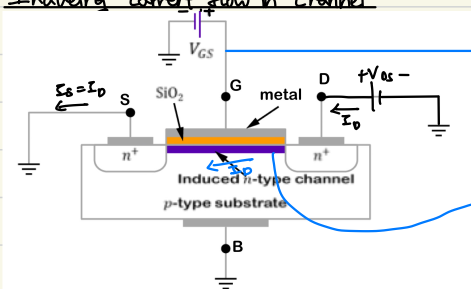
1. NMOS (n-channel type) → heavily-doped n region created on p-type substrate
2. PMOS (p-channel type) → heavily-doped p region created on n-type substrate





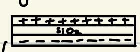


Inducing current flow in Channel



→ value of V_{GS} > threshold voltage (V_t)
↳ before no. of sufficient electrons can accumulate to form conducting channel

magnitude of I_D dependent on V_{GS}



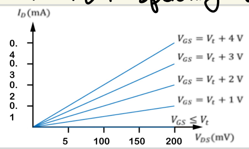
↳ caused by +ve charge → with +ve induces an electric field → controls amount of charge in channel & determine channel conductivity flowing through source & drain

$I_D = \mu_n C_{ox}(\frac{W}{L})[(V_{GS}-V_t)V_{DS} - 0.5V_{DS}^2]$

- ↳ W (width of induced channel), L (length of induced channel), μ_n (electron mobility ~$580cm^2/Vs$), C_{ox} (oxide capacitance F/um^2)
- ↳ $k_n' = \mu_n C_{ox}$ → process transconductance parameter ($\sim 20-100\mu A/V^2$)
- ↳ when $V_{GS} \approx V_t$ → channel is just induced → $I_D \approx 0$. $V_{GS} > V_t$ → more e^- attracted to induced channel → $I_D \uparrow$

Triode Region

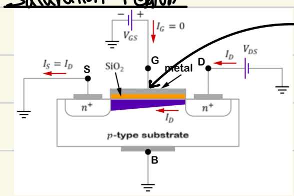
↳ MOSFET operating with small V_{DS} → operating in triode region

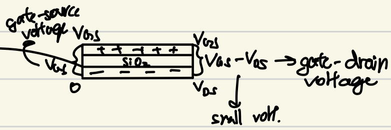


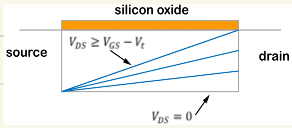
→ V_{DS} is small → ignore V_{DS}^2 → $I_D \approx \mu_n C_{ox}(\frac{W}{L})[(V_{GS}-V_t)V_{DS}]$

→ linear resistance $r_{DS} = \frac{V_{DS}}{I_D} = [\mu_n C_{ox}(\frac{W}{L})(V_{GS}-V_t)]^{-1}$

Saturation Region







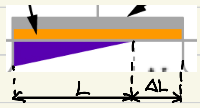
→ $V_{GD} = 0$ → channel said to be pinched off

→ $V_{DS} \geq V_{GS} - V_t = V_{DS_{sat}}$ → drain current saturates

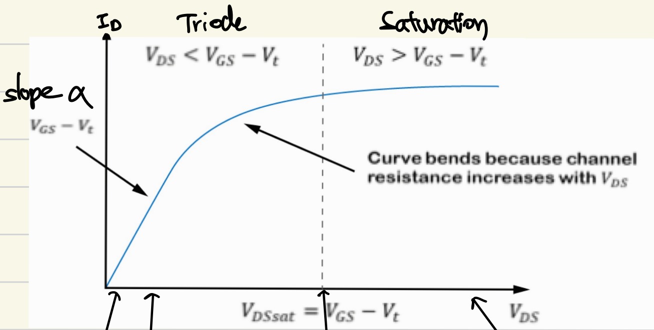
→ $I_D = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})(V_{GS}-V_t)^2$ → sub $V_{DS} = V_{GS} - V_t$

→ As $V_{DS} \uparrow$ above $V_{DS\,sat}$, $\Delta L \uparrow$ & effective L reduces → $I_D = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})(V_{GS}-V_t)^2(1+\lambda V_{DS})$

↳ λ → channel length modulation parameter ($0.001V^{-1} < \lambda < 0.10V^{-1}$)



Summary

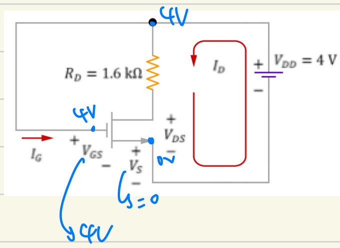


$I_D \approx \mu_n C_{ox}(\frac{W}{L})[(V_{GS}-V_t)V_{DS}]$

$I_D = \mu_n C_{ox}(\frac{W}{L})[(V_{GS}-V_t)V_{DS} - 0.5V_{DS}^2]$

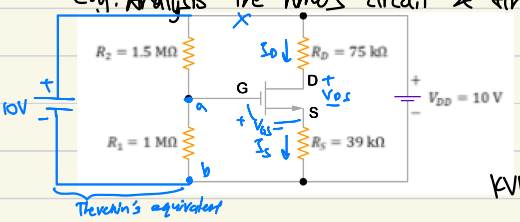
$I_D = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})(V_{GS}-V_t)^2$

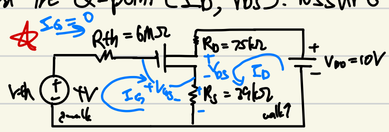
$I_D = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})(V_{GS}-V_t)^2(1+\lambda V_{DS})$



Large Signal Analysis

E.g. Analysis the NMOS circuit & find the Q-point (I_D, V_{DS}). Assume $V_t = 1V$ & $\mu_n C_{ox}(\frac{W}{L}) = 25\mu A/V^2$





↳ Assuming in saturation region first

① KVL

② $I_D = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})(V_{GS}-V_t)^2$

KVL : $4 - V_{GS} - 39k(I_D) = 4 - V_{GS} - 39k(\frac{1}{2}(25\mu))(V_{GS}-1)^2$

$V_{GS} = -2.71,\ 2.66$ ✓

$I_D = \frac{1}{2}(25\mu)(2.66-1)^2 = 34.4\,mA$

$10 - 75k(I_D) - V_{DS} - 39k(I_D) = 0$

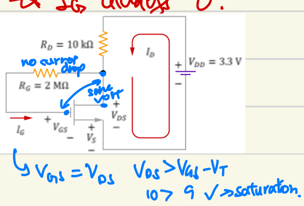
$V_{DS} = 6.08V$

check if saturation correct: $V_{DS} > V_{GS} - V_T$ (satisfied) ✓

$6.08V$ $\quad$ $2.66V$ $\quad$ $1V$

↳ if wrong, Δ I_D formula & redo calculation

★ I_G always 0.



↳ $V_{GS} = V_{DS}$ $\quad$ $V_{DS} > V_{GS} - V_T$

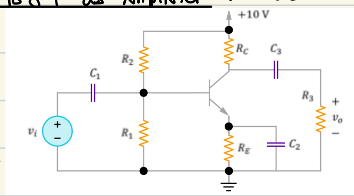
$10 > 9$ ✓ → saturation

AC Analysis

- inductors —(L)— → open circuit $i_L = 0$
- capacitors —| |— → short circuit $V_C = 0$

DC circuit is opposite.

BJT as Amplifier (0.008V → small signal)

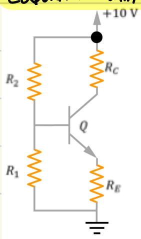


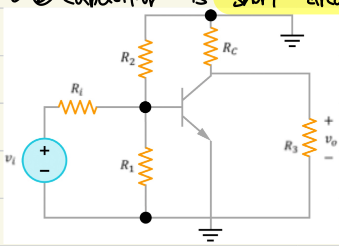
→ Capacitor C_1 & C_3 are known as coupling capacitors → use to inject AC input signal & extract output signal without disturbing the Q-point

→ Capacitor C_2 bypass capacitor → bypass R_E for AC (higher gain) → needed for good Q-point stability

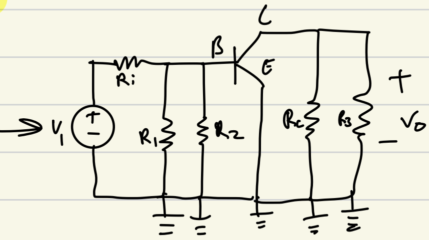
DC equivalent { ① kill a.c. source
② replace capacitor with open circuit

AC equivalent { ① kill DC & connect DC to ground (+10V gone)
② capacitor is short circuit

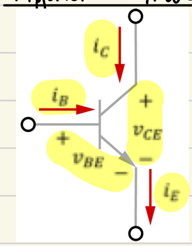




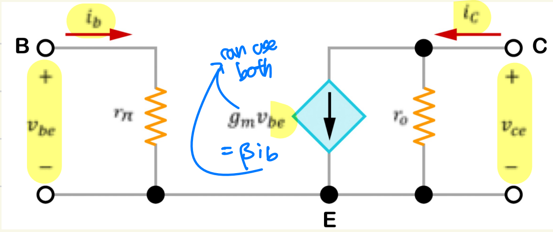
→



Hybrid-π Model of BJT



→



★ $i_c = \beta i_b$ → BJT in forward-active mode

Transconductance : $g_m = \frac{I_C}{V_T}$, where $V_T = \frac{kT}{q} \approx 25mV$

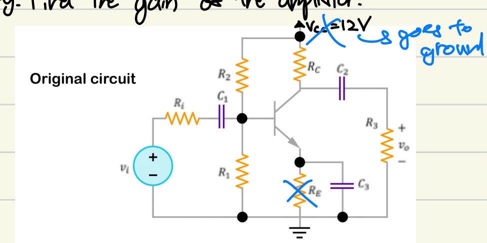
Input resistance : $r_\pi = \frac{\beta}{g_m}$ → small signal current gain = large signal current gain $(\beta = \frac{I_C}{I_B} \approx \frac{i_c}{i_b})$

Output resistance : $r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$ if $V_A >> V_{CE}$, V_A = Early voltage → someone name

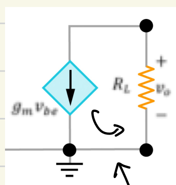
- For small signal operation to work, $|V_{be}| \leq 5mV$
  ↳ $I_C + g_m V_{be} = I_C + i_c$

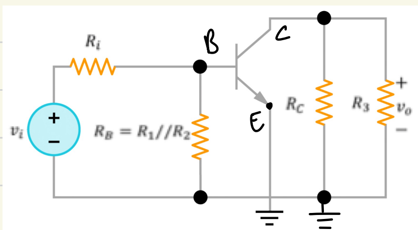
Small-Signal Analysis

E.g. Find the gain of the amplifier.

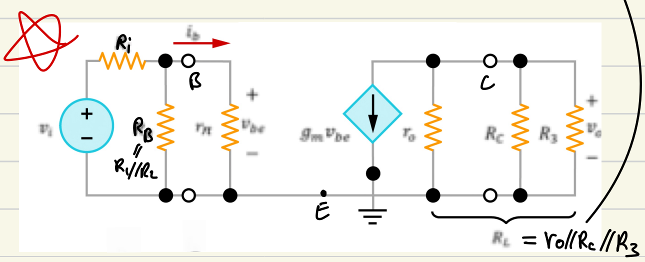


1. Simplified AC circuit
2. Hybrid-π model
3. Find amplifier gain





→



$$\text{Gain} = \frac{V_o}{V_i} = \frac{V_o}{V_{BE}} \frac{V_{BE}}{V_i}$$

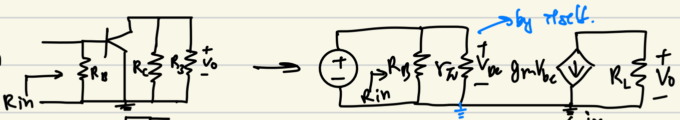
$$V_o = -g_m V_{BE} R_L \longrightarrow \frac{V_o}{V_{BE}} = -g_m R_L \quad (1)$$

$$V_{BE} = \frac{r_\pi // R_1 // R_2}{R_i + r_\pi // R_1 // R_2} \times V_i \longrightarrow \frac{V_{BE}}{V_i} = \frac{r_\pi // R_1 // R_2}{R_i + r_\pi // R_1 // R_2} \quad (2)$$

$$\text{Gain} = A_V = -g_m R_L \left(\frac{R_B // r_\pi}{R_i + (R_B // r_\pi)} \right)$$

→ When looking for Rin

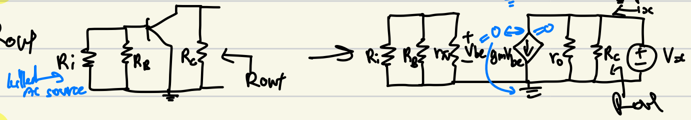
$R_{in} = R_B // r_\pi$



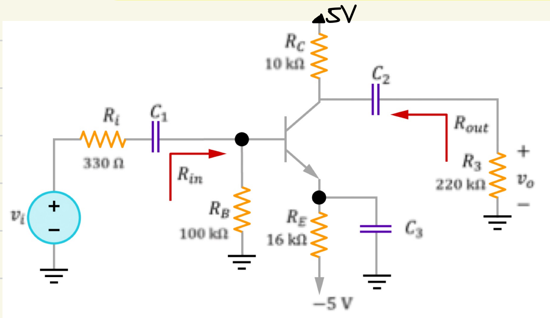
→ when looking for Rout

$R_{out} = \frac{V_x}{i_x}$ — killed AC source

↳ $R_{out} = r_o // R_c$



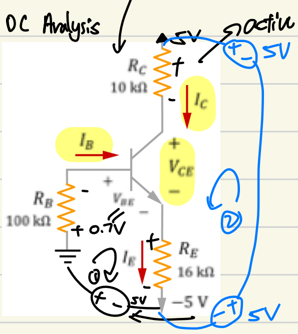
E.g. Given that common emitter current gain is $\beta = 65$ & early voltage $V_A = 50V$, find the voltage gain, input & output resistances of circuit. Assume that $V_{BE} = 0.7V$, $V_T = 25mV$ & that the BJT is biased for small-signal operation.



(1) DC analysis → Q point (I_B, I_C, V_{CE})

(2) AC analysis ← ↳ r_o, r_π, g_m

DC Analysis



(1) KVL for loop 1

$-I_B\,100k - V_{BE} - I_E\,16k + 5 = 0$ —— (2)

BJT active mode:

$I_C = \beta I_B$, $I_E = I_B + I_C = (\beta + 1) I_B$ —— (1) (β → 65)

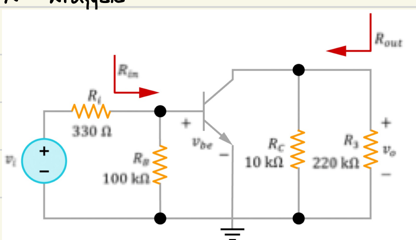
Sub (1) into (2), get I_B, then I_C, I_E

$3.71\mu A$ $241\mu A$ $245\mu A$

(2) KVL for loop 2

$5 - I_C\,10k - V_{CE} - I_E\,16k + 5 = 0$ → get $V_{CE} = 3.67V$

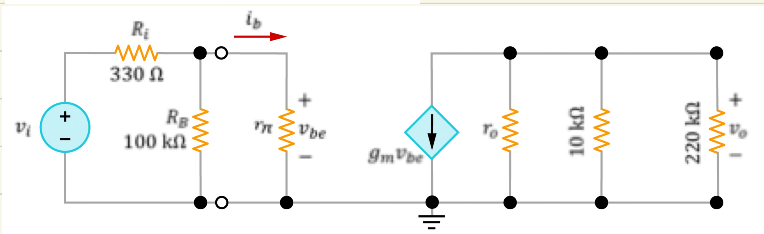
AC Analysis



$$g_m = \frac{I_C}{V_T} = 40 I_C = 9.64 \times 10^{-3} S$$

$$r_\pi = \frac{\beta}{g_m} = 6.64k\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = 223k\Omega$$



$$\text{Gain} = \frac{V_o}{V_i}$$

$$V_o = -g_m V_{BE} (r_o // 10k // 220k)$$

$$\frac{V_o}{V_{BE}} = -g_m (r_o // 10k // 220k)$$

$$V_{BE} = \frac{r_\pi // R_B}{R_i + r_\pi // R_B} V_i$$

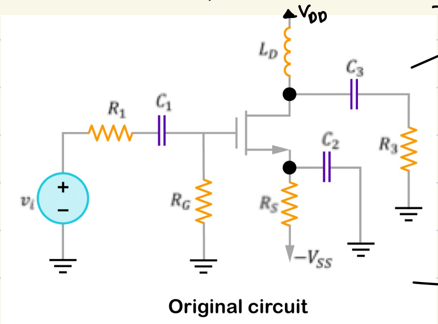
$$\frac{V_{BE}}{V_i} = \frac{r_\pi // R_B}{R_i + r_\pi // R_B}$$

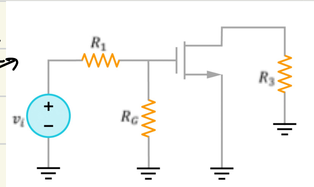
$$\text{Gain} = -g_m (r_o // 10k // 220k) \left(\frac{r_\pi // R_B}{R_i + r_\pi // R_B} \right)$$

$$= -83.8\ V/V$$

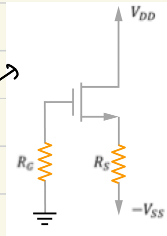
MOSFET as Amplifier (0.5V small signal)

A.C. Analysis $\dashv\vdash \rightarrow$ —•— cancel D.C volt

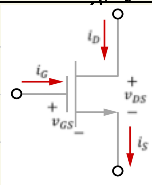


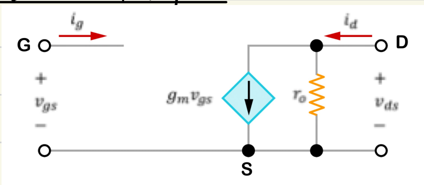


D.C. Analysis $\dashv\vdash \rightarrow$ — • — cancel AC voltage



Small-signal Model of MOSFET Amplifier





Transconductance: $g_m \approx \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_t)$

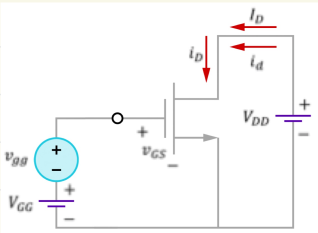
$\approx \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D}$

Output resistance: $r_o = \frac{\lambda^{-1}+V_{DS}}{I_D} \approx \frac{1}{\lambda I_D}$

$\lambda \rightarrow$ channel length modulation parameter

same

1. NMOS circuit



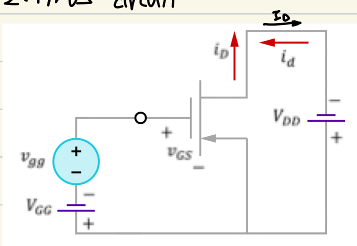
$V_{GS} = V_{GG} + v_{gg}$

$i_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS}-V_t)^2$

$i_D = I_D + i_d$

— $\uparrow v_{gg}$ causes $\uparrow$ in I into drain terminal

2. PMOS circuit

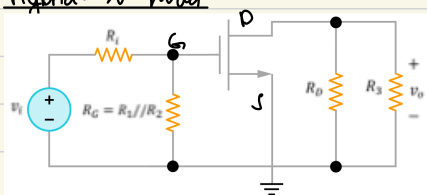


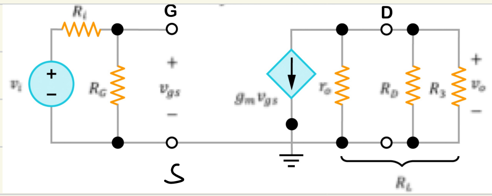
$v_{SG} = V_{GG} - v_{gg}$

$i_D = I_D - i_d$

— $\uparrow v_{gg}$ reduce source-gate voltage $\rightarrow \downarrow i_D$ exiting drain terminal $\rightarrow \uparrow i_d$ entering

Hybrid-π model





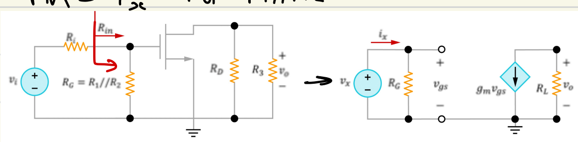
$A = \frac{V_o}{V_i} = \frac{V_o}{V_{GS}} \cdot \frac{V_{GS}}{V_i}$

$V_o = -g_m V_{GS}(r_o // R_D // R_3)$

$V_{GS} = \frac{R_1 // R_2}{R_i + R_1 // R_2}$

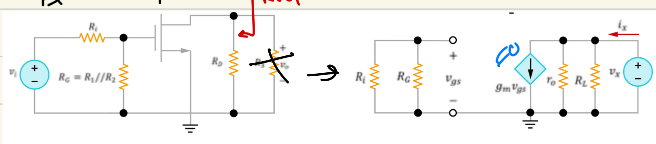
$\rightarrow$ Rin & Rout created like above.

$R_{in} = \frac{V_x}{i_x} = R_G = R_1 // R_2$



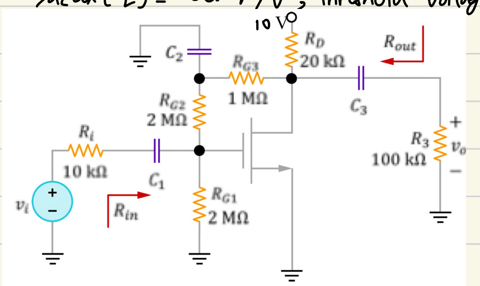
$\rightarrow$ Rout

$\frac{V_x}{i_x} = R_x = R_{out} = r_o // R_L$

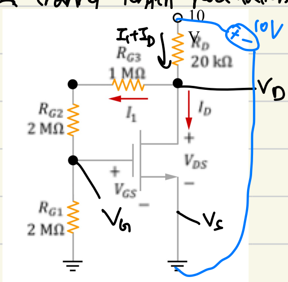


Small-signal analysis

E.g. Find the voltage gain, input & output resistance of the NMOS circuit as shown below. Assume that $\mu_n C_{ox}\left(\frac{W}{L}\right) = 500\mu A/V^2$, threshold voltage $V_t = 1V$ & channel length modulation parameter $\lambda = 0.0167V^{-1}$!



DC $\rightarrow$ analysis



Q point $\rightarrow V_{DS}, V_{GS}, I_D$?

$V_D - V_S = V_{DS}$

$V_G - V_S = V_{GS}$

$V_{DS} = I_1(1M + 2M + 2M)$

$V_{GS} = R_1(2M)$

$\frac{V_{DS}}{V_{GS}} = \frac{5}{2} = 2.5$

$V_{GS} = 0.4 V_{DS}$ ——— ①

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2 \text{ — ②}$$

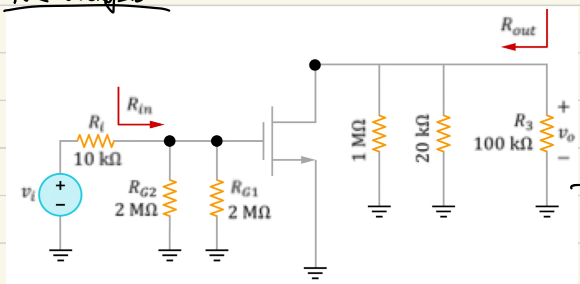
(1)

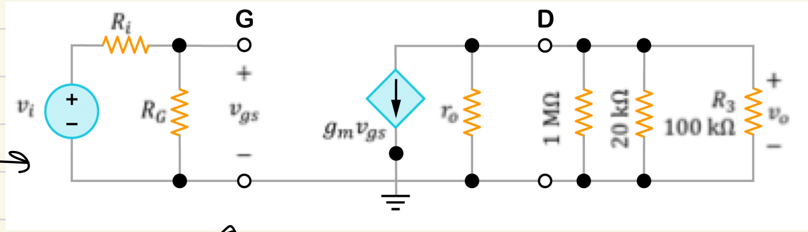
$$-10V + R_D(I_1 + I_D) + V_{GS} = 0 \text{ — ③ (KVL loop)}$$

$$I_1 = \frac{V_{GS}}{5M} \text{ — ④}$$

sub ①, ②, ④ into ③ & solve & get the rest

AC analysis





$A = \frac{v_o}{v_i}$ & get R_{in} & R_{out} by replacing them in