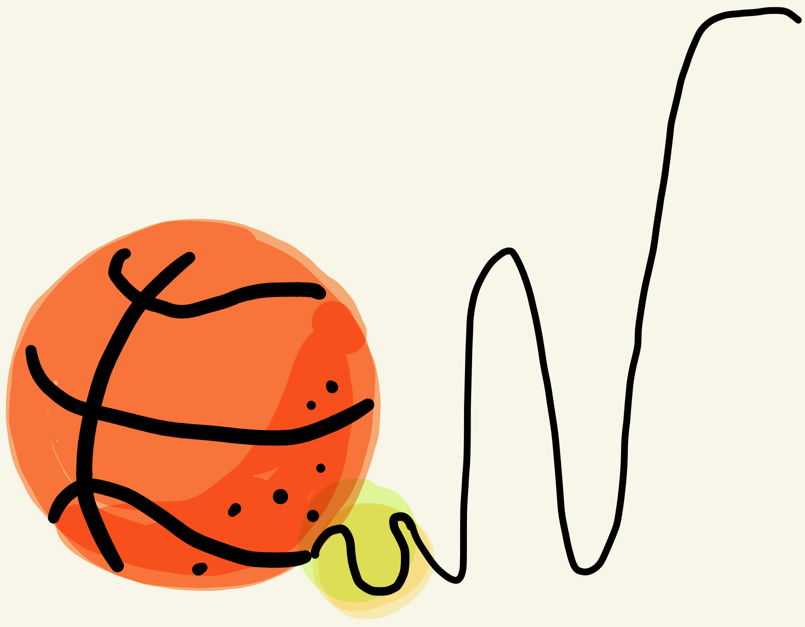
PHYSICS



Constant acceleration

$v = u + at$

$v^2 - u^2 = 2as$

$s = ut + \frac{1}{2}at^2 + s_0 \Rightarrow s = \frac{1}{2}(u+v)t$

zero acceleration

$v = u$

$s = ut + s_0$

General motion

Displacement = Final position − Initial position

Average velocity $= \frac{\text{displacement}}{\text{time elapsed}} = \frac{\Delta x}{\Delta t}$

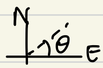
Average speed $= \frac{\text{distance travelled}}{\text{time elapsed}}$

Average acceleration $= \frac{\Delta V}{\Delta t}$

Instantaneous velocity $= \frac{dx}{dt}$

Instantaneous acceleration $= \frac{d^2x}{dt^2}$

* north of <u>east</u> → where angle starts from



Relative velocity

(denote $\vec{V}_{AR}$ as velocity of A from Roadside)

$\vec{V}_{AB} = \vec{V}_{AR} - \vec{V}_{BR}$, $\vec{V}_{BA} = \vec{V}_{BR} - \vec{V}_{AR}$

Velocity of A as seen from B (in B, minus B) → A relative to B

Circular Motion

$360° = 2\pi$ rad

Angular displacement, $\theta = \frac{s}{r}$ [angle which object turns w.r.t center of path]

Angular velocity, $\omega = \frac{\Delta\theta}{\Delta t} = 2\pi f = \frac{2\pi}{T}$

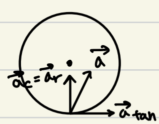
Angular acceleration, $\alpha = \frac{\Delta\omega}{\Delta t}$ $\neq a_c/a_r$

] → instantaneous / average

* Centripetal acceleration, $a_c/a_r = \frac{v^2}{r} = r\omega^2 = v\omega$ [radial a same as centripetal a]

Uniform circular motion → $\vec{a}_{tan} = 0$ ∴ $\vec{a} = \vec{a}_r$

Non-uniform circular motion: $\vec{a} = \vec{a}_r + \vec{a}_{tan}$ →



Δ direction of v

Δ magnitude of v $\left[a_{tan} = \frac{dV_{tan}}{dt}\right]$

linear (r = radius)	angular
$s = r\theta$	θ
$v_{tan} = r\omega$	ω
$a_{tan} = r\alpha$	α
$v = u + at$	$\omega_f = \omega_i + \alpha t$
$v^2 - u^2 = 2as$	$(\omega_f)^2 - (\omega_i)^2 = 2\alpha\theta$
$s = \frac{1}{2}(u+v)t$	$\theta = \frac{1}{2}(\omega_i + \omega_f)t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

Newton's First Law (N1L)

An object at rest remains at rest and an object in motion will continue in motion with constant velocity unless it experiences a non-zero net external force.

- Inertia → SI unit : kg

Newton's Second Law (N2L)

The rate of change of linear momentum of an object is proportional to the net force acting on it and occurs in direction of net force.

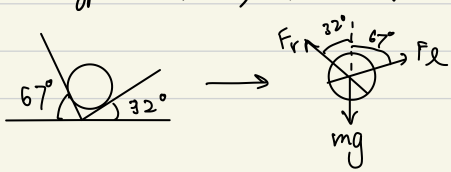
$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \rightarrow \vec{p} = m\vec{v}$$

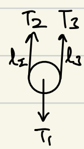
- if mass is constant → $\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\vec{a}$

Newton's Third Law (N3L)

If object A exerts a force on object B, then body B exerts a force on object A. These two forces are equal in magnitude but opposite in direction.

↳ same type of force, act on different bodies





$l_2 = l_3$ (length same)

$T_2 = T_3$ (massless pulley)

$T_2 + T_3 = T_1$

$2T_2 = \omega$

Hooke's law $\longrightarrow$ $F = k\Delta x$

Stress $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A}$

Strain $= \dfrac{\text{change in length}}{\text{original length}} = \dfrac{\Delta l}{l_0}$

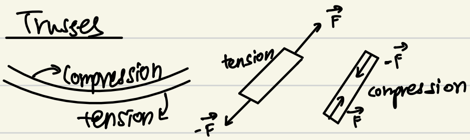
$\longrightarrow$ Young Modulus $E = \dfrac{\text{stress}}{\text{strain}} = \dfrac{\frac{F}{A}}{\frac{\Delta l}{l_0}} = \dfrac{kl_0}{A}$

only dependent on material

Translational Equilibrium: zero net force $\rightarrow F_{net} = 0$

Rotational Equilibrium: zero net torque $\rightarrow \tau_{net} = 0$

Trusses



E.g. Figure 1 shows a simple truss that carries a load at the center C of 1.35×10^4 N.

a) calculate the force on each strut at the pins, A, B, C, D.

b) Determine which struts (ignore their mass) are under tension & compression

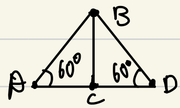
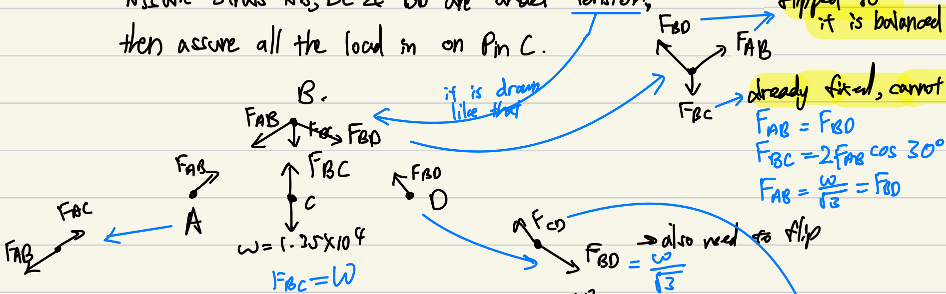


Figure 1

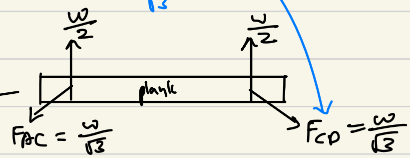
Assume struts AB, BC & BD are under tension, then assume all the load is on pin C.



BC – tension

AB, BD – compression (if got flipped)

strut ACD – tension



Static friction, $f_s = \mu_s F_N$ (opp. anticipated motion)

Kinetic friction, $f_k = \mu_k F_N$ (opp. direction of motion)

For cases where box is stationary now $\longrightarrow$ box about to slide down $\rightarrow f_s$ up

$\searrow$ box about to slide up $\rightarrow f_s$ down

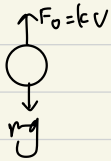
Upthrust, $U = \rho_{fluid} V_{submerged}\, g = m_{fluid\ displaced}\, g$

Drag Force (Only in Liquid/gas)

→ Low speed: $F = kv$

→ High speed: $F = bv^2$

- When 2 forces become equal, terminal velocity occurs. Derivation:



N2L: $mg - kv = ma$

$$mg - kv = m\frac{dv}{dt}$$

$$g - \frac{k}{m}v = \frac{dv}{dt}$$

$$\int dt = \int \frac{1}{g - \frac{k}{m}v}\,dv$$

$$t = \left(-\frac{m}{k}\right)\ln\left(g - \frac{k}{m}v\right) + c$$

$$-\frac{k}{m}t = \ln\left(g - \frac{k}{m}v\right) + c$$

$$e^{-\frac{k}{m}t - c} = g - \frac{k}{m}v$$

$$e^{-\frac{k}{m}t - c} - g = -\frac{k}{m}v$$

$$-\frac{m}{k}e^{-\frac{k}{m}t - c} + \frac{mg}{k} = v$$

$$v = \frac{mg}{k} - Ae^{-\frac{k}{m}t}, \quad A = -\frac{m}{k}e^{-c}$$

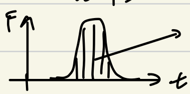
sub $t=0, v=0$ → $v = \frac{mg}{k}\left(1 - e^{-\frac{k}{m}t}\right)$

★ $V(t) = \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right)$

Impulse (vector quantity, same direction as F)

$\vec{I} = \vec{F}\Delta t$

$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$



$I = \int_{t_i}^{t_f} \vec{F}\,dt = F_{avg}(t_f - t_i)$

$\Delta p = F_{avg}\Delta t$

Collisions

1. Perfectly Elastic Collisions

Conservation of Kinetic Energy [COKE → $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$]

2. Completely Inelastic Collision → objects get stuck together

3. Explosion → a single mass disintegrating into many pieces

Conservation of linear momentum (COLM) applies to all.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

↳ In some cases, may need to split it into horizontal & vertical components

★ [Explosions]

Average Force brought by moving mass from 0 to v at rate of $\frac{dm}{dt}$,

$$F = v\frac{dm}{dt}$$

$\frac{dm}{dt} \rightarrow$ mass flow rate

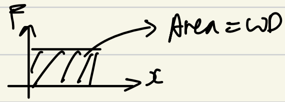
$v \rightarrow$ velocity

$$F = v\frac{dm}{dt} = v\frac{d}{dt}(\rho V)$$
$$= v\rho A\left(\frac{dh}{dt}\right)$$

Work done (path independent)

Constant force: $W = \vec{F}\cdot d = |\vec{F}||\vec{d}|\cos\theta$

Non-constant force: $W = \int \vec{F}\cdot d\vec{x}$



Energies

Gravitational potential energy, $GPE = mgh$

Non-constant force on object moving through large vertical distance, $WD = \int_{r_a}^{r_b} \frac{GMm}{r^2}\,dr$

Elastic potential energy, $EPE = \frac{1}{2}kx^2$

Kinetic potential energy, $KPE = \frac{1}{2}mv^2$

$$\left[\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \int \vec{F}\cdot d\vec{x}\right]$$

Relationship b/w PE & forces

a) Non-conservative forces $\Rightarrow$ cannot define a PE (friction, drag)

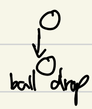
b) Conservative forces $\Rightarrow$ can define a PE

$$\Delta U = \Delta PE = -W_{conservative} = -\int_{initial}^{final} \vec{F}\cdot d\vec{x}$$

$$\Delta KE = W = W_{conservative} + W_{non\text{-}conservative}$$

$$\Delta KE = -\Delta PE + W_{non\text{-}con}$$

$$F = -\frac{d(PE)}{dx} \longrightarrow \ominus\text{ve sign}$$



① mg // displacement, WD by $mg > 0$

② $\Delta V = \Delta mgh < 0$ as $h\downarrow$

☆ COE: $KE_f + PE_f = KE_i + PE_i + W_{non\text{-}con}$

Power

$$P = \frac{\Delta WD}{\Delta t} = \frac{F\Delta x}{\Delta t} = Fv$$

$P = F\frac{dx}{dt}$ (instantaneous power)

Centre of mass

↳ point where all the mass is supported & net moment about point is zero

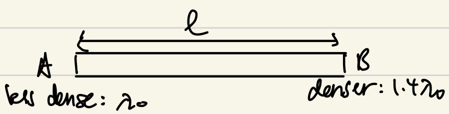
Discrete mass: $x_{cm} = \frac{1}{M} \sum_{i=1}^{N} m_i x_i$ where M is total mass of objects

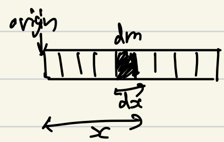
Continuous mass: $x_{cm} = \frac{1}{M} \int x \, dm$ where $M = \int dm$

Writing dm:

3D large mass : $m = \rho V$	3D small mass : $dm = \rho dV$ (not differentiation.)
2D large mass : $m = \mu A$ (density/area)	2D small mass : $dm = \mu dA$
1D large mass : $m = \lambda l$ (density/length)	1D small mass : $dm = \lambda dl$

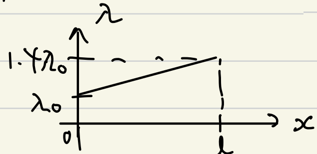
Eg. A non-uniform rod of length l is shown in figure 1. The mass density varies linearly from λ_0 at the end A to $1.4\lambda_0$ at the end B. You are given that the mass of the rod is $M = 1.2\lambda_0 l$. Show that the centre of mass of rod is $\frac{19}{36} l$ from end A.





1D as problem only has length ← $x_{cm} = \frac{1}{M} \int x \, \underbrace{dm}$ & take A as the origin

$dm = \lambda \, dx$



$y = mx + c$

$\lambda = \frac{1.4\lambda_0 - \lambda_0}{l - 0} x + \lambda_0$

$\lambda = \frac{0.4\lambda_0}{l} x + \lambda_0$

$x_{cm} = \frac{1}{1.2\lambda_0 l} \int x \lambda \, dx$

$x_{cm} = \frac{1}{1.2\lambda_0 l} \int x \left(\frac{0.4\lambda_0}{l} x + \lambda_0\right) dx$

$x_{cm} = \frac{19}{36} l$

Moment of Inertia (I)

Discrete masses : $I = \sum_i m_i r_i^2$

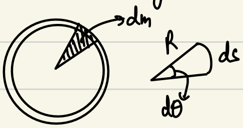
Continuous masses : $I = \int r^2\, dm$ → continuous summation of small slices of discrete masses

① Find a small slice of the shape

② Use density to get mass of small slice

③ Sub the formula gotten from on top into integration formula & solve

Eg. Compute the moment of inertia of a thin hoop of mass M & radius R about the cylindrical axis.



$dm = \lambda\, ds \qquad \lambda = \frac{\text{mass}}{\text{length}}$

$= \lambda R\, d\theta$

$= \frac{M}{2\pi \cancel{R}} \cancel{R}\, d\theta$

$= \frac{M}{2\pi}\, d\theta$

$I = \int r^2\, dm$

$= \int_0^{2\pi} R^2 \frac{M}{2\pi}\, d\theta$

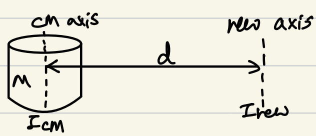
$= \frac{MR^2}{2\pi} \int_0^{2\pi} d\theta$

$= MR^2$

Parallel axis theorem

↳ Relates I_{cm} to another I

$I_{new} = I_{cm} + Md^2$ → M: mass of object, d: distance b/w 2 axis



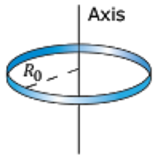
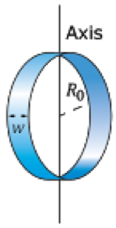
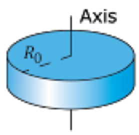
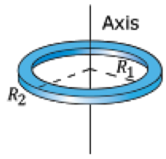
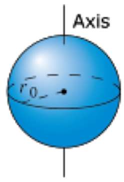
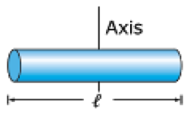
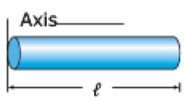
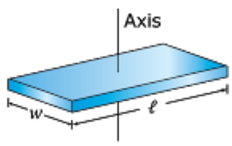
Perpendicular axis theorem

$I_z = I_x + I_y$

I_x, I_y & I_z are moment of inertia about x, y, z axes.

Linear		Rotational
mass, m		moment of inertia, I
$p = mv$	COLM	$L = I\omega$ (angular momentum)
$F = ma = \frac{dp}{dt}$		$\tau = I\alpha = \frac{dL}{dt}$ (torque)
$KE = \frac{1}{2}mv^2$		$RKE = \frac{1}{2}I\omega^2$

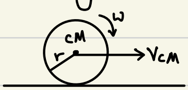
Moment of Inertia >> Example: (provided)
Table of Moment of Inertia for various objects with axis of rotation about CM

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R_0	Through center		MR_0^2
(b)	**Thin hoop,** radius R_0 width w	Through central diameter		$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R_0	Through center		$\frac{1}{2}MR_0^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius r_0	Through center		$\frac{2}{5}Mr_0^2$
(f)	**Long uniform rod,** length ℓ	Through center		$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end		$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center		$\frac{1}{12}M(\ell^2 + w^2)$

Rotational Dynamics

- Torque is defined by $\vec{\tau} = \vec{r} \times \vec{F}$ — $\vec{r}$: radius, $\vec{F}$: force
- Angular Momentum $\rightarrow$ $\vec{L} = \vec{r} \times \vec{p}$ — $\vec{p}$: momentum, $\vec{r}$: radius
- COAM $\rightarrow$ $\vec{L_i} = \vec{L_f}$

Rolling without slipping



$v_{cm} = r\omega$

$a_{cm} = r\alpha$

$\rightarrow$ only true for rolling without slipping, not the same as non-uniform circular motion

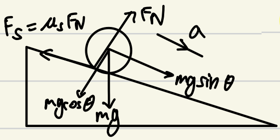
① Rotation about CM $\rightarrow$ combination of forces that does not pass through CM

$\tau = I\alpha$

② Translation of CM $\rightarrow$ normal $F = ma$

Total $KE = \frac{1}{2} I_{cm} \omega^2 + \frac{1}{2} m v^2_{cm}$

E.g. Represent the acceleration of hollow ball with radius R and mass m rolling down slope in θ which is the angle of slope and g which is acceleration



Hollow ball: $I = \frac{2}{3} m R^2$

$a = R\alpha \Rightarrow \alpha = \frac{a}{R}$

Rotation about CM: $\tau = I\alpha$

$$F_s \cancel{R} = \frac{2}{3} m R^{\cancel{2}} \left(\frac{a}{\cancel{R}}\right)$$

$$F_s = \frac{2}{3} m a$$

$$\cancel{m} g \cos\theta \, \mu_s = \frac{2}{3} \cancel{m} a$$

$$\mu_s = \frac{2a}{3g\cos\theta} \text{ — ①}$$

Translation of CM: $F = ma$

$$F - f_s = ma$$

$$\cancel{m} g \sin\theta - \cancel{m} g \cos\theta \, \mu_s = \cancel{m} a$$

$$g \sin\theta - g\cos\theta \, \mu_s = a \text{ — ②}$$

Combine ① & ②

$$g \sin\theta - \cancel{g\cos\theta}\left(\frac{2a}{3\cancel{g\cos\theta}}\right) = a$$

$$g \sin\theta = a + \frac{2}{3}a = \frac{5}{3}a$$

$$a = \frac{3}{5} g \sin\theta$$

Gravitational Field (mass is the source)

1. Force: $\vec{F} = m\vec{g}$ (general formula)

$\vec{F} = \frac{GMm}{r^2}(-\hat{r})$ unit vector (radially inwards) $\hat{r} \rightarrow$ radially outwards

only for point masses

2. Field: $\vec{g} = \frac{GM}{r^2}(-\hat{r})$ [only for point masses]
3. Potential: $V/\phi = -\frac{GM}{r}$ [only for point masses]
4. Potential Energy: $U = m\phi$ [general]

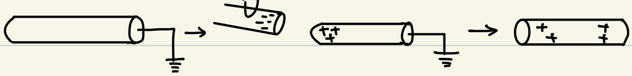
$U = -\frac{GMm}{r}$ [only for point masses]

$\vec{F} = -\frac{dU}{dr}$ $\qquad \Delta U = -\int_{initial}^{final} \vec{F} \cdot d\vec{r}$

$\vec{g} = -\frac{dV}{dr}$ $\qquad \Delta V = -\int_{initial}^{final} \vec{g} \cdot d\vec{r}$

Electric Field (charge is the source)

- Coulomb (C) → amount of charge that flows in 1 second when there is a steady current of 1 ampere.
- Charges: $e = 1.60217733 \times 10^{-19}$ C. Electron charge → $-e$. Proton charge → e
- Earth neutralises charges



1. Electric Force: $\vec{F} = q\vec{E}$ [general]

$\vec{F} = \frac{kQq}{r^2}(\hat{r}) = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ [only for point charges]

Permittivity of free space (vacuum): $\varepsilon_0 = 8.8541878 1762 \times 10^{-12} \frac{C^2}{Nm^2}$

Constant of proportionality k in vacuum: $k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \ Nm^2/C^2$

$\vec{F}_{12} \leftarrow (+)_1 \qquad (+)_2 \rightarrow \vec{F}_{21}$ [Replusion]

(force on 1 due to 2) $\qquad$ (force on 2 due to 1)

2. Field: $\vec{E} = \frac{kQ}{r^2}\hat{r} = \frac{Q}{4\pi\varepsilon_0 r^2}$ [only for pt charges]
3. Potential: $V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$ [only for pt charges]
4. Potential Energy: $U = qV = \frac{Qq}{4\pi\varepsilon_0 r}$ [general]

$$U = \frac{kQq}{r}$$ [only for pt charges]

$$\vec{F} = -\frac{dU}{dr} \qquad \Delta U = -\int_{initial}^{final} \vec{F} \cdot d\vec{r}$$

$$\vec{E} = -\frac{dV}{dr} \qquad \Delta V = -\int_{initial}^{final} \vec{E} \cdot d\vec{r}$$

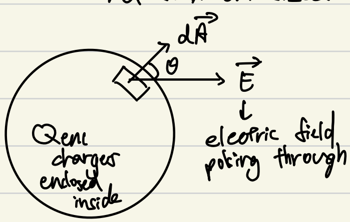
- To find $\vec{E}$ (resultant field) for a group of N points charges, add them tgt
  $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$
- Parallel Plates: E-field is uniform → $E = \frac{V}{d}$, V is potential difference across plate & d is plate separation.

Gauss Law

$$\underbrace{\oint \vec{E} \cdot d\vec{A}}_{\text{net flux over closed imaginary surface}} = \frac{Q_{enc}}{\varepsilon_0}$$

$\oint$ → integration over closed surface

Q_{enc} → charge enclosed inside the imaginary surface



$|d\vec{A}|$ = small area

direction of $d\vec{A}$ → normal vector (pointing outwards)

small flux = $\underbrace{\vec{E} \cdot d\vec{A}}_{\text{3 cases}} = E\cos\theta \, dA$

→ flux "poke out" → positive dot pdt (θ less than 90°)
→ flux "poke in" → negative dot pdt (θ more than 90°)
→ flux neither in nor out → zero dot pdt ($\theta = 90°$)

Conductors (in equilibrium) → no net motion of charges
- E-field is zero everywhere inside the conductor material (if conductor isolated → charges reside on surface)
- can be solid/hollow conductor but E may not be zero in hollow space
- If excess charges added to conductor:

Remember 3 examples 4 [GAUSS LAW]

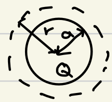
charge / volume

1. Spherical

Insulating solid sphere of radius a has a uniform volume charge density ρ & carries a total positive charge Q. Calculate electric field at point

a) Choose spherical Gaussian surface outside sphere

↳ $\vec{E}$ & $d\vec{A}$ parallel over surface → $\vec{E} \cdot d\vec{A} = E\,dA \cos 0$



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$ → charge enclosed

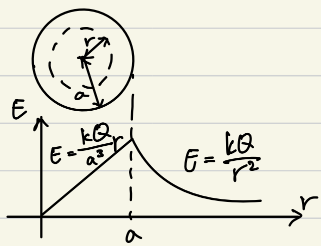
$$\oint E\,dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{Q_{enc}}{\varepsilon_0}$$ → $|\vec{E}|$ same value of the surface → constant wrt integration

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$ → $Q_{enc} = Q$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$

b) Choose spherical Gaussian surface inside sphere



$$Q_{enc} = \rho \times \frac{4}{3}\pi r^3$$

$$= \frac{Q}{\frac{4}{3}\pi a^3} \times \frac{4}{3}\pi r^3 = \frac{Q r^3}{a^3}$$

$$E(4\pi r^2) = \left(\frac{Q r^3}{a^3}\right) \div \varepsilon_0$$

$$E = \frac{Qr}{4\pi\varepsilon_0 a^3} = \frac{kQ}{a^3} r$$

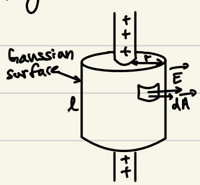
2. Infinite line of charge (cylindrical)

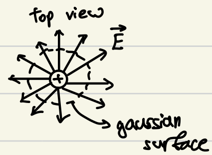
Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ.

↳ Choose a cylindrical imaginary Gaussian surface →

↳ $\vec{E}$ & $d\vec{A}$ // over surface → $\vec{E} \cdot d\vec{A} = E\,dA$

↳ 2 circular faces $\vec{E} \perp d\vec{A}$ → $\vec{E} \cdot d\vec{A} = 0$





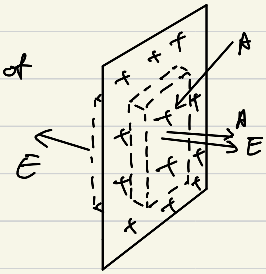
$$\oint E\,dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi r \ell) = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda \ell}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

3. Infinite charged plate

Find the electric field due to an infinite insulating plate of positive charge with uniform surface charge density σ.



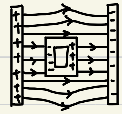
- → Choose box-shape imaginary Gaussian surface
- → $\vec{E}$ & $d\vec{A}$ // on 2 faces of box, $\vec{E} \cdot d\vec{A} = EdA$
- → Other 4 faces, $\vec{E} \perp dA \Rightarrow \vec{E} \cdot d\vec{A} = 0$

$$E \oint dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2A) = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2A) = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$



1. Excess charges move to surfaces of conductor
2. Excess charges distributed on surfaces so that all points on conductor — whether on surface or inside → come to same potential.

Microscopic Model of Electrical Conduction

$I = nqV_dA$ → cross sectional area

n → carrier density, q → charge, V_d → drift velocity

$n_e = \frac{N_A}{V}$ (N_A → advogadro constant, V → volume)

$J = \frac{I}{A}$ → current density

- With no electric field, electron has frequent collisions with ions, but no net displacement

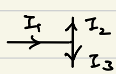
- Ohm's law
  i) $V = IR$
  ii) $J = \sigma E$ (J → current density (A/m²), σ → conductivity, E → E field due to battery)

- Definition of resistance
  i) $R = \frac{V}{I}$ (circuit definition)
  ii) $R = \frac{\rho l}{A}$ (material definition) — resistivity ($\rho = \frac{1}{\sigma}$)

- Power: $P = I^2R = VI = \frac{V^2}{R}$

Kirchhoff's Law

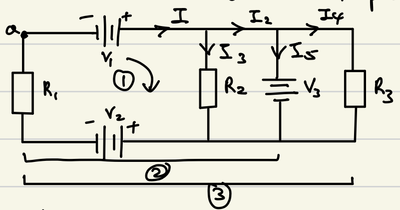
1) Junction Rule: at any circuit junction, assign



$I_1 = I_2 + I_3$ ← consistent choice

2) Voltage Rule: Choose any closed circuit loop ("walk" around loop)
   → experience rise in PD, put positive voltage term
   → experience drop in PD, put negative voltage term
   → at the end of "walking" put 0 at RHS

E.g. There is a circuit with $V_1 = 5V$, $V_2 = 10V$, $V_3 = 1V$, $R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$. Calculate current passing through resistor R_3



$I = I_2 + I_3$
$I_2 = I_4 + I_5$ → 3 junction equations
$-6 = I_4 + I_5 + I_3$

Voltage equation start from a:

Loop ①: $V_1 - I_3R_2 - V_2 - IR_1 = 0$ (from ⊕ to ⊖ in battery → drop)

$5 - 2I_3 - 10 - I = 0 \longrightarrow -5 - 2I_3 - I = 0$

Loop ②: $5 - V_3 - 10 - I = 0$ (if walk in same direction as current into $R \Rightarrow -IR$)

$5 - 1 - 10 - I = 0 \longrightarrow -6 = I$

Loop ③: $5 - I_4R_3 - 10 - I = 0$

$5 - 3I_4 - 10 - I = 0 \longrightarrow -5 - I - 3I_4 = 0$

$I_3 = -\frac{1}{2}A$, $I_2 = -\frac{11}{2}A$, $I_4 = -\frac{1}{3}A$, $I_5 = -\frac{31}{6}A$

$\therefore I_4 = \frac{1}{3}A$ from loop ③

Resistors: a) Series resistors → $R_{eq} = R_1 + R_2 + \ldots$

b) Parallel resistors → $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$

Capacitance (material/geometric properties)

Capacitance, C of capacitor: $C = \frac{Q}{\Delta V}$ (circuit definition)

1. Parallel plate capacitor: $C = \frac{\varepsilon_0 A}{d}$ → A: area of plate; d → spacing b/w plate

★ 2. Cylindrical capacitor: $C = \frac{2\pi\varepsilon_0 R}{\ln\left(\frac{R_{outer}}{R_{inner}}\right)}$ → R: length of capacitor

Energy stored in capacitor: $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

Capacitors: a) Series capacitor: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$

b) Parallel capacitor: $C_{eq} = C_1 + C_2 + \ldots$

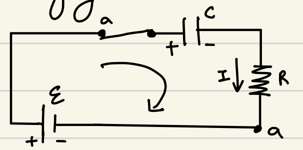
Dielectric ($\kappa = 80$ for water & $\kappa \sim 300$ for $SrTiO_3$)

- Can change medium between plates to change capacitance → can add insulating material between plates known as dielectric
- Charges remain unchanged in both cases but when diaelectric inserted b/w plates, p.d. ↓, capacitance ↑

$$\Delta V = \frac{\Delta V_0}{\kappa} \qquad C = \kappa C_0$$

κ → diaelectric constant

Charging for circuit with 1R1C → q & I Δ w time



Using Kirchhoff's loop law from a,

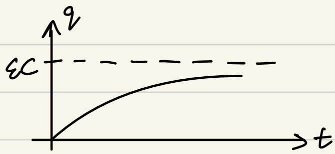
$$\varepsilon - \frac{q}{c} - IR = 0$$

$$\varepsilon - \frac{q}{c} - R\frac{dq}{dt} = 0$$

$$\int \frac{1}{RC}\,dt = \int \frac{1}{\varepsilon C - q}\,dq$$

$$\frac{t}{RC} = -\ln(\varepsilon C - q) + A$$

$$q = \varepsilon C - e^{A} e^{-\frac{t}{RC}}$$



assume $t=0$, $q=0$

$$0 = \varepsilon C - e^{A}$$

$$e^{A} = \varepsilon C$$

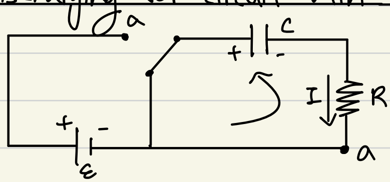
$$q(t) = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right)$$ → charging eqn

only for 1R1C:

$$I(t) = \frac{dq(t)}{dt} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

$$V(t) = \frac{q(t)}{C} = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

Discharging for circuit with 1R1C



Kirchhoff's loop law from a:

$$-IR + \frac{q}{C} = 0$$

$$-\left(-\frac{dq}{dt}\right)R + \frac{q}{C} = 0$$

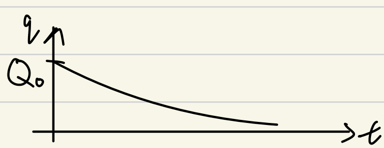
current decreasing

$$\frac{dq}{dt} = -\frac{q}{RC}$$

$$\int \frac{1}{q}\,dq = \int dt \left(-\frac{1}{RC}\right)$$

$$\ln q = -\frac{t}{RC} + A$$

$$q = e^{-\frac{t}{RC}} e^{A}$$



1R1C only $\leftarrow$ $\begin{cases} I(t) = \dfrac{dq(t)}{dt} = -\dfrac{Q_0}{RC}e^{-\frac{t}{RC}} \\ V(t) = \dfrac{q(t)}{C} = \dfrac{Q_0}{C}e^{-\frac{t}{RC}} \end{cases}$

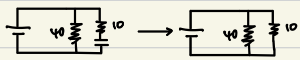
Assume $t=0$, $q=Q$

$q(t) = Qe^{-\frac{t}{RC}}$ → discharging eq$\overline{n}$

☆ Try to reduce problems to 1R1C

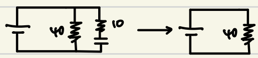
RC circuit tricks (charging/discharging)

Trick ①: Short time trick (for charging)



↳ capacitor as wire → immediately after switch is closed → current flows freely with no resistance

Trick ②: Long time trick (for charging)



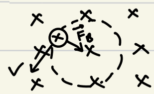
↳ treat capacitor as open circuit → long after switch is closed → capacitor is fully charged & no current flows through capacitor

Magnetic Force

Magnetic Force on a) a moving charge: $\vec{F} = q\underbrace{\vec{v}}_{\text{velocity}} \times \underbrace{\vec{B}}_{\text{magnetic flux density}} = qvB\sin\theta$
(Deflection force)

↳ $\vec{F} \perp$ to both $\vec{v}$ & $\vec{B}$

↳ magnitude of velocity no Δ, F_B towards center



$F_B = qvB = \dfrac{mv^2}{r} = mr\omega^2$

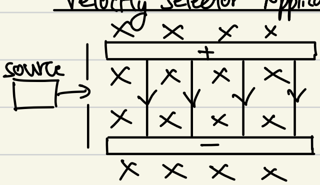
$r = \dfrac{mv}{qB}$ $\omega = \dfrac{qB}{m}$

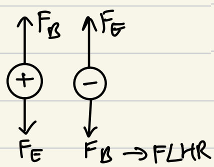
mass-charge ratio: $\dfrac{m}{q} = \dfrac{rB}{v}$

b) on a current carrying conductor: $\vec{F} = I\vec{L} \times \vec{B} = BIL\sin\theta$

($\vec{L}$: length of wire → in direction of current)

Velocity selector Application



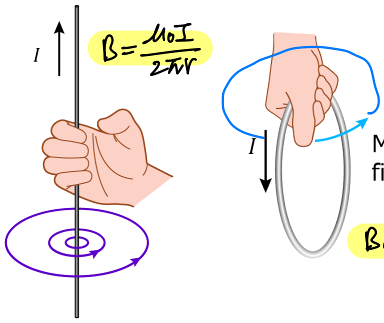


$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

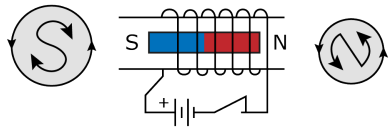
$0 = qE - qvB$ (charge does not deflect)

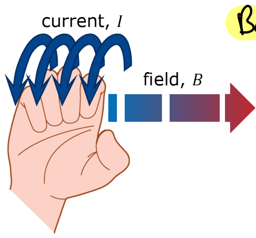
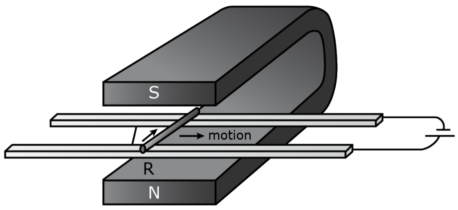
$v = \dfrac{E}{B}$

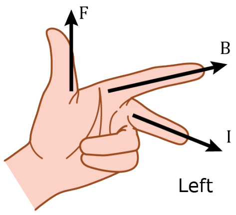
Magnetic Fields >> Summary of ALL hand rules:

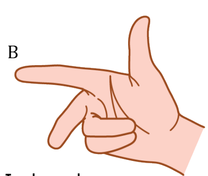
<table>
<tr><th></th><th>Left Hand</th><th>Right Hand</th></tr>
<tr>
<td>Magnetic Field Problem</td>
<td>No left hand rule.
permeability of free space
$\mu_0 = 4\pi \times 10^{-7}\ T\cdot m/A$</td>
<td>Right Hand Grip Rule:


$B=\frac{\mu_0 I}{2\pi r}$
$B_{center}=\frac{\mu_0 I}{2r}$
radius of coil

Solenoid Rule:


$B_{center}=\mu_0 n I$

</td>
</tr>
<tr>
<td>Magnetic Force Problem</td>
<td>Fleming's Left Hand Rule:



</td>
<td>No right hand rule.
For an arbitrary shaped wire in $\vec{B}$ field, should consider small force $d\vec{F}_B$ exerted on small segment vector length $d\vec{s}$
$d\vec{F}_B = I\, d\vec{s}\times\vec{B}$
$\vec{F}_B = I\int_a^b d\vec{s}\times\vec{B}$
(a, b represent end-point of wire)

</td>
</tr>
<tr>
<td>Induction Problem</td>
<td>No left hand rule.</td>
<td>Fleming's Right Hand Rule:

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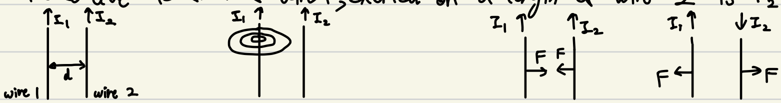
</table>

E.g. Magnetic force produced at position of wire 2 due to the current in wire 1 distance d away is $B = \frac{\mu_0 I_1}{2\pi d}$

Force due to field of wire 1, exerted on a length of wire 2 is $F_2 = B_1 I_2 \ell = \frac{\mu_0 I_1 I_2}{2\pi d}\ell$



Biot–Savart's Law

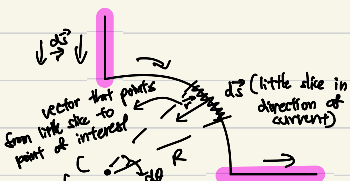
↳ How magnetic field look like due to currents:

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$d\vec{s}$ = small slice of current with magnitude = small length of wire & direction = direction of current

$\hat{r}$ = unit vector that points from the small slice of current to the point of interest

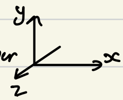
E.g 1. One quarter of a circular loop of wire carries a current I. The current I enters & leaves on a straight segment of wire, as shown; the straight wires are along the radial direction from the center C of the circular portion. Find the magnetic field at point C.



↳ $ds \parallel$ to $\hat{r} \rightarrow d\vec{s} \times \hat{r} = 0$

For quarter circle segment

$d\vec{s} \times \hat{r} = R\, d\theta\, (-\hat{k}) \rightarrow$ points into paper



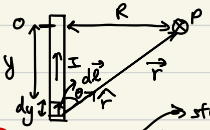
$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{R\, d\theta}{R^2}(-\hat{k})$

$= \frac{\mu_0 I}{4\pi R}\int_0^{\frac{\pi}{2}} d\theta\, (-\hat{k})$

$= \frac{\mu_0 I}{8R}$

Eg 2. For the field near a long straight wire carrying a current I, show that the Biot–Savart's law gives the familiar result (P) $B = \frac{\mu_0 I}{2\pi r}$.

$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times \hat{r}}{r^2}$



start from 0

try to ∆ all variable into one

Let $y = -\frac{R}{\tan\theta}$

$dy = R\,\mathrm{cosec}^2\theta\, d\theta$

$= \frac{R\,d\theta}{\sin^2\theta} = \frac{R\,d\theta}{(\frac{R}{r})^2}$

$= \frac{r^2\,d\theta}{R}$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{|dy||\hat{r}|\sin\theta}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\int_0^{\pi} \frac{\sin\theta}{\cancel{r^2}}\,\frac{\cancel{r^2}\,d\theta}{R}$$

$$= \frac{\mu_0 I}{4\pi R}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{\mu_0 I}{2\pi R}$$

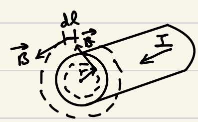
Ampere's law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

net B lying on an imaginary closed Amperian loop

E.g 1. A long straight cylindrical wire conductor of radius R carries a current I of uniform current density in the conductor. Assume that r, the radial distance from axis is much less than length of wire. Determine magnetic field due to this current at

a) points outside conductor $(r>R)$:



since // can write as

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\oint B\,dl = \mu_0 I$$

perimeter $B\oint dl = \mu_0 I$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

b) points inside conductor $(r<R)$:

$$B(2\pi r) = \mu_0 I_{enc}$$

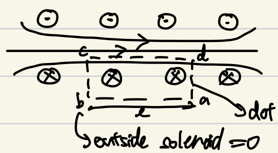
$$B = \frac{\mu_0}{2\pi r}\,\frac{Ir^2}{R^2} = \left(\frac{\mu_0 I}{2\pi R^2}\right) r$$

current density $= \frac{I}{\pi R^2}$

$I_{enc} = \frac{I}{\pi R^2} \times \pi r^2$ → area enclosed by loop

Eg 2. A solenoid is a coil of wire containing many loops. To find the field inside, we use Ampere's law along the path indicated in the figure. The magnetic field at the side of the solenoid is approximately half of that in the middle,

$B_{end} \approx \frac{1}{2} B_{centre}$



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$B(cd) + 0 + 0 + 0 = \mu_0 N I$ → number of wires

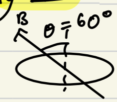
$Bl = \mu_0 N I$

$B = \mu_0 \frac{N}{l} I = \mu_0 n I$

Electromagnetic Induction (Making electricity from changing magnetic fields)

Magnetic flux (ϕ) [Unit: Weber, Wb]

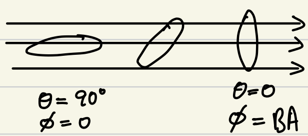
↳ product of an area & the component of the magnetic flux density ⊥ to that area



$\phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta$

magnetic flux density (T)

area (m^2)



$\rightarrow N\phi = NBA\cos\theta$

Faraday's law of electromagnetic induction

↳ e.m.f induced in a conductor is directly proportional to the rate of change of magnetic flux linkage / rate of cutting of magnetic flux

induced EMF ← $\mathcal{E} = -\frac{d\phi_B}{dt}$ → magnetic flux linkage

↳ due to lenz's law

2 Interpretations of Faraday's law:

either [1) Change in flux linkage : $\mathcal{E} = -\frac{d\Phi_B}{dt}$

2) Flux cutting (motional EMF) : $\mathcal{E} = Blv$ → velocity of cutting flux

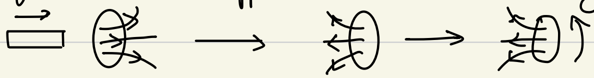
→ length of conductor cutting flux

$V = \frac{\mathcal{E}}{R}$

Lenz's law

↳ the induced current produces a magnetic field to oppose the magnetic change

a) Magnet moved towards coil → ↑ magnetic flux → induced current in coil induces a magnetic field to oppose Δ in flux caused by magnet



→ use right-hand grip rule → determine direction of current

b) Shrinking coil (coil area ↓) in magnetic field pointing into paper → $\phi \downarrow$ → induced current will be clockwise to produce its own magnetic field into page to make up for this $\downarrow \phi$

Eddy currents

↳ As conducting plates enters the field → flux cutting induces an e.m.f on plate → causes free electrons in plate to move → produce swirling currents (eddy currents)

↳ slow down plate & bring back to original position while heating up the plate.

Temperature, Thermal Expansion & Ideal Gases

Zeroth Law of Thermodynamics: If systems A & B are each in thermal equilibrium with a third system C, then A & B are in thermal equilibrium with each other

1. Linear expansions

↳ temperature of solid Δ → change in length $\Delta L \propto$ change in temperature ΔT

$$\Delta L = \alpha L_0 \Delta T \quad \& \quad L = L_0(1+\alpha\Delta T)$$

L_0 → original length, α → coefficient of linear expansion (unit K^{-1} / $°C^{-1}$)

2. Volume expansions

↳ $\Delta V \propto \Delta T$

$$\Delta V = \beta V_0 \Delta T \quad \& \quad V = V_0(1+\beta\Delta T)$$

V_0 → original volume, β → coefficient of volume expansion

→ $\beta \approx 3\alpha$ $[V_0 = l_0 w_0 h_0]$

E.g. 70ℓ steel gas tank of a car is filled to the top with gasoline at 20°C. The car sits in the Sun & the tank reaches a temperature of 40°C How much gasoline do you expect to overflow from the tank? $\alpha_{steel} = 12 \times 10^{-6}\ °C^{-1}$ & $\beta_{gas} = 950 \times 10^{-6}\ °C^{-1}$

$$\Delta V_{steel} = 3\alpha_{steel} \times 70 \times (40-20)$$

$$\Delta V_{gas} = \beta_{gas} \times 70 \times (40-20)$$

$$\therefore \text{overflow} = \Delta V_{gas} - \Delta V_{steel} = 1.28\ L$$

Fixed points

1. Ice point → pure ice & water co-exist in equilibrium at atmospheric pressure [0°C; ~101kPa]
2. Steam point → pure water and steam co-exist in equilibrium at atmospheric pressure [100°C; ~101kPa]

3. Triple point of water → pure ice, water & water vapour can exist tgt in equilibrium [273.16K, 0.01°C; 611.73 Pa]

$$\theta(°C) + 273.15 = T(K)$$

Ideal gas

$$Pa \leftarrow PV = nRT \rightarrow K$$

m^3, mol

$$n = \frac{\text{mass}}{\text{molar mass}} \Big/ \frac{\text{no. of particles}}{\text{Avogadro's constant}}$$

6.022×10^{23}

, $R = 8.314\ J\ mol^{-1}\ K^{-1}$

The Scientific Kelvin Scale $\lim_{P\to 0}$ (real gases)

- At triple point (using ideal gas): $P_{tp} = \frac{nR}{V} T_{tp}$
- At unknown temperature (using ideal gas): $P = \frac{nR}{V} T$
- $T = T_{tp} \frac{P}{P_{tp}} = 273.16 \frac{P}{P_{tp}}$. For real gas, $T = 273.16 \lim_{P_{tp}\to 0} \frac{P}{P_{tp}}$

☆ E.g. A particular resistance thermometer has a resistance of 30.00Ω at ice point, 41.58Ω at steam point and 34.59Ω when immersed in a boiling liquid. A constant gas thermometer gives readings of 1.333×10^5 Pa, 1.821×10^5 Pa & 1.528×10^5 Pa at the same three temperatures. Calculate the temperature at which the liquid is boiling.

a) On the scale of the gas thermometer (Kelvin scale)

b) On the scale of the resistance thermometer (Celsius scale)

steam pt

a) $273.15 = 273.16 \lim_{P_{tr}\to 0} \frac{1.333 \times 10^5}{P_{tr}}$ —— ①

ice pt

$273.15 + 100 \leftarrow 373.15 = 273.16 \lim_{P_{tr}\to 0} \frac{1.821 \times 10^5}{P_{tr}}$ —— ②

$T = 273.16 \lim_{P_{tr}\to 0} \frac{1.528 \times 10^5}{P_{tr}}$ —— ③

③ ÷ ① → $T = 313.12K$

③ ÷ ② → $T = 313.11K$

b) $T = \frac{34.59 - 30.00}{41.58 - 30.00} \times 100$

$= 39.64°C = 312.79K$

Gallery of Thermometers:

<table>
<tr><th></th><th>Properties:</th><th>Range/Response/Accuracy</th></tr>
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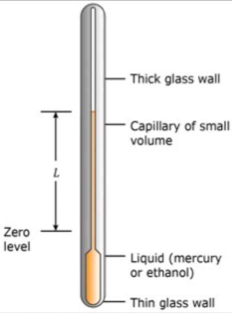

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Mercury in glass thermometer

Relies on variation of volume of mercury with temperature. Mercury is opaque and easily seen; good conductor of heat; does not stick to glass.

$V = V_0(1 + b\Delta T)$

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a) -39°C to 357°C (can be increased with modifications).

b) Slow response (relatively large heat capacities).

c) Typically 0.1°C. (Non-uniform bore, expansion of glass.) Affects temp of object it is measuring.

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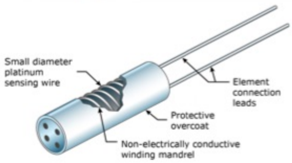

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Platinum Resistance thermometer

Rely on the fact that the electrical resistance of metals are temp dependent. Platinum has temp coef of resistance; high melting pt (1773°C).

$R(T) = R_0(1 + aT + bT^2)$

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a, c) Extremely accurate from -200°C to 1200°C.

b) Relatively large heat capacities so takes longer time to come in thermal equilibrium with surroundings – slow response.

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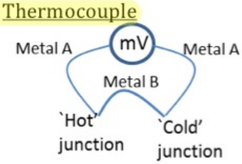
Thermocouple



Two fine wires of different metals – EMF E in millivoltmeter depends on temp diff at junction

$E(T) = aT + bT^2$

</td><td>

Can set cold junction in ice/water (0°C).

a) Using several combinations of metals can get from -269°C to 2300°C.

b) Small heat capacity – fast response and can measure temp even at embedded pt.

c) Accurate over wide range.

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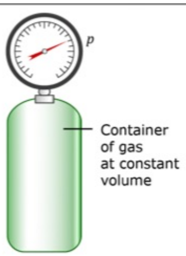

(b) Changes in temperature cause the pressure of the gas to change.
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Constant Vol Gas thermometer

Kept at constant vol, the pressure of the gas varies with temp. For ideal gas, $PV = nRT$. All other thermometers depend critically on the nature and purity of materials used. Under the right conditions, behaviour of gas thermometer is independent of gas.

</td><td>

Seldom used any more as thermometers - used to define meaning of temp. Used as standard reference for temp.

a) About -271°C to 1100°C

b) Very slow response (large vol of gas used).

c) Accurate over a wide range

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Heat and heat transfer

1. Heat capacity

↳ heat capacity (C) of a body is defined as being the heat required to produce a unit temperature change. Units: $J\ K^{-1}$ or $J\ °C^{-1}$ $[Q = C\Delta T]$

↳ heat capacity per unit mass of substance (c). Units: $J\ kg^{-1}\ K^{-1}$ / $J\ kg^{-1}\ °C^{-1}$

$\Delta Q = mc\Delta T$ $C = mc$

2. Latent Heat

↳ the specific latent heat (l) of fusion (vaporisation/sublimation) of a substance is the energy required to cause unit mass of substance to change between solid & liquid without temperature change

$Q = ml_v$ (liquid → gas) $Q = ml_f$ (solid → liquid)

E.g. If $200 cm^3$ of tea at 95°C is poured into a 150g glass cup initially at 25°C, what will be the the common final temperature T of the tea & cup when equilibrium is reached, assuming no heat flows to the surroundings?

$c_{water} = 4189 J\ kg\ K^{-1}$; $c_{glass} = 840 J\ kg\ K^{-1}$

Density of water $= 1 g/cm^3$

Mass of tea $= 200 \times 1 = 200g$

heat lost by tea = heat gained by cup

Let final temp $= T$,

$$mc(95 - T) = mc(T - 25)$$

$$\left(\frac{200}{1000}\right)(4189)(95 - T) = \frac{150}{1000}(840)(T - 25)$$

$$631.674 - 6.6492T = T - 25$$

$$T = 86°C$$

Kinetic Theory of Gases

- If there are N molecules in the vessel of volume V, the number of molecules per unit volume is $\frac{N}{V}$. For a given cross sectional area A, the number of gas molecules colliding with wall per second is $\frac{1}{2}Av_x \times 1s \times \frac{N}{V}$. Half of the molecules moving towards wall & half are moving away.
- Total change in momentum per second for gas molecules in area A is

$$F = \frac{1}{2}Av_x \times 1 \times \frac{N}{V} \times (2m|v_x|) = m|v_x|^2 \frac{N}{V}$$

$$p = \frac{F}{A} = m|v_x|^2 \frac{N}{V}$$

- molecules can move freely in 3 directions

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \qquad v_{r.m.s.} = \sqrt{\langle v^2 \rangle}$$

$$p = \frac{F}{A} = \frac{1}{3}\left(\frac{N}{V}\right) m \langle v^2 \rangle$$

$$pV = \frac{1}{3}Nm\langle v^2 \rangle = nRT$$

- Total kinetic energy of all gas molecules in container:

$$\frac{1}{2}M\langle v^2 \rangle = \frac{3}{2}nRT$$

- Average ke of a single molecule:

$$\frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}kT$$

$$k = \frac{R}{N_A}$$

Assumptions made:

1. Container has a large number of identical molecule & walls of the contain are rigid & do not move
2. Molecule are point size particles $\Rightarrow$ volume they occupy negligible compared to volume of container
3. Molecules are in random motion & they obey Newton's laws
4. All collisions are perfectly elastic
5. The molecules do not exert any force on each other except during collisions
6. The effects of gravity can be ignored.

Law of Thermodynamics

1. Internal energy (U): Ideal gas only has kinetic energy
2. Heat (Q): $Q>0 \to$ when heat enters the system. $Q<0$ when heat leaves the system. Wrong to assume: $Q=0 \to$ ~~no Δ in temp~~ no heat transfer
3. Work done by gas (W): $W>0 \to$ work done by gas to surrounding (expand)
   $W<0 \to$ work done on gas by surrounding (contract)

1st law of thermodynamics

$$\Delta U = Q - W$$

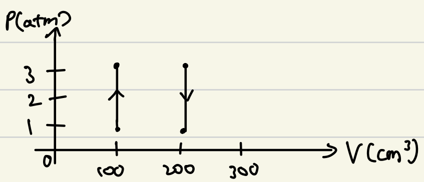
Q is the heat supplied to system, ΔU is the change in internal energy & W is the work done by system on surrounding.

4 Standard Processes:

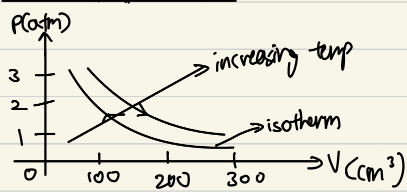
$W = P(V_f - V_i)$

	ΔU	Q	W
Isochoric no Δ in volume (V)	$\Delta U = \underbrace{\frac{3}{2}nR\Delta T}_{\text{monoatomic}} = \underbrace{\frac{5}{2}nR\Delta T}_{\text{diatomic}}$	$Q = \begin{cases} \frac{3}{2}nR\Delta T \text{ for mono} \\ \frac{5}{2}nR\Delta T \text{ for dia} \end{cases}$	0
Isobaric no Δ in pressure (P)	$\Delta U = \underbrace{\frac{3}{2}nR\Delta T}_{\text{monoatomic}} = \underbrace{\frac{5}{2}nR\Delta T}_{\text{diatomic}}$	$Q = \begin{cases} \frac{5}{2}nR\Delta T \text{ for mono} \\ \frac{7}{2}nR\Delta T \text{ for dia} \end{cases}$	$W = P(V_f - V_i)$
Isothermal no Δ in T	0	$Q = W = nRT\ln\frac{V_f}{V_i}$	$W = nRT\ln\frac{V_f}{V_i}$
Adiabatic no Δ in Q (heat transfer)	$\Delta U = \underbrace{\frac{3}{2}nR\Delta T}_{\text{monoatomic}} = \underbrace{\frac{5}{2}nR\Delta T}_{\text{diatomic}}$	0	$W = -\frac{1}{\gamma - 1}(P_fV_f - P_iV_i)$

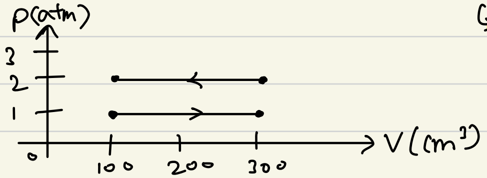
Isochoric Process



Isothermal Process



Isobaric Process



For isobaric process, common to write $Q = nC_p\Delta T$

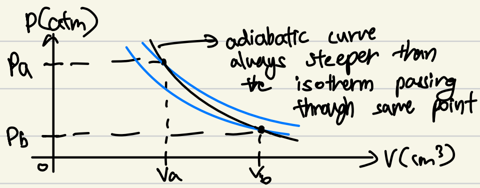
↳ C_p → constant pressure molar heat capacity → heat required to Δ temperature of 1 mol of ideal gas by 1K at constant temp

$C_p = \frac{5R}{2}$ (mono) $C_p = \frac{7R}{2}$ (dia)

- Mayer's relation: $C_p - C_v = R$
- ratio γ → $\gamma = \frac{C_p}{C_v}$ (used in adiabatic process)

$\frac{5\cancel{R}}{\cancel{2}} : \frac{7\cancel{R}}{\cancel{2}} = \frac{5}{7}$

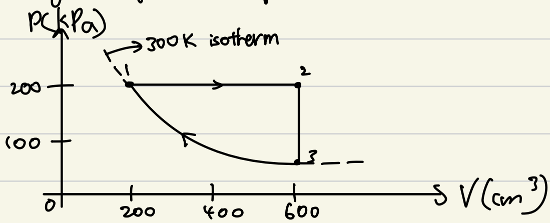
Adiabatic Process



$P_iV_i^{\gamma} = P_fV_f^{\gamma}$

$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$ → to find ΔT

E.g. Analyze the heat engine (monoatomic ideal gas) of PV diagram to determine: a) net work done per cycle, b) engine's thermal efficiency c) engine's power output if it runs at 600rpm.



thermal efficiency: $\eta = \frac{W_{net}}{Q_{in}}$

$n = \frac{P_1V_1}{RT_1} = 0.016$ moles

a) Net WD $= W_{12} + W_{23} + W_{31}$

$= P\Delta V + 0 + nRT_3 \ln\frac{V_1}{V_3}$

$= 80 - 43.8 = 36\text{J}$

b) $Q_{12} = nC_p\Delta T = n\frac{5R}{2}(T_2 - T_1)$

$= n\frac{5R}{2}\left(\frac{P_2V_2}{nR} - 300\right) = 200\text{ J (in)}$

$Q_{23} = n C_v \Delta T$

$= n\left(\frac{3R}{2}\right)(300 - T_2) = -120J$ (out)

$Q_{31} = \Delta U + W = 0 + W_{31}$

$= -43.8J$ (out)

$\eta = \frac{36}{200} = 0.18 = 18\%$

c) 600rpm = 10 cycles/s

$P = WD$ in 1 second

$= 36 \times 10 = 360W$