w_1 0.15, w_2 0.25, w_3 0.2, w_4 0.3, w_5, w_6 0.5, w_7 0.45, w_8 0.55

$target_{o_1} = 0.01$

$target_{o_2} = 0.99$

$b_1 = 0.35$ $b_2 = 0.6$

（一）向前传播

① 输入层 → 隐藏层输入的加权和：

$$Z_{h_1} = 0.15 \; w_1 * i_1 + w_2 * i_2 + b_1 * 1$$

$$= 0.15 \times 0.05 + 0.2 \times 0.1 + 0.35 \times 1$$

$$= 0.3775$$

② 代入激活函数后：

$$a_{h_1} = \frac{1}{1+e^{-z_{h_1}}}$$

$$= \frac{1}{1+e^{-0.3775}} = 0.593269992$$

同理可得 $a_{h_2} = 0.596884378$

（三）② 隐藏层 → 输出层输入的加权和：

$$Z_{o_1} = w_5 * a_{h_1} + w_6 * a_{h_2} + b_2 * 1$$

$$= 0.4 \times 0.593269992 + 0.45 \times 0.596884378 + 0.6 \times 1$$

$$= 1.105905967$$

代入激活函数后：

$$a_{o_1} = \frac{1}{1+e^{-z_{o_1}}}$$

$$= \frac{1}{1+e^{-1.105905967}} = 0.751365069$$

用同理可得 $a_{o_2} = 0.772928465$

（二）计算损失函数

$$E_{total} = \sum \frac{1}{2}(target - a_o)^2$$

$$E_{total} = \frac{1}{2}(0.01 - 0.751365069)^2 + \frac{1}{2}(0.99 - 0.772928465)^2 = 0.274811083 + 0.023560026 = 0.298371109$$
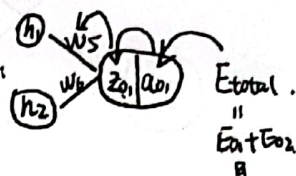
（三）反向传播.

① 隐藏层到输出层的权值更新. (以 w_5 为例).

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial a_{o_1}} \cdot \frac{\partial a_{o_1}}{\partial z_{o_1}} \cdot \frac{\partial z_{o_1}}{\partial w_5}$$

（w_5 只影响了 o_1, 没影响 o_2.）

分别计算各偏导：

$\frac{\partial E_{total}}{\partial a_{o_1}}$:

$$E_{total} = \frac{1}{2}(target_{o_1} - a_{o_1})^2 + \frac{1}{2}(target_{o_2} - a_{o_2})^2$$

$$\frac{\partial E_{total}}{\partial a_{o_1}} = -(target_{o_1} - a_{o_1}) = 0.75136507 - 0.01 = 0.74136507$$

$\frac{\partial a_{o_1}}{\partial z_{o_1}}$:

$$a_{o_1} = \frac{1}{1+e^{-z_{o_1}}}$$

$$\frac{\partial a_{o_1}}{\partial z_{o_1}} = \frac{e^{-z_{o_1}}}{(1+e^{-z_{o_1}})} = a_{o_1} \cdot (1 - a_{o_1}) = 0.7513657 \times (1 - 0.75136507) = 0.18681560$$

$\frac{\partial z_{o_1}}{\partial w_5}$:

$$z_{o_1} = w_5 * a_{h_1} + w_6 + a_{h_2} + b_2 * 1$$

$$\frac{\partial z_{o_1}}{\partial w_5} = a_{h_1} = 0.593269992.$$

综上：$\frac{\partial E_{total}}{\partial w_5} = 0.74136507 \times 0.18681560 \times 0.593269992 = 0.082167041.$

由此可简化 $\frac{\partial E_{total}}{\partial w} = -(target - o)$

更新 w_5 ： $w_5' = w_5 - \eta \times \frac{\partial E_{total}}{\partial w_5} = 0.4 - 0.5 \times 0.082167041 = 0.35891648$ （η 学习率 $=0.5$）
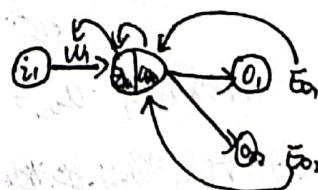
同理更新 ： $w_6' = 0.408666186$

$w_7' = 0.511301270$

$w_8' = 0.561370121.$

② 隐藏层到隐藏层的权值更新. (以 w_1 为例))

$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial a_{h_1}} * \frac{\partial a_{h_1}}{\partial z_{h_1}} * \frac{\partial z_{h_1}}{\partial w_1}$$

$$= \left(\frac{\partial E_{o_1}}{\partial a_{h_1}} + \frac{\partial E_{o_2}}{\partial a_{h_1}}\right) * \frac{\partial a_{h_1}}{\partial z_{h_1}} * \frac{\partial z_{h_1}}{\partial w_1}$$

（w_1 影响了 o_1 和 o_2）

分别计算各偏导：

$\dfrac{\partial E_{o_1}}{\partial a_{h_1}}$: $\quad \dfrac{\partial E_{o_1}}{\partial a_{h_1}} = \dfrac{\partial E_{o_1}}{\partial a_{o_1}} * \dfrac{\partial a_{o_1}}{\partial z_{o_1}} * \dfrac{\partial z_{o_1}}{\partial a_{h_1}}$

$$= 0.74136507 \times 0.18681562 \times 0.4.$$

$$= 0.055399425.$$

同理可得：$\quad \dfrac{\partial E_{o_2}}{\partial a_{h_2}} = -0.019049119.$

$\dfrac{\partial E_{total}}{\partial a_{h_1}} = 0.055399425 - 0.019049119 = 0.036350306.$

$\dfrac{\partial a_{h_1}}{\partial z_{h_1}}$: $\quad a_{h_1} = \dfrac{1}{1+e^{-z_{h_1}}}$

$$\dfrac{\partial a_{h_1}}{\partial z_{h_1}} = a_{h_1}(1-a_{h_1}) = 0.59326999 2(1-0.59326999 2) = 0.2413007086.$$

$\dfrac{\partial z_{h_1}}{\partial w_1}$: $\quad z_{h_1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$

$$\dfrac{\partial z_{h_1}}{\partial w_1} = i_1 = 0.05.$$

综上：$\dfrac{\partial E_{total}}{\partial w_1} = 0.036350303 \times 0.2413007086 \times 0.05 = 0.000438568.$

更新 w_1 的权值：$\quad w_1' = w_1 - \eta \times \dfrac{\partial E_{total}}{\partial w_1} = 0.15 - 0.05 \times 0.000438568 = 0.149780716.$

同理更新：$\quad w_2' = 0.199956143$

$\quad w_3' = 0.24975114.$

$\quad w_4' = 0.29950229.$

(四) 把更新的权值重新计算。不停迭代至总误差 E_{total} 满足要求.