Moscow Aviation Institute
(National Research University)
Institute 1 «Aviation engineering»
Department 101 "Design and Certification of Aviation technic"

Laboratory work 1

Of “Parts and assemblies” course

Design a cantilever beam for given load, length and materials

Fifth case

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Accepted by Dukhnovskiy D.A. ____________________

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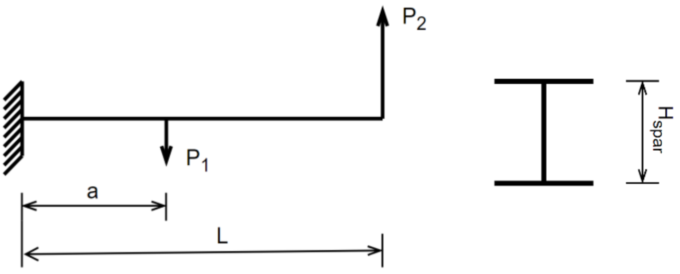
1. Terms of reference



$\mathrm{L} = 5000\ \mathrm{mm}$

$\mathrm{P_1} = 4000\ \mathrm{N}$

$\mathrm{P_2} = 30000\ \mathrm{N}$

$\mathrm{H_{spar}} = 350\ \mathrm{mm}$

$\mathrm{a} = 1000\ \mathrm{mm}$

fig. 1, Load scheme and beam cross-section

2. Material choosing

Д16Т as material for web was chosen, because it`s easier in production then web out of ВТ20 and 30ХГСА, moreover, web from this material will be thicker, so risk of buckling will be lower. For caps, 30ХГСА was chosen, because it has higher ultimate tensile stress then Д16Т, so standard cross-section can be chosen.

3. Load calculating

$$\mathrm{P_2 \cdot L - P_1 \cdot a = M(0) = 146 \times 10^6\ N \cdot mm}$$

$$\mathrm{P_2 \cdot (L - a) = M(a) = 120\ \times 10^6\ N \cdot mm}$$

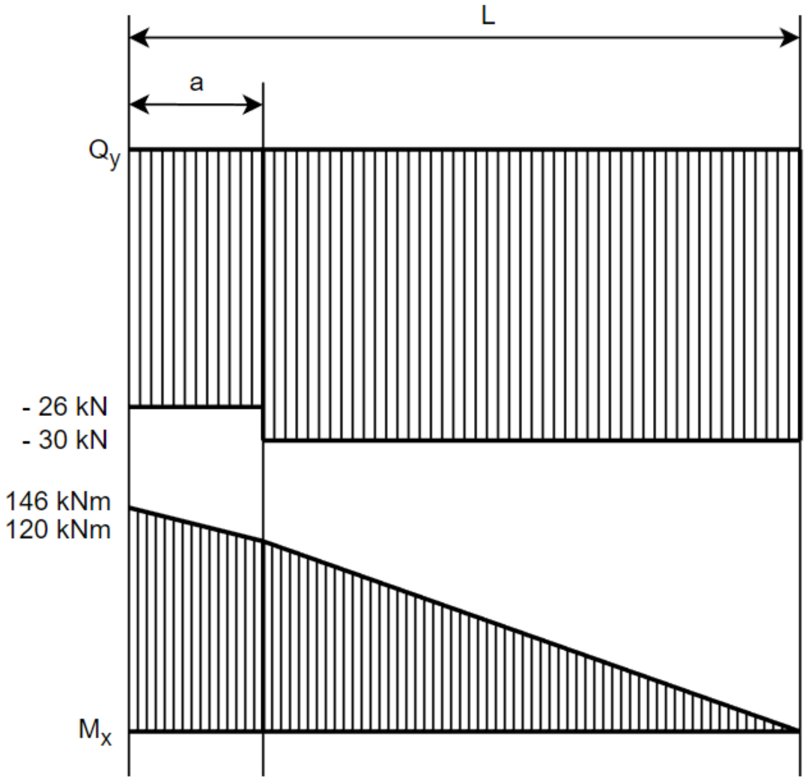


fig. 2, load diagrams

$$M(0.5m) = P_2 \cdot (L - 0.5) - P_1(a - 0.5) = 133 \times 10^6\ N \cdot mm$$

$$M(3m) = P_2(L - 3) = 60 \times 10^6\ N \cdot mm$$

$$F(0) = \frac{M(0)}{0.95 \cdot H_{spar}} = 439000\ N$$

$$F(0.5m) = \frac{M(0.5m)}{0.95 \cdot H_{spar}} = 400000\ N$$

$$F(3m) = \frac{M(3m)}{0.95 \cdot H_{spar}} = 180450\ N$$

4. Calculation of caps area

$$[\sigma_{ult,cap}] = 1100\ MPa$$

$$A(0) = \frac{F(0)}{[\sigma_{ult,cap}]} = 399\ mm^2$$

$$A(0.5) = \frac{F(0.5m)}{[\sigma_{ult,cap}]} = 364\ mm^2$$

$$A(3) = \frac{F(3m)}{[\sigma_{ult,cap}]} = 164\ mm^2$$

Thus, following standard T-Bar sections for caps were chosen:

Tab. 1, standard sections parameters

<table>
<tr><th colspan="11">Rectangular Equilateral T-Bar Sections State Standard 13622 - 90</th></tr>
<tr><th rowspan="2">№</th><th rowspan="2">Number of Profile</th><th>H</th><th>B</th><th>S</th><th>S1</th><th>R</th><th>r</th><th rowspan="2">Cross-sectional Area, mm²</th><th colspan="2">Moment of Inertia, cm⁴</th></tr>
<tr><th colspan="6">mm</th><th>Ix</th><th>Iy</th></tr>
<tr><td>5</td><td>420159</td><td>26</td><td>40</td><td>3,0</td><td>4,5</td><td>2,5</td><td>0,0</td><td>247.2</td><td>1,44</td><td>1,05</td></tr>
<tr><td>9</td><td>420209</td><td>30</td><td>45</td><td>4,0</td><td>6,5</td><td>5,0</td><td>0,0</td><td>397.2</td><td>2,13</td><td>4,96</td></tr>
<tr><td>10</td><td>420325</td><td>40</td><td>50</td><td>3,5</td><td>6,5</td><td>4,0</td><td>0,0</td><td>449.1</td><td>4,65</td><td>6,78</td></tr>
</table>

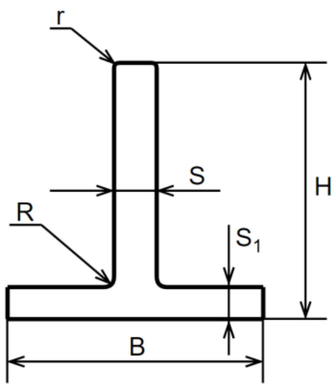


fig. 3, section parameters

Checking the stress safety factor:

$$\sigma = \frac{F}{A} \quad \eta_{stress} = \frac{[\sigma_{ult,cap}]}{\sigma}$$

$$\eta_{stress,1} = \frac{A_{10}[\sigma_{ult,cap}]}{F(0)} = 1.13$$

$$\eta_{stress,2} = \frac{A_9[\sigma_{ult,cap}]}{F(0.5m)} = 1.10$$

$$\eta_{stress,3} = \frac{A_5[\sigma_{ult,cap}]}{F(3m)} = 1.51$$

Thus, each spar cap will be produced out of three steel profiles with different cross-section, and then welded into one part.

5. **Calculation of spar cap local buckling stress**

$$\sigma_{loc,cap} = \frac{0.9kE_{cap}}{(b/S_1)^2}, \ b = \frac{B-S}{2}, \ k \approx 0.45, \ E_{cap} = 210000 \text{ MPa}$$

Local buckling stress of first section (from 0 to 0.5 meters):

$$\sigma_{loc,cap,1} \approx 6650 \text{ MPa}$$

Local buckling stress of second section (from 0.5 to 3 meters):

$$\sigma_{loc,cap,2} \approx 8550 \text{ MPa}$$

Local buckling stress of third section (from 3 to 5 meters):

$$\sigma_{loc,cap,3} \approx 5000 \text{ MPa}$$

Checking the safety factor for caps:

$$\eta_{buckling} = \frac{\sigma_{loc}}{\sigma}$$

$$\eta_{buckling,1} = \frac{A_{10}\sigma_{loc,cap,1}}{F(0)} = 6.8$$

$$\eta_{buckling,2} = \frac{A_9\sigma_{loc,cap,2}}{F(0.5m)} = 8.5$$

$$\eta_{buckling,3} = \frac{A_5\sigma_{loc,cap,3}}{F(3m)} = 6.85$$

Thus, local buckling stress of spar caps much higher than working stress.

6. **Calculation of web thickness and stiffeners quantity at tip section**

Shear stress in web:

$$\tau_{web} = \frac{F}{A_{web}} = \frac{Q_{tip}}{H_{eq}\delta_{web}}$$

Local buckling shear stress in web with stiffeners:

$$\tau_{loc,web} = \frac{0.9kE_{web}}{(b/\delta_{web})^2}, \quad k = 5.6 + \frac{3.8}{\left({}^{a}/_{b}\right)^2}$$

Where a is web high, b is distance between stiffeners, optimal ${}^{a}/_{b} = 1$

$$E_{web} = 72000 \ MPa$$

Nearest possible $^{a}/_{b}$ very near to one, if 13 stiffeners at tip section will be used. Optimum safety factor $\eta_{buckling,web} = \frac{\tau_{loc,web}}{\tau_{\mathrm{web}}} = 1.2$

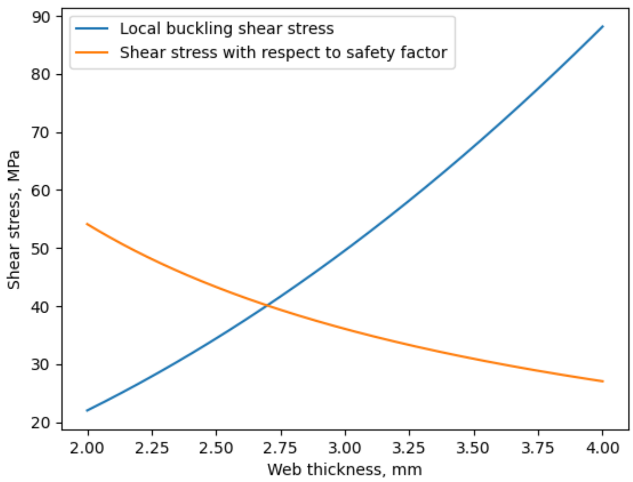


fig. 4, web thickness optimization

$\tau_{loc,web} = \eta_{buckling,web} \cdot \tau_{\mathrm{web}}$

Thus, optimum web thickness $\delta_{\mathrm{web}} = 2.8\ mm$

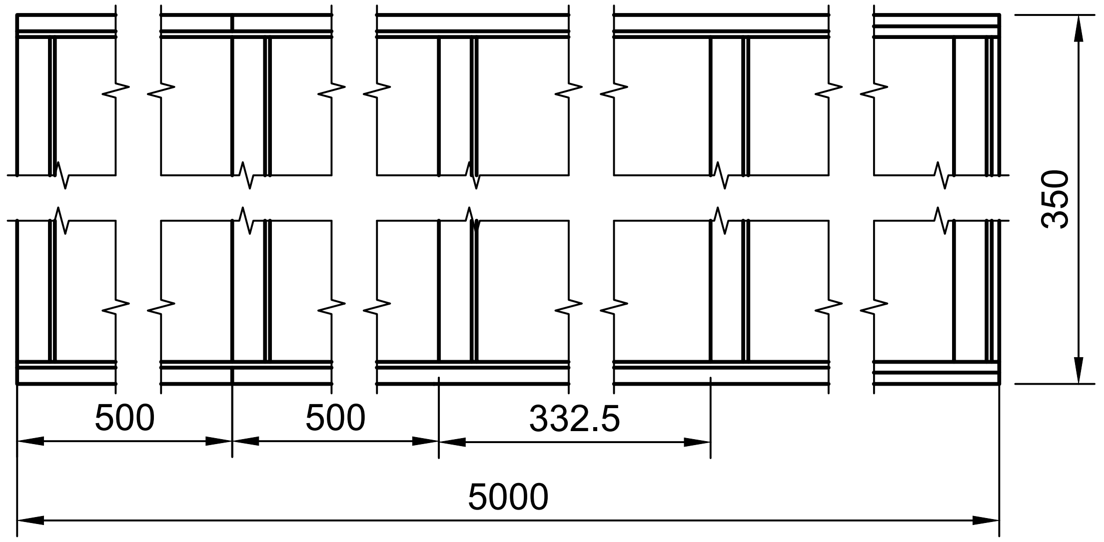
7. Calculation of stiffeners quantity at the engine section

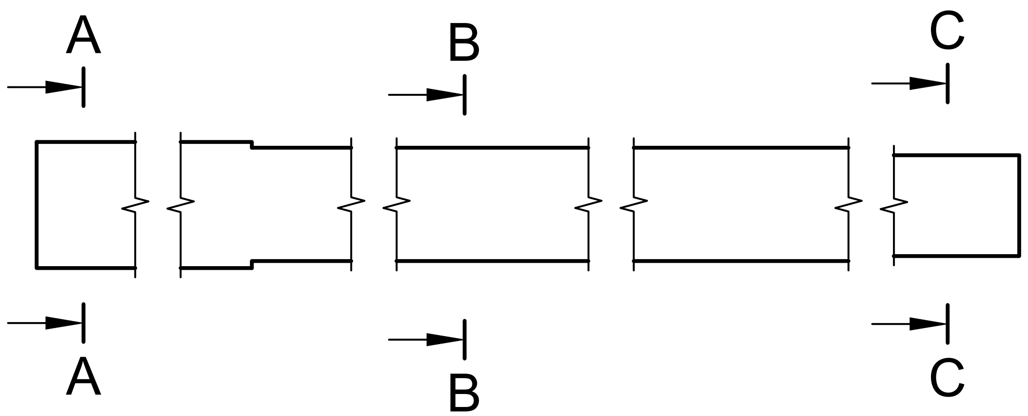
Using founded web thickness:

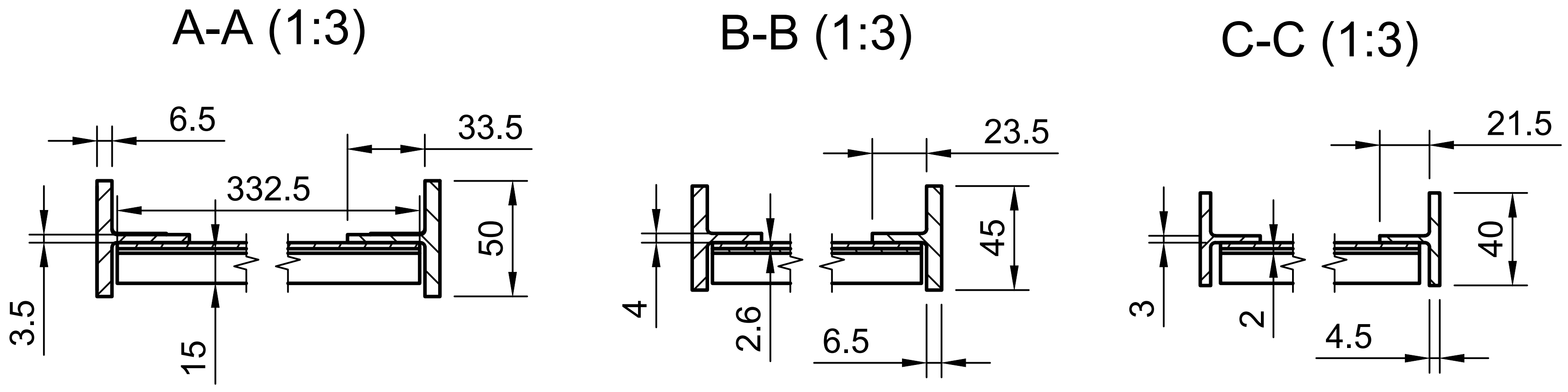
$$\tau_{\mathrm{web}} \cdot \eta_{buckling,web} = \frac{Q_{engine}\eta_{buckling,web}}{\mathrm{H_{eq}}\delta_{\mathrm{web}}} = \tau_{loc,web} = \frac{0.9kE_{web}}{(b/\delta_{web})^2}$$

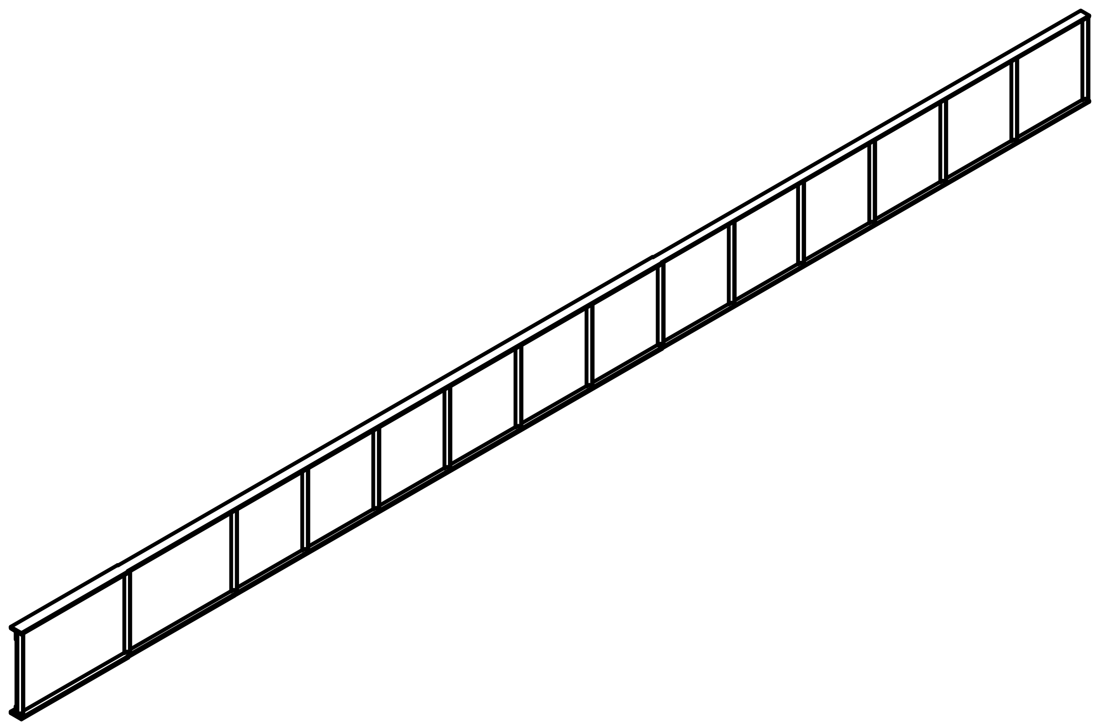
$$5.6 + \frac{3.8}{\left(^{\mathrm{a}}/_{\mathrm{b}}\right)^2} = \frac{Q_{engine}b^2\eta_{buckling,web}}{0.9\mathrm{H_{eq}}\delta_{web}^3 E_{web}}$$

$b \approx 420mm$, thus, in engine section two stiffeners is near to optimal.









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