# Minimal Proof System of Matching Logic

Formal Systems Laboratory

November 10, 2017

| | |
|---|---|
| Propositional$_1$ | $\varphi_1 \to (\varphi_2 \to \varphi_1)$ |
| Propositional$_2$ | $(\varphi_1 \to (\varphi_2 \to \varphi_3)) \to ((\varphi_1 \to \varphi_2) \to (\varphi_1 \to \varphi_3))$ |
| Propositional$_3$ | $(\neg\varphi_1 \to \neg\varphi_2) \to (\varphi_2 \to \varphi_1)$ |
| Variable Substitution | $\forall x.\varphi \to \varphi[y/x]$ |
| Distribution$_\forall$ | $\forall x.(\varphi_1 \to \varphi_2) \to (\varphi_1 \to \forall x.\varphi_2)$ |
| | if $x$ does not occur free in $\varphi_1$ |
| K | $\sigma(\ldots, \varphi_1 \vee \varphi_2, \ldots) \leftrightarrow \sigma(\ldots, \varphi_1, \ldots) \vee \sigma(\ldots, \varphi_2, \ldots)$ |
| Barcan | $\sigma(\ldots, \exists x.\varphi, \ldots) \leftrightarrow \exists x.\sigma(\ldots, \varphi, \ldots)$ |
| | if $x$ does not occur free |
| | in the left-hand side of the double implication |
| *Membership Symbol | $x \in \sigma(\ldots, \varphi_i, \ldots) \to \exists y.y \in \varphi_i \wedge x \in \sigma(\ldots, y, \ldots)$ |
| *Membership $\neg$ | $x \in \neg\varphi_1 \to \neg(x \in \varphi_1)$ |
| *Membership $\exists$ | $x \in \forall y.\varphi \to \forall y.(x \in \varphi)$ |
| | where $x$ is distinct from $y$ |
| *Definedness | $\lceil x \rceil$ |
| N | From $\varphi$ deduce $\neg\sigma(\neg\varphi)$ |
| Modus Ponens | From $\varphi_1$ and $\varphi_1 \to \varphi_2$ deduce $\varphi_2$ |
| Universal Generalization | From $\varphi$ deduce $\forall x.\varphi$ |
| Equational Substitution | From $\varphi_1 \leftrightarrow \varphi_2$ deduce $\varphi_3[\varphi_1/x] \leftrightarrow \varphi_3[\varphi_2/x]$ |
| Framing | From $\varphi_1 \to \varphi_2$ deduce $\sigma(\varphi_1) \to \sigma(\varphi_2)$ |
| *Introduction$_\in$ | From $\varphi$ deduce $x \in \varphi$ |
| | If $x$ does not occur free in $\varphi$ |
| *Elimination$_\in$ | From $x \in \varphi$ deduce $\varphi$ |
| | If $x$ does not occur free in $\varphi$ |