# Towards an Efficient and Economic Deductive System of Matching Logic

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We aim for a Hilbert style deductive system which has a relatively large number of axioms but only a few inference rules.

- (K1) $P \to (Q \to P)$
- (K2) $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$
- (K3) $(\neg P \to \neg Q) \to (Q \to P)$
- (K4) $\forall x.(P \to Q) \to (P \to \forall x.Q)$ if $x$ does not occur free in $P$
- (K5) $\forall x.P \to P$ if $x$ does not occur free in $P$
- (K6) $\forall x.P(x) \to P(y)$
- (K7) $P = P$
- (K8) $P_1 = P_2 \to (Q[P_1/x] \to Q[P_2/x])$
- (K9) $\exists y.Q = y \to (\forall x.P(x) \to P[Q/x])$ if $Q$ is free for $x$ in $P$

Inference rules include

- (Modus Ponens) From $P$ and $P \to Q$, deduce $Q$.
- (Universal Generalization) From $P$, deduce $\forall x.P$.

**Proposition 1** (Deduction Theorem). *If $\Gamma \cup \{P\} \vdash Q$ and the proof does not use $\forall x$-Generalization where $x$ is free in $P$, then $\Gamma \vdash P \to Q$.*