Minimal Proof System of Matching Logic

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We conjecture that the following is a sound and complete proof system of matching logic.

PROPOSITIONAL$_1$	$\varphi_1 \to (\varphi_2 \to \varphi_1)$
PROPOSITIONAL$_2$	$(\varphi_1 \to (\varphi_2 \to \varphi_3)) \to ((\varphi_1 \to \varphi_2) \to (\varphi_1 \to \varphi_3))$
PROPOSITIONAL$_3$	$(\neg\varphi_1 \to \neg\varphi_2) \to (\varphi_2 \to \varphi_1)$
VARIABLE SUBSTITUTION	$\forall x.\varphi \to \varphi[y/x]$
$\forall$	$\forall x.(\varphi_1 \to \varphi_2) \to (\varphi_1 \to \forall x.\varphi_2)$
	if x does not occur free in φ_1
PROPAGATE$_\vee$	$\sigma(\dots, \varphi_1 \vee \varphi_2, \dots) \leftrightarrow \sigma(\dots, \varphi_1, \dots) \vee \sigma(\dots, \varphi_2, \dots)$
PROPAGATE$_\bot$	$\neg\sigma(\bot)$
PROPAGATE$_\exists$	$\sigma(\dots, \exists x.\varphi, \dots) \leftrightarrow \exists x.\sigma(\dots, \varphi, \dots)$
	if x does not occur free
EXISTENCE	$\exists x.x$
MODUS PONENS	From φ_1 and $\varphi_1 \to \varphi_2$ deduce φ_2
UNIVERSAL GENERALIZATION	From φ deduce $\forall x.\varphi$

Figure 1: Minimal Proof System $\mathcal{S}$

Matching logic is proved to have a complete proof system in the presence of definedness symbols. Notice the above minimal proof system $\mathcal{S}$ does not depend on definedness symbols. In the following, we show how to establish all axioms and rules of the old proof system using the new minimal proof system $\mathcal{S}$ plus the axiom $\forall x.\lceil x \rceil$ for definedness symbols.

Proposition 1. $\vdash \varphi = \varphi$

Proof. content... $\square$