Minimal Proof System of Matching Logic

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The following inference rules are shown to constitute a sound and complete proof system of matching logic. We conjecture that all rules marked with "$*$" are derivable from the rest plus the axiom (Definedness). In other words, we conjecture that all rules that are *not* marked with "$*$" constitute a sound and complete proof system of matching logic.

Propositional$_1$	$\varphi_1 \to (\varphi_2 \to \varphi_1)$
Propositional$_2$	$(\varphi_1 \to (\varphi_2 \to \varphi_3)) \to ((\varphi_1 \to \varphi_2) \to (\varphi_1 \to \varphi_3))$
Propositional$_3$	$(\neg\varphi_1 \to \neg\varphi_2) \to (\varphi_2 \to \varphi_1)$
Variable Substitution	$\forall x.\varphi \to \varphi[y/x]$
$\forall$	$\forall x.(\varphi_1 \to \varphi_2) \to (\varphi_1 \to \forall x.\varphi_2)$
	if x does not occur free in φ_1
K	$\sigma(\dots, \varphi_1 \vee \varphi_2, \dots) \leftrightarrow \sigma(\dots, \varphi_1, \dots) \vee \sigma(\dots, \varphi_2, \dots)$
Barcan	$\sigma(\dots, \exists x.\varphi, \dots) \leftrightarrow \exists x.\sigma(\dots, \varphi, \dots)$
	if x does not occur free
	in the left-hand side of the double implication
*Membership Symbol	$x \in \sigma(\dots, \varphi_i, \dots) \to \exists y.y \in \varphi_i \wedge x \in \sigma(\dots, y, \dots)$
*Membership $\neg$	$x \in \neg\varphi \to \neg(x \in \varphi)$
*Membership $\exists$	$x \in \forall y.\varphi \to \forall y.(x \in \varphi)$
	where x is distinct from y
*Definedness	$\lceil x \rceil$
N	From φ deduce $\neg\sigma(\neg\varphi)$
Modus Ponens	From φ_1 and $\varphi_1 \to \varphi_2$ deduce φ_2
Universal Generalization	From φ deduce $\forall x.\varphi$
Equational Substitution	From $\varphi_1 \leftrightarrow \varphi_2$ deduce $\varphi_3[\varphi_1/x] \leftrightarrow \varphi_3[\varphi_2/x]$
Framing	From $\varphi_1 \to \varphi_2$ deduce $\sigma(\varphi_1) \to \sigma(\varphi_2)$
*Introduction$_\in$	From φ deduce $x \in \varphi$
	If x does not occur free in φ
*Elimination$_\in$	From $x \in \varphi$ deduce φ
	If x does not occur free in φ