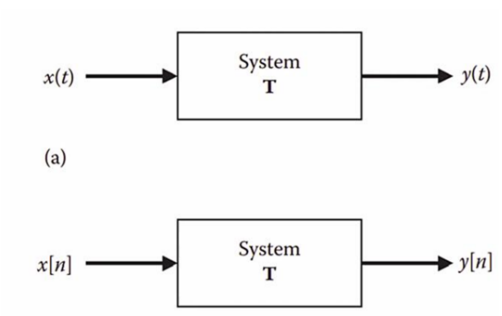
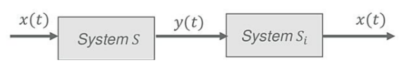
Signals Analysis Lect:2

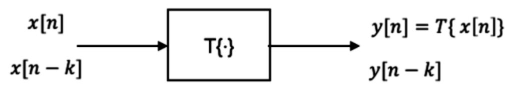
- **System Model**
  - Continoud-Time and Discrete-Time Systems

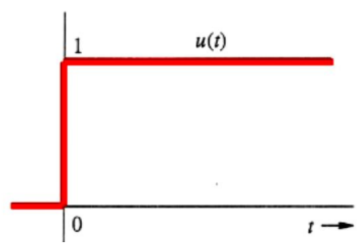
  - Causal and Noncausal System
    - $y[n] = x\left[\frac{n}{2}\right]$ — $y(t) \doteq x(t-1)$
    - A Causal System depends only on the present and past values of input x(t)
    - noncausal — $y(t) = x(t+2)$ $\quad y[n] = x[1-n]$
  - Linear and Nonlinear systems
    - $\mathbf{T}\{k_1x_1 + k_2x_2\} = k_1y_1 + k_2y_2$
    - If Tx_1 = y_1 and Tx_2 = y_2, then $\mathbf{T}\{x_1 + x_2\} = y_1 + y_2$
    - Linear Discrete time systems — System is Linear if $y[n] = T\{a_1x_1[n] + a_2x_2[n]\} = a_1T\{x_1[n]\} + a_2T\{x_2[n]\}$
  - Time-Vaying and Time-Invarying systems
    - Continous Time-Invariant system: $T\{x(t-\tau)\} = y(t-\tau)$
    - Discrete Time-Invarient System: $T\{x[n-m]\} = y[n-m]$
  - Memory Systems and Memoryless Systems
    - A memoryless system is one in which the current output depends only on the current input; it does not depend on the past or future inputs.
    - A system with a memory is also called a dynamic system:
    - A memoryless system is called a static system
  - Invertible and non-invertible systems
    - Let a system S produce an output y(t) with input x(t). If there exists another system Si, which produces x(t) from y(t), then S is an invertible system

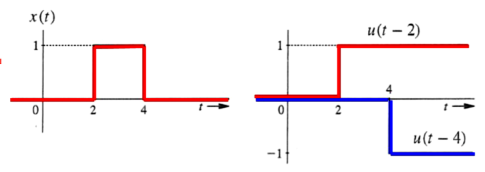
  - Stable and Unstable System
    - A system is said to be bounded-input bounded-output stable (BIBO stable) iff every bounded input results in a bounded output
    - $\forall t \quad |x(t)| \le M_x < \infty \rightarrow \forall t \quad |y(t)| \le M_y < \infty$
  - Shiift-Invarient Discrete time systems
    - System is Shift-Invariant if $y[n] = T\{a_1x[n-k] + a_2x_2[n-k]\} = y[n-k]$

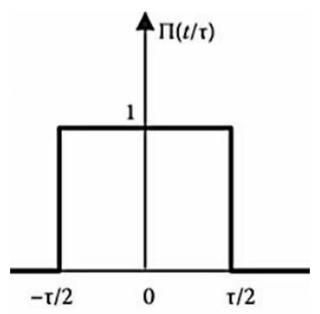
- **Signal Model**
  - Unit step Function u (t)

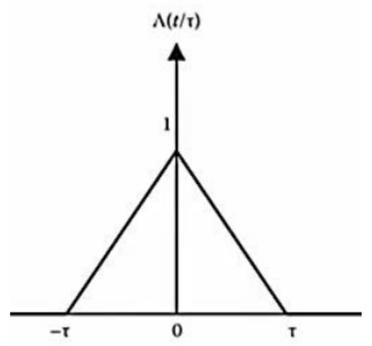
    - Defined By — $u(t) = \begin{cases} 1 & t \ge 0 \\ 0 & t < 0 \end{cases}$
    - Useful to describe causal signal that begins at t=0 — Ex. the causal for the exponential signal can be described as: $e^{-at}u(t)$
  - Pulse Signal

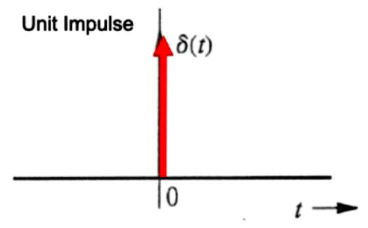
    - Defined by — x(t) = u(t-2) - u(t-4)
    - Rectangular Pulse Signal

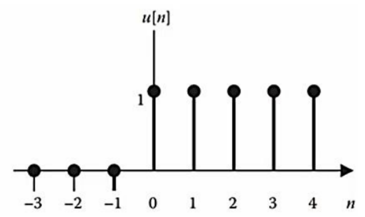
      - Unit Rectangular Pulse signal defined by — $\Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| \le \tau/2 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} 1, & -\tau/2 < t < \tau/2 \\ 0, & \text{otherwise} \end{cases}$
      - Rectangular Signal in terms of the unit step function — $\Pi\left(\frac{t}{\tau}\right) = u\left(t + \frac{\tau}{2}\right) - u\left(t - \frac{\tau}{2}\right)$
    - Triangular pulse function

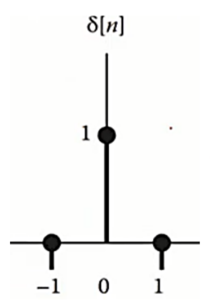
      - the unit triangular function is defined by — $\Lambda\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \frac{|t|}{\tau}, & -\tau < t < \tau \\ 0, & \text{otherwise} \end{cases} = \begin{cases} 1 + \frac{t}{\tau}, & -\tau < t < 0 \\ 1 - \frac{t}{\tau}, & 0 < t < \tau \\ 0, & \text{otherwise} \end{cases}$
  - Unit Impulse Function

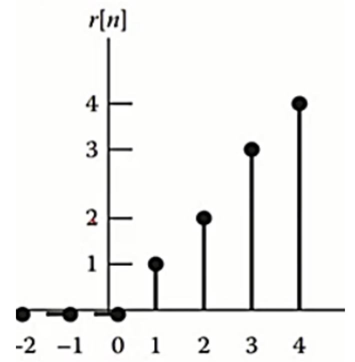
    - Defined by — $\delta(t) = 0 \quad t \ne 0$, $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$
    - Useful for Multiply a function — $\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$
    - Sampling Property of Unit Impulse Function — $\int_{-\infty}^{\infty} \phi(t)\delta(t-T)dt = \phi(T)$
  - Discrete/Sequence Signals
    - Unit Step Sequence

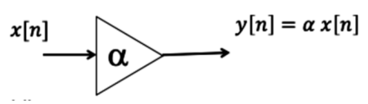
      - Defined by — $u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \ge 0 \end{cases}$
    - Unit Impulse Sequence

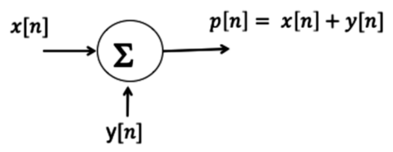
      - Defined by — $\delta[n] = \begin{cases} 0, & n \ne 0 \\ 1, & n = 0 \end{cases}$
    - Unit Ramp Sequence

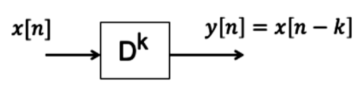
      - Defined by — $r[n] = \begin{cases} n, & n \ge 0 \\ 0, & n < 0 \end{cases}$
- **Basic Building Blocks in a discrete linear system**
  - Scalling

  - Adding

  - Delay (Time Shift)

- **Moving Average Filter**
  - y[n]= 0.25 (x[n] + x[n-1]+x[n-2]+x[n-3])