1\. Q (a) There are 4 states ~~there~~

(b) Sprinkler and WetGrass are always true

$p(c|r,s) = \alpha P(c)p(r|c)p(s|c) = 0.04\alpha$

$p(\neg c|r,s) = 0.05\alpha \quad P(c|r,s) = \frac{4}{9}, \ P(\neg c|r,s) = \frac{5}{9}$

$p(c|\neg r,s) = 0.01\alpha, \quad P(\neg c|\neg r,s) = 0.2\alpha$

$P(C|\neg r,s) = \langle \frac{1}{21}, \frac{20}{21} \rangle$

$p(R|c,s,w) = \alpha P(R|c)P(w|s,R)$

$= \alpha(0.8, 0.2)(0.99, 0.9) = \alpha(\frac{22}{27}, \frac{5}{27})$

$P(R|\neg c,s,w) = \alpha P(R|\neg c)P(w|s,R) = \alpha(\frac{11}{51}, \frac{40}{51})$

possible y and y' 1: (c,r) 2: $(\neg c, r)$, 3: $(c, \neg r)$ 4: $(\neg c, \neg r)$

$q(1\to 1) = 0.5p(c|r,s) + 0.5p(r|c,s,w) = \frac{17}{27}$

$q(1\to 3) = 0.5P(\neg r|c,s,w) = \frac{5}{54}$

$q(1\to 2) = 0.5P(\neg c|r,s) = \frac{5}{18}$

$q(1\to 4) = 1 - \sum_{i=1}^{3} q(1\to i) = 0$

$q(2\to 1) = 0.5p(c|r,s) = \frac{2}{9}$

$q(2\to 2) = \frac{59}{153} \quad q(2\to 3) = 0 \quad q(2\to 4) = \frac{20}{51}$

$q(3\to 1) = \frac{11}{27} \quad q(3\to 2) = 0 \quad q(3\to 3) = \frac{22}{189} \quad q(3\to 4) = \frac{10}{21}$

$q(4\to 1) = 0 \quad q(4\to 2) = \frac{11}{102} \quad q(4\to 3) = \frac{1}{42} \quad q(4\to 4) = \frac{310}{357}$

transition matrix Q that change from row to column

	1	2	3	4
1	$\frac{17}{27}$	$\frac{5}{18}$	$\frac{5}{54}$	0
2	$\frac{2}{9}$	$\frac{59}{153}$	0	$\frac{20}{51}$
3	$\frac{11}{27}$	0	$\frac{22}{189}$	$\frac{10}{21}$
4	0	$\frac{11}{102}$	$\frac{1}{42}$	$\frac{310}{357}$

(c) Q^2 reprents the transition matrix where probability of transition from i to j is $Q^2[i,j]$

(d) Q^{∞} represent the long term probability of being in state j for $Q^{\infty}[i,j]$, ~~because~~. For an irreducible aperiodic Q, $\lim_{n\to\infty} Q^n[i,j]$ is fixed, long term probability is independent of initial state

(e) a 2x2 matrix with fixed value element

(f) constant $O(1)$

(g) not. ~~Q^{∞} is~~ Suppose Q^{∞} converges in large ~~is~~ s, so we need to compute Q^{s}, for compute Q with n ~~ram~~ random variable and size k, there are k^n states, time complexity for compute Q is $O(k^n \times k^n) = O(k^{2n})$

time complexity to compute Q^s is $\Theta(\log_2 s)$

Total time complexity is $O(k^{2n} \log_2 s)$

so this is unpractical

2. (a) second order:

$X_{t-2} \rightarrow X_{t-1} \rightarrow X_t \rightarrow X_{t+1} \rightarrow X_{t+2} \rightarrow$

transition model $P(X_t \mid X_{t-2}, X_{t-1})$

Let X_t' be the parent of X_{t+1}, and its parent is X_{t-1}

so that $X_{t-1} \rightarrow X_t' \rightarrow X_{t+1}$ ~~(let $X_t' = X_{t-1}$~~ ($X_{t-1} = X_t'$ in value)

(b) ~~Then $P(X_{t+1} \mid X_t'$~~ Not, because we only change the variables, the parameters does not change in the model.

3. Instruction:
File: kmean.py generate_test.py
Language:python3.7

For Step2-4: python kmeans.py

Step 2 function: count
Step 3 function: step3, compute_error
Step 4 function: step4

For Step5,bonus1,2: python generate_test.py

Step 5 function: step5, generate, compute_error
Bonus 1 function: bonus1, generate2
Bonus 2 function: bonus2, generate