

1. (a) There are 4 states ~~that~~

(b) Sprinkler and Wet Grass are always true

$$p(c|r,s) = d p(c) p(r|c) p(s|c) = 0.04d$$

$$p(r|c,s) = 0.05d \quad p(c|r,s) = \frac{4}{9}, \quad p(r|c,s) = \frac{5}{9}$$

$$p(c|r,s) = 0.01d \quad p(r|c,s) = 0.2d$$

$$p(c|7r,s) = \langle \frac{1}{2}, \frac{1}{2} \rangle$$

$$p(r|c,s,w) = d p(r|c) p(w|s,R)$$

$$= d(0.8, 0.2)(0.99, 0.9) = d(\frac{22}{27}, \frac{5}{27})$$

$$p(r|7c,s,w) = d p(r|7c) p(w|s,R) = d(\frac{11}{51}, \frac{40}{51})$$

possible y and y' : 1. (c,r) 2. $(7c,r)$ 3. $(c,7r)$ 4. $(7c,7r)$

$$q(1 \rightarrow 1) = 0.5 p(c|r,s) + 0.5 p(r|c,s,w) = \frac{17}{27}$$

$$q(1 \rightarrow 2) = 0.5 p(r|c,s,w) = \frac{5}{54}$$

$$q(1 \rightarrow 3) = 0.5 p(r|c,s) = \frac{5}{18}$$

$$q(1 \rightarrow 4) = 1 - \sum_{i=1}^3 q(1 \rightarrow i) = 0$$

$$q(2 \rightarrow 1) = 0.5 p(c|r,s) = \frac{4}{9}$$

$$q(2 \rightarrow 2) = \frac{59}{153} \quad q(2 \rightarrow 3) = 0 \quad q(2 \rightarrow 4) = \frac{20}{51}$$

$$q(3 \rightarrow 1) = \frac{11}{27} \quad q(3 \rightarrow 2) = 0 \quad q(3 \rightarrow 3) = \frac{22}{189} \quad q(3 \rightarrow 4) = \frac{10}{21}$$

$$q(4 \rightarrow 1) = 0 \quad q(4 \rightarrow 2) = \frac{11}{162} \quad q(4 \rightarrow 3) = \frac{1}{42} \quad q(4 \rightarrow 4) = \frac{310}{357}$$

transition matrix Q that change from row to column

	1	2	3	4
1	$\frac{17}{27}$	$\frac{5}{54}$	$\frac{5}{18}$	0
2	$\frac{4}{9}$	$\frac{59}{153}$	0	$\frac{20}{51}$
3	$\frac{11}{27}$	0	$\frac{22}{189}$	$\frac{10}{21}$
4	0	$\frac{11}{162}$	$\frac{1}{42}$	$\frac{310}{357}$

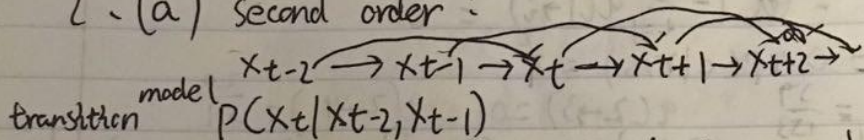
(c) Q^2 represents the transition matrix where the probability of transition from i to j is $Q^2[i,j]$

(d) Q^{∞} represent the long term probability of being in state j for $Q^{\infty}[i,j]$, ~~because~~ For an irreducible aperiodic Q , $\lim_{n \rightarrow \infty} Q^n[i,j]$ is fixed, long term probability is independent of initial state

(e) a 2×2 matrix with fixed value element
 (f) constant $O(1)$

(g) not. ~~Q^{∞}~~ is Suppose Q^{∞} converges in large S , so we need to compute Q^S , for compute Q with n random variable and size k , there are k^n states, time complexity for compute Q is $O(k^n \times k^n) = O(k^{2n})$
 time complexity to compute Q^S is $O(\log_2 S)$
 Total time complexity is $O(k^{2n} \log_2 S)$
 so this is impractical

2. (a) second order:



Let $x_{t'}$ be the parent of x_{t+1} , and its parent is x_{t-1}
 so that $x_{t-1} \rightarrow x_{t'} \rightarrow x_{t+1}$ (~~Let $x_{t'} = x_t$~~ $(x_{t-1} = x_{t'}$ in value)

(b) ~~Then $P(x_{t+1} | x_{t'})$~~ Not, because we only change the variables, the parameters does not change in the model.

3. Instruction:

File: kmean.py generate_test.py

Language:python3.7

For Step2-4: python kmeans.py

Step 2 function: count

Step 3 function: step3, compute_error

Step 4 function: step4

For Step5,bonus1,2: python generate_test.py

Step 5 function: step5, generate, compute_error

Bonus 1 function: bonus1, generate2

Bonus 2 function: bonus2, generate