2.1). The false-positive rate $= (1-e^{-km/n})^k$.

where k is number of hash functions, n is the bit-array length, m is the number of members of S.

Thus: Three hash function false-positive rate $: (1-e^{-3\cdot 2/10})^3 \approx 0.0918 = 9.18\%$

Four hash function false-positive rate $= (1-e^{-4\cdot 2/10})^4 \approx 0.09195 = 9.195\%$

2.2). Suppose we are given the ratio m/n and want to min the false-positive rate with the optimal k.

We have false-positive function: $(1-e^{-km/n})^k$.

let $g = \ln(f) = k\ln(1-e^{-km/n})$

Suppose $p = e^{-km/n}$, Thus $k = -\frac{n}{m}\ln(p)$. Then:

$g = k\ln(1-e^{-km/n}) = -\frac{n}{m}\ln(p)\ln(1-p)$.

Thus $k = -\frac{n}{m}\ln(1/2) = \ln(2)\cdot\frac{n}{m}$.

We Assume $n = 10$ billion bits, $m = 2$ billion members of set S.

The number of hash function mins the false positive rate is:

$k = \ln(2)\cdot\frac{10}{2}$.

≈ 3.45.

Thus the Three or four hash function