16.8421

4.1. a) $h(x) = (2x+1) \bmod 32$.

$h(3) = (2\cdot3+1) = 7$.

00111 = 7 the tail length is 0. estimate $= 2^0 = 1$.

$h(1) = (2+1) = 3$

00011 = 3 the tail length is 0. estimate $= 2^0 = 1$.

$h(4) = (2\cdot4+1) = 9$.

01001 = 9 → tail length is 0 estimate $= 2^0 = 1$.

$h(6) = (2\cdot6+1) = 13$

01101 = 13. → tail length is 0 estimate $= 2^0 = 1$.

$h(5) = (2\cdot5+1) = 11$

01011 = 11 → tail length 0. estimate $= 2^0 = 1$.

$h(9) = (2\cdot9+1) = 19$.

10011 = 19 → tail length 0 estimate $= 2^0 = 1$.

the max trial length is 0.
the estimate number of distinct elements is 1.

b) $h(x) = (3x+7) \bmod 32$.

$h(3) = (3\cdot3+7) = 16$

10000 = 16 → tail length 4 estimate $= 2^4 = 16$.

$h(1) = (3\cdot1+7) = 10$

01010 = 10 → tail length 1 estimate $= 2^1 = 2$.

$h(4) = (3\cdot4+7) = 19$

10011 = 19 → tail length 0 estimate $= 2^0 = 1$

$h(6) = (3\cdot6+7) = 25$

11001 = 25 → tail length 0 estimate $= 2^0 = 1$.

$h(5) = (3\cdot5+7) = 22$

10110 = 22 → tail length 1 estimate $= 2^1 = 2$.

$h(9) = (3\cdot9+7) = 34 \bmod 32 = 2$.

00010 = 2. → tail length 1 estimate $= 2^1 = 2$.

the max trial length for $h(x) = (3x+7)$ is 4.
the estimate number of distinct elements is 16.

c) $h(x) = 4x \bmod 32$

$h(3) = 12$

01100 → trial length 2 estimate $= 2^2 = 4$.

$h(1) = 4$

00100 → trail length 2 estimate $= 2^2 = 4$.

$h(4) = 16$

10000 → trail length 4 estimate $= 2^4 = 16$.

$h(6) = 24$

11000 → trail length 3 estimate $= 2^3 = 8$.

$h(5) = 20$.

10100 → trail length 2 estimate $= 2^2 = 4$

$h(9) = 36$ in mode $32 = 4$.

00100 = 4 → trial length 2 estimate $= 2^2 = 4$

the max trial length for $h(x) = 4x$ is 4.
The estimate number of distinct elements is 16.

4.2 a) $h(6x+2) \bmod 32$

$h(4) = (6x4+2) = 26$.

11010 = 26 → trial length 1 estimate $2^1 = 2$.

$h(5) = (6x5+2) = 32 \bmod 32 = 0$.

00000 → trial length 0. estimate $2^0 = 1$.

$h(6) = (6x6+2) = 38 \bmod 32 = 6$

00110 → trial length 1 estimate $2^1 = 2$.

$h(7) = (42+2) = 44 \bmod 32 = 12$.

01100 = 12 → trial length 2 estimate $2^2 = 4$.

$h(10) = 62 \bmod 32 = 30$.

11110 = 30 → trial length 1 estimate $2^1 = 2$.

$h(15) = 92 \bmod 32 = 28$.

11100 = 28 → trial length 2 estimate $2^2 = 4$.

The max trial length for $h(x) = (6x+2)$ is 2
The estimate number of distinct elements is 4.

b) $h(x) = (2x+5) \bmod 32$.

$h(4) = 13$

01101 = 13 → trial length 0 estimate $2^0 = 1$

$h(5) = 15$

01111 = 15 → trial length 0 estimate $2^0 = 1$

$h(6) = (2x6+5) = 17$

10001 = 17 → trial length 0 estimate $2^0 = 1$

$h(7) = (2x7+5) = 19$

10011 = 19 → trial length 0 estimate $2^0 = 1$

$h(10) = (2x10+5) = 25$

11001 = 25 → trial length 0 estimate $2^0 = 1$

$h(15) = (2x15+5) = 35 \bmod 32 = 3$.

00011 = 3 → trial length 0 estimate $2^0 = 1$

The max trial length for $h(x) = (2x5)$ is 0
The estimate number of distinct elements is 1.

) $h(x) = 2x \bmod 32$.

$h(4) = 2 \times 4 = 8$

01000 = 8 → trail length is 3, estimate $2^3 = 8$.

$h(5) = 2 \times 5 = 10$

01010 = 10 → trial length is 1 estimate $2^1 = 2$.

$h(6) = 2 \times 6 = 12$

01100 = 12 → trial length is 2 estimate $2^2 = 4$.

$h(7) = 2 \times 7 = 14$

01110 = 14 → trial length is 1 estimate $2^1 = 2$

$h(10) = 2 \times 10 = 20$

10100 = 20 → trial length is 2 estimate $2^2 = 4$

$h(15) = 2 \times 15 = 30$

11110 = 30 → trial length is 1 estimate $2^1 = 2$

The max trial length for $h(x) = 2x$ is 3.
The estimate number of distinct elements is 8.