Fast Two-Dimensional Otsu's Thresholding Method Based on Integral Image

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Abstract—**The traditional two-dimensional Otsu's threshold method using gray level-average gray level two-dimensional histogram,in view of the obvious deficiencies of the regional division of the traditional two-dimensional histogram, an improved fast two-dimensional Otsu's thresholding method with integral image based on gray level-gradient two-dimensional histogram is proposed. Using integral image to reduce the computational complexity in searching the best threshold value of two-dimensional histogram, thereby reducing the amount of calculation, the results show that the method has a good segmentation effect which greatly and significantly increased the computing speed. Therefore, it is a fast and effective thresholding segmentation method with a good real-time quality.**

Keywords-image segmentation; threshold selection; two-dimensional Otsu's method; integral image

I. INTRODUCTION

Image segmentation subdivides image into its sub-areas or objects, and it is a major step of extracting image properties in the image processing methods. Owing to its realization of intuitiveness and simplicity, the image threshold processing method plays an important role in the image segmentation[1]. In many threshold selection methods based on gray histogram, the maximum between-class variance method proposed by the Japanese scholar Otsu[2] becomes widely used in image processing for its better segmentation results, easy computation and wide scope of application. Based on the one-dimensional gray histogram of image, this method regards the largest inter-class variance between target and background as a principle to choose the best segmentation threshold. In the case of high quality of the image and stable change of the background, One-dimensional Otsu's method can obtain better segmentation results. But as one-dimensional histogram of image can't reflect the space-related information among the image pixel, it is difficult to obtain satisfactory segmentation results when the image contains noise. For this reason, Liu Jianzhuang et al[3] introduced the two-dimensional histogram, which is composed of the pixel gray-level distribution and neighborhood average gray-level distribution. The two-dimensional Otsu's method above the cross lines separately parallel with the two axes of coordinate system to divides the two- dimensional histogram into four regions, only considering the two regions along the diagonal when computing the threshold. As for this method, it is assumed that " the total probability of the target area and the background area is approximately 1". This assumption has a certain degree of rationality. However, it is obviously impractical to simply consider the probability of the half region as 0, wich is next to threshold and has similar pixel gray level and neighborhood average gray level.

In addition, a one-dimensional Otsu's method is extended to a two-dimensional, segmentation effect is significantly improved. However, the introduction of two-dimensional histogram has increased the computational complexity and extended the run time. In order to reduce the amount of computation, Gong Jian et al[4]start from the point of reducing double counting, developed a fast two-dimensional Otsu's thresholding method. Based on a re-classify for the area of two-dimensional histogram, He Yinming [5]proposed a fast algorithm which turn two-dimensional thresholding into one-dimensional thresholding. Both fast algorithm can save computational time or improve run speed in some degrees, but still can not meet the requirement of relatively high real-time occasion.

When the two-dimensional method search the best threshold, it needs to traverse the whole two-dimensional histogram, and do a great deal of mean computation in its rectangular regions, however, it is very convenient to compute on the basis of integral image[6]which can avoid the traverse of the region. This paper applies the integral image into two-dimensional Otsu method, and turn the loop sum operation when using the exhaustive search method into a small amount of addition and subtraction operations. It reduces the amount of computation greatly and has a good real-time feature.

This paper points out the weakness of traditional gray level - an average of two-dimensional gray-level histogram in region segmentation, introduced gray level - gradient two-dimensional histogram, and a fast two-dimensional Otsu's threshold segmentation method based on gray level-gradient two-dimensional histogram are given as well.

II. IMPROVED TWO-DIMENSIONAL OTSU'S METHOD

Suppose the size of an image as $M \times N$, gray level as L .By calculating neighborhood average gray level of $k \times k$ at each pixel, we can obtain a smoothing image $g(x, y)$,with gray level L .

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A. Traditional Two-Dimensional Histogram

Abscissa for the traditional two-dimensional histogram is the pixel's gray level $f(x,y)$,and vertical axis is its neighborhood average gray level $g(x,y)$. Suppose the threshold vector is (s,t), we can divide two-dimensional histogram into four regions as shown in Figure1.

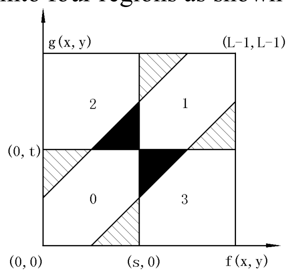

Figure 1. Traditional regional division of two-dimensional histogram

For the internal pixels of the target or background, its gray level and neighborhood average gray level are similar, but for the pixels of the target and the edge of the background, its gray level and neighborhood average gray level vary greatly. Therefore the region 0 and region 1 separately represent target and background, and the region 2 and 3 represent edge and noise. Suppose the appearance probability of the region 0 and the region 1 is approximately 1 when calculating the threshold, then all the pixels probability of the region 2 and region 3 is approximately 0. There are following deficiencies in this histogram regional division: there may be edge pixels and noise pixels are seen as interior target or background processing within the pixels when calculating the optimal threshold in the slash areas of the region 0 or region 1 (Figure1) due to the difference between gray level of pixels and the neighborhood average gray level. At the same time, the dark areas where are next to the threshold vector and similar pixels gray level and neighborhood average gray level (Figure1) will be neglected. That is, some interior target and background pixels are divided into noise and edge pixels and then removed. Obviously, this assumption does not match the fact, leading to the segmentation result inaccurate.

The improved two-dimensional histogram presented in this paper reflects the gray level differences between center pixels and the neighborhood pixels, and makes full use of the edge information and the detailed characteristics of the image, significantly improving the inaccurate segmentation results.

B Improved Two-dimensional Histogram

Abscissa for the improved two-dimensional histogram remains central pixel's gray level $f(x,y)$, vertical axis is changed into pixel's neighborhood gray gradient, that is the absolute value for the difference between pixel gray level and its neighborhood average gray level $|f(x,y)-g(x,y)|$. If gray level i and neighborhood gray gradient j of each pixel in the image form a binary group (i,j),the corresponding associated probability density is $p_{ij}=\frac{c_{ij}}{M\times N},0\le i,j\le L-1$ （1）

among which c_{ij} is the times for the appearance of (i,j),

$$\sum_{i=0}^{L-1}\sum_{j=0}^{L-1}p_{ij}=1 \quad .$$

$$c_{ij}=\sum_{x=1}^{M}\sum_{y=1}^{N}\delta\big(f(x,y)-i\big)\delta\big(|f(x,y)-g(x,y)|-j\big)$$

$$i=0,1,\dots,L-1,\quad j=0,1,\dots,L-1 \qquad (2)$$

According to the definition of two-dimensional histogram above, any thresholding vectors (s,t) ,can divide two-dimensional histogram into four regions as shown in Figure 2. Among which，regions 0 corresponding to the target，region 1 corresponding to the background，region 2 and 3 stand for the edge and the noise.

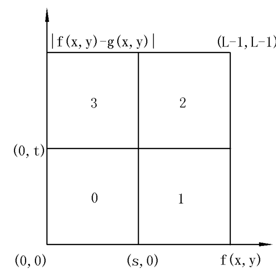

Figure 2. Regional division of two-dimensional histogram

As the gradient value reflects the gray difference in a neighborhood, small neighborhood gradient magnitude corresponding to some gray level shows more appearances of this gray level within the target or background, and less pixels on the edge, large neighborhood gradient magnitude indicates more appearances on the edge and less with the target and background. Therefore in this two-dimensional histogram region 0 is corresponding to the target, region 1 to the background and region 2 and 3 represent edge and noise. In this way, region 0 and 1 contain all the target pixels and background pixels as much as they can. This can overcome the shortcoming of the noisy pixels that may exist in region 0 and 1 in the traditional histogram division method.

C Improved Two-dimensional Otsu's method

According to the improved two-dimensional histogram, this paper presents an Otsu's thresholding method and its fast recursive algorithm. Suppose there are two types of C_0 and C_1 represent target and background separately in the two-dimensional histogram, if P_0 、 P_1 represent the probably of two types, therefore：

$$P_0(s,t)=\sum_{i=0}^{s}\sum_{j=0}^{t}p_{ij}\ ,\quad P_1(s,t)=\sum_{i=s+1}^{L-1}\sum_{j=0}^{t}p_{ij} \qquad (3)$$

Two types of the corresponding mean vector $\boldsymbol{\mu}_0$ 、 $\boldsymbol{\mu}_1$ are ：

$$\begin{aligned}\boldsymbol{\mu}_0&=(\boldsymbol{\mu}_{0i},\boldsymbol{\mu}_{0j})^{\mathrm{T}}=\left(\sum_{i=0}^{s}\sum_{j=0}^{t}\frac{ip_{ij}}{P_0},\sum_{i=0}^{s}\sum_{j=0}^{t}\frac{jp_{ij}}{P_0}\right)^{\mathrm{T}}\\ \boldsymbol{\mu}_1&=(\boldsymbol{\mu}_{1i},\boldsymbol{\mu}_{1j})^{\mathrm{T}}=\left(\sum_{i=s+1}^{L-1}\sum_{j=0}^{t}\frac{ip_{ij}}{P_1},\sum_{i=s+1}^{L-1}\sum_{j=0}^{t}\frac{jp_{ij}}{P_1}\right)^{\mathrm{T}}\end{aligned} \qquad (4)$$

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The total mean vector $\boldsymbol{\mu}_T$ or the two-dimensional histogram is：

$$\boldsymbol{\mu}_T = \left(\boldsymbol{\mu}_{Ti}, \boldsymbol{\mu}_{Tj}\right)^{\mathrm{T}} = \left(\sum_{i=0}^{L-1}\sum_{j=0}^{L-1} i p_{ij}, \sum_{i=0}^{L-1}\sum_{j=0}^{L-1} j p_{ij}\right)^{\mathrm{T}} \quad (5)$$

According to the assumption that the total probability of region 2 and 3 is approximately 0,Dispersion matrix $\mathbf{S}_B$ between the two types is：

$$\begin{aligned}\mathbf{S}_B(s,t) = P_0(s,t)\left[\left(\boldsymbol{\mu}_0(s,t)-\boldsymbol{\mu}_T\right)\left(\boldsymbol{\mu}_0(s,t)-\boldsymbol{\mu}_T\right)^{\mathrm{T}}\right] \\ +P_1(s,t)\left[\left(\boldsymbol{\mu}_1(s,t)-\boldsymbol{\mu}_T\right)\left(\boldsymbol{\mu}_1(s,t)-\boldsymbol{\mu}_T\right)^{\mathrm{T}}\right]\end{aligned} \quad (6)$$

Adopt the vestige of $\mathbf{S}_B$ as among –class discrete as follows:

$$\begin{aligned}Tr\mathbf{S}_B(s,t) = P_0(s,t)\left[\left(\boldsymbol{\mu}_{0,0}-\boldsymbol{\mu}_{T,0}\right)^2+\left(\boldsymbol{\mu}_{0,1}-\boldsymbol{\mu}_{T,1}\right)^2\right] \\ +P_1(s,t)\left[\left(\boldsymbol{\mu}_{1,0}-\boldsymbol{\mu}_{T,0}\right)^2+\left(\boldsymbol{\mu}_{1,1}-\boldsymbol{\mu}_{T,1}\right)^2\right]\end{aligned} \quad (7)$$

$$\left(s^*, t^*\right) = \arg\max_{0\le(s,t)\le L-1}\left\{Tr\mathbf{S}_B(s,t)\right\} \quad (8)$$

When $Tr\mathbf{S}_B(s,t)$ obtained the maximum value,($s^* t^*$)is the best thresholding value for the two-dimensional Otsu's method.

III. INTEGRAL IMAGE

Any gray value *ii*(*x,y*) for point (*x,y*) of integral image stand for the total gray value for the shadow area of Original images as shown in Figure 3,

$$ii(x,y) = \sum_{x'\le x, y'\le y} f(x', y') \quad (9)$$

And $f(x', y')$ is the gray value of point (x', y') for Original images.

ii(*x,y*) is calculated by iteration from formula (10) and (11)：

$$s(x,y) = s(x,y-1) + f(x,y) \quad (10)$$

$$ii(x,y) = ii(x-1,y) + s(x,y) \quad (11)$$

Among which ,s(x,y) stands for integral for one list, and

$$s(x,-1)=0, ii(-1,y)=0$$

To calculate the integral image, only one time traverse among the original images,and need very little computation.

The total gray value of window D is shown in Figure 4, however big the window D is, and four corresponding points 1,2,3,4 of integral image can be calculated quickly. The total gray value of window D is 1+4-(2+3).

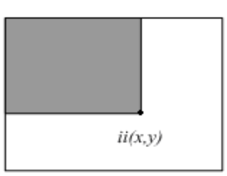


Figure 3 Schematic diagram of integral images

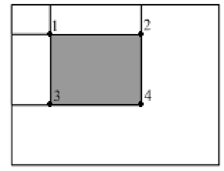


Figure 4 Schematic diagram of integral images

IV. FAST TWO-DIMENSIONAL OTSU'S METHOD

Two-dimensional Otsu's method in a large number of summary calculation for two-dimensional histogram of the rectangular region. If calculated from the first two-dimensional histogram of the corresponding integral image, we can easily calculate the total values. The proposed algorithm as follows：

Firstly,calculate the two-dimensional histogram by formula (1),and traverse the whole image.

Then， from formula (7),we can know that calculated dispersion matrix trace the emergence of two first calculate the probability and the two types of gray mean vector. This would calculate the number of pixels contained in the respective categories， and gray sum of the original images and gray gradient sum of its neighborhood. Therefore, we need calculate three integral images，separately are：

Integral image of the pixels number ：

$$ii_N(x,y) = \sum_{i=0}^{x}\sum_{j=0}^{y} c_{ij} \quad (12)$$

Gray integral image for the original image：

$$ii_O(x,y) = \sum_{i=0}^{x}\sum_{j=0}^{y} i c_{ij} \quad (13)$$

Integral image for the neighborhood gray gradient image：

$$ii_G(x,y) = \sum_{i=0}^{x}\sum_{j=0}^{y} j c_{ij} \quad (14)$$

The computation of three integral images above need only one time traverse to the two-dimensional histogram.

Lastly, Search for the optimal threshold by the formula (7),using integral image for fast calculation. Then：

$$\begin{aligned}F(s+1,t+1) = ii(s,t) + ii(L-1,0) - ii(L-1,t) \\ -ii(s,0)\end{aligned} \quad (15)$$

Put the formula (9) into the formula (3),then：

$$\begin{aligned}P_0(s,t) &= \frac{1}{M\times N} ii_N(s,t) \\ P_1(s,t) &= \frac{1}{M\times N} F(s+1,t+1)\end{aligned} \quad (16)$$

Put the formula (1) , (3)into the formula (4),then：

$$\boldsymbol{\mu}_0(s,t) = (\boldsymbol{\mu}_{0,0}, \boldsymbol{\mu}_{0,1})^{\mathrm{T}} = \left(\sum_{i=0}^{s}\sum_{j=0}^{t} \frac{ip_{ij}}{P_0}, \sum_{i=0}^{s}\sum_{j=0}^{t} \frac{jp_{ij}}{P_0} \right)^{\mathrm{T}}$$

$$= \left(\sum_{i=0}^{s}\sum_{j=0}^{t} \frac{ic_{ij}}{\sum_{i=0}^{s}\sum_{j=0}^{t} c_{ij}}, \sum_{i=0}^{s}\sum_{j=0}^{t} \frac{jc_{ij}}{\sum_{i=0}^{s}\sum_{j=0}^{t} c_{ij}} \right)^{\mathrm{T}} \quad (17)$$

Put the formula (12) , (13)into the formula (17),then：

$$\boldsymbol{\mu}_0(s,t) = \left(\frac{ii_O(s,t)}{ii_N(s,t)}, \frac{ii_G(s,t)}{ii_N(s,t)} \right)^{\mathrm{T}} \quad (18)$$

Similarly,

$$\boldsymbol{\mu}_1(s,t) = (\boldsymbol{\mu}_{1,0}, \boldsymbol{\mu}_{1,1})^{\mathrm{T}}$$

$$= \left(\frac{F_O(s+1,t+1)}{F_N(s+1,t+1)}, \frac{F_G(s+1,t+1)}{F_N(s+1,t+1)} \right)^{\mathrm{T}} \quad (19)$$

$$\boldsymbol{\mu}_T = (\boldsymbol{\mu}_{T,0}, \boldsymbol{\mu}_{T,1})^{\mathrm{T}}$$

$$= \left(\frac{ii_O(s+1,t+1)}{M \times N}, \frac{ii_G(s+1,t+1)}{M \times N} \right)^{\mathrm{T}} \quad (20)$$

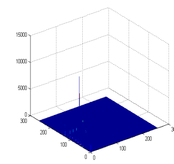
After calculating the mean vector by the formula (18),(19),(20),we can get divergence trace of matrix by the formula (7). The method in this paper has reduced the dimension of the results. The confirmation of the best threshold (s^*t^*) need traverse the space once only, and make the computation complex from O(L^4) to O(L^2).

V. THE EXPERIMENT RESULTS

The simulated experiment is operated in 1.8 GHz Micro Processing unit with Conroe Dual Core. And compared with the segmentation effect and operation time in Reference[3]and Reference[4]with the algorithm proposed in this paper. The experimental image is a girl's picture with the size of 256×256 and adds noise 0.005. When the neighborhood of gradient is reflected in the window 3×3 , the experiment result is shown in Figure 5.

The best thresholding valve of every algorithm are given in Table 1. It can be seen from Figure 5(c)、(d),that the algorithm in reference[3],[4] can produce better segmentation results for large targets, but in the background, gray level varying parts are mistaken as small targets and are recognized. It can be seen from Figure 5(e) that compared to the algorithm in reference[3],[4], the improved two-dimensional Otsu's method has better noise-resistance. With this method, the obtained regions are more consistent, and the edge of the obtained target is clearer. As regards to the efficiency of computation（Table 2）, it can be found that the improved method reduced the operation time of image segmentation, properly fulfilling the requirements of real-time system..

(a) Original images



(b) two-dimensional histogram

(c) algorithm of reference [3]

(d) algorithm of reference [4]

(e) algorithm of This paper

Figure 5 Segmentation result comparison of three methods

TABLE I. THE BEST THRESHOLDING VALVE OF EVERY ALGORITHM

algorithm	girl's picture
reference [3]	（74,78）
reference [4]	（74,78）
This paper	（74,87）

TABLE II. THE RUNNING TIME OF EVERY ALGORITHM (S)

algorithm	girl's picture
reference [3]	67.539
reference [4]	2.551
This paper	0.072

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