A Hierarchical Hough Transform for Fingerprint Matching

Chaoqiang Liu[1], Tao Xia[2], and Hui Li[1]

[1] Temasek Laboratories and Centre for Wavelets
Approximation and Information Processing
National University of Singapore,
{tslliucq,tsllh}@nus.edu.sg,

[2] Centre for Wavelets, Approximation and Information Processing
National University of Singapore
iesxiat@nus.edu.sg

Abstract. This paper addresses the improvement on the matching accuracy and speed of generalized Hough transform for the biometric identification applications. The difficulties encountered in generalized Hough transform are investigated and a new hierarchical Hough transform algorithm is proposed, which is faster and more accurate compared to conventional generalized Hough transform.

1 Introduction

The biometrics is a technology that (uniquely) identifies a person based on his physiological or behavioral characteristics. Nowadays, biometrics identification is commonly in use for example at banks, airports, and government agencies. For identification signatures at present around the world, the popular methods are face images, fingerprint, voice print and iris pattern. Among them, the fingerprint is the most favorite one for its uniqueness for the individual. Accurate automatic personal fingerprint identification is becoming more and more important to the operation of our increasingly electronically interconnected information society.

An automatic fingerprint authentication system has four main design components: acquisition, representation (template), feature extraction, and matching. Among them the feature extraction including the fingerprint image processing problem is well studied [1]–[3]. However, no perfect feature extraction method to guarantee all fingerprint features are extracted exactly, especially for the latent fingerprints with poor quality and incompleteness. Therefore, the better matching algorithm for the fingerprint authentication is a big problem in this society.

Fingerprint matching has been approached from several different strategies, such as image-based [4], ridge pattern matching, graph-based fingerprint matching [5] and structural matching [6]. Also, given the minutiae representation of fingerprints, matching a fingerprint against a database converts to the problem of point matching [7]. In recent years, the Hough Transform and generalized Hough

D. Zhang and A.K. Jain (Eds.): ICBA 2004, LNCS 3072, pp. 373–379, 2004.

Transform [8], GHT, has become an extremely popular method of 2D shape detection. The most exciting properties of Hough transform are its immunity to noise. The GHT-based approach converts point pattern matching [9] to a problem of detecting peaks in the Hough space of transformation parameters. The best transformation parameters of matching two point sets corresponding the peak is attained by accumulating the evidence in the discretized Hough space. Several novel approaches have been proposed to improve its computational efficiency and practicability [10],[11].

However, the traditional GHT used in fingerprint matching may lead to some errors due to the coarse quantization in the rotation parameter space. Based on the analysis of this kind of errors, the GHT in the translation parameter space is modified to ensure our candidate optimal solutions in the coarser level includes the real optimal solution in finer level. Therefore we develop a novel hierarchical Hough transform (HHT).

This paper is organized as follows. The fingerprint matching problem is briefly introduced in Section 2. Our method is described in Section 3. The experimental results and discussions are presented in Section 4. Conclusions are drawn in Section 5.

2 Fingerprint Alignment and Matching

As mentioned above, an automatic fingerprint authentication system is composed of four main design components. Although there are other representations such as pore [12], common representations of fingerprints are still minutiae (fingerprint features of ridge ending and ridge bifurcation [13]) based, and each minutia is described by its location (x,y coordinates) and the orientation. A feature extractor detects the minutiae for automated feature extraction and matching. In order to match one against another, the two points sets must be registered with respect to each other. The rotation, scale, and translation parameters of alignment transform are estimated using GHT. The actual transformation parameters are obtained by detecting peak in Hough space. The transformations $F_{s,\delta_x,\delta_y,\delta_\theta} : R^2 \to R^2$ given below is the most common one for alignment.

$$F_{s,\delta_x,\delta_y,\delta_\theta}\begin{pmatrix} x \\ y \end{pmatrix} = sR_{\delta_\theta}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} \tag{1}$$

where $R_{\delta_\theta} = \begin{pmatrix} \cos\delta_\theta & \sin\delta_\theta \\ -\sin\delta_\theta & \cos\delta_\theta \end{pmatrix}$ is the rotation transform. s, (δ_x, δ_y), and δ_θ are the scale, shift, and rotation parameters, respectively.

In the fingerprint identification/authentication application, the scale difference is mainly caused by the dpi for different kind of sensors and it is supposed to be known in advance. Therefore in this paper we deal with other two parameters only, namely rotation and translation.

In practice, the parameter space is partitioned into the uniform cubic according to the specified precision of each parameters. Therefore, the accuracy of alignment transformation depends on the quantization of the Hough space. Apparently, finer precision will cause more cubics and it will lead to the huge burden of computation cost. On the other hand, coarser precision sometimes will lead to the wrong result, especially for the fingerprint authentication applications. Therefore, the proper trade-off between computation cost and accuracy is indeed an important problem for the real application.

Mainly, there are two kinds of errors/difficulties of the existing fingerprint alignment and matching,

– Missing the matched pair due to the different tolerance bounds for different alignment precision and minutiae location.
– Duplication count of matched minutiae, i.e. there are more than one minutiae are matched against one minutia in the template and vice versa.

In this paper, we proposed a hierarchical Hough transform (HHT) to overcome these two problems. For simplicity, in the following contents, the denotation $v_1 \leq v_2$ for two vectors means for every element, this inequality holds and sometimes we use v to represent its x, y components. Also, when we state $\theta_1 \leq \theta_2$ for two angles, we always compare θ_1, θ_2 under the modular 2π.

3 Hierarchical Hough Transform

The accuracy of alignment transform $F_{\delta_x,\delta_y,\delta_\theta}$ plays an important role in fingerprint matching. To achieve higher accuracy, the precision of all parameters should be high even though that will lead to burden of computation cost. In this application, the parameter space consists of the transform of rotation and translation by which the two coordinate systems will be aligned.

Let U be the feature set of template fingerprint saved in database, and V be the feature set of latent fingerprint. The element in a feature set is a triplet consisting of coordinate and orientation of one minutia. Suppose the template feature set has M minutiae, i.e. $U = \{u_i,\ i = 1, \cdots, M\}$ and the latent feature set has N minutiae, i.e. $V = \{v_i,\ i = 1, \cdots, N\}$. Let $M_{xy} = (\max(|v_x|),\ \max(|v_y|))^T$ be the maximum value of the fingerprint image of set V. $\triangle_{xy} = (\triangle_x, \triangle_y)^T, \triangle_\theta$ are the expected precisions of alignment parameters of the translation and rotation respectively. $\triangle_{xy}, \triangle_\alpha$ is the tolerate bound of location and orientation difference of two matched minutiae respectively. $[x_{\min},\ x_{\max}], [y_{\min},\ y_{\max}]$ and $[\theta_{\min},\ \theta_{\max}]$ are the intervals of possible translation and rotation angles for the latent fingerprint against the template fingerprint in database.

One typical error in popular GHT is that it will miss some matched pairs of minutiae. This is caused by the coarse rotation precision of alignment transform. The following theorem will give the necessary condition for such case.

Theorem 1 The Missing Pair of the Matched Minutiae
Suppose $u \in U, v \in V$ and the tolerance bounds for matching two fingerprints are $\triangle_x, \triangle_y$, the rotation angle for alignment transform of V against U is θ, if the transform imposed on V is $R_{\theta'}$, i.e. $v \xrightarrow{R_{\theta'}} v^1$ then, $\|v^1 - u\| \leq \|(\triangle_x,\ \triangle_y)^t\| + 2\sin(\frac{|\theta' - \theta|}{2})\|v\|$.

Theorem 1 shows that the actual error bounds should be different for different rotation precisions and different minutia in V. Same error bound for all cases will lead to the missing pairs. For the sake of simplicity, only alignment precision is considered here and $M_{xy,\rho} := 2\sin(\rho/2)M_{xy}$ is used to replace $2\sin(\frac{|\theta'-\theta|}{2})\|v\|$, in Theorem 1, where ρ is the rotation precision, i.e. $|\theta' - \theta| \leq \rho$. That will lead to algorithm 1.

Given an array of precision sequence $(\rho_1, \rho_2, \cdots, \rho_K)$, where $\rho_1 > \rho_2 > \cdots > \rho_K = \triangle_\theta$ and the threshold T of the minimum number of matched minutiae of two matched fingerprints.

Algorithm 1 *Hough Transform for orientation angle*

1. *for* $j = 1$ *to* K,
2. $\Theta = \{[\theta_{\min}, \theta_{\max}]\}, \rho = \rho_j, \Theta_r = \emptyset$, *n=0.*
3. *For each interval* $[\beta, \beta'] \in \Theta$
   (a) *Partition* $[\beta, \beta'] = \bigcup_{i=0}^{L-1} [\beta_i, \beta_{i+1}]$ *with step size* $\rho = \beta_i - \beta_{i-1}, \forall i$, .
   (b) *for* $i = 0$ *to* $L - 1$,
      i. $Q = \{(k, l) \,|\, |(u_{k,\theta} - v_{l,\theta}) - (\beta_i + \frac{\rho}{2})| \leq \rho + \triangle_\alpha,\ u_k \in U,\ v_l \in V\}$
      ii. *If* $j < K$
         using algorithm 2 to compute $m = H_1(Q, \triangle_{xy} + M_{xy,\frac{\rho}{2}}, \beta_i + \frac{\rho}{2})$
         If $m \geq T$ *then* $\Theta_r = \Theta_r \bigcup \{[\beta_i, \beta_{i+1}]\}$
         else
         Using algorithm 3 to compute $m = H_2(Q, \triangle_{xy} + M_{xy,\frac{\rho}{2}}, \beta_i + \frac{\rho}{2}, n)$
      iii. *If* $m > n$, *then* $m \to n$,
4. *if* $\Theta_r = \emptyset$ *or* $n < T$ *These two fingerprint can not be matched, end.*
5. $\Theta_r \to \Theta$.
6. *These two fingerprint are matched with n pairs of minutiae, corresponding score is computed accordingly.*

Remarks

1. For different angle precisions, we keep all possible candidates to avoid local maximum solution.
2. If $K = 1$, then this algorithm will be the generalized Hough transform.
3. The precision sequence can be chosen arbitrarily to meet the requirement of trade-off between the computation cost and the accuracy.

The algorithm proposed here is a hierarchical one in the sense of generating different parameter spaces for the translation in algorithm 1. The approximation of the exact number of matched pairs of two fingerprints U and V will be evaluated by coarse matching algorithm 2 and fine matching algorithm 3 respectively depending on where the rotation angle precision is coarse or not.

Algorithm 2 *Coarse Matching Algorithms of* $H_1(Q, D, \beta)$

1. *Quantizing* $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$ *into an accumulator* $P(i, j) = 0$, $i = \lceil \frac{x_{\min}}{q_x} \rceil, \cdots, \lceil \frac{x_{\max}}{q_x} \rceil, j = \lceil \frac{y_{\min}}{q_y} \rceil, \cdots, \lceil \frac{y_{\max}}{q_y} \rceil$, q_x, q_y *are quantization sizes.*
2. $Q' = \{(x, y) | (x, y) = u_i - R_\beta v_j,\ (i, j) \in Q\}$

3. *For each point* $(x, y) \in Q'$,*Let* $\delta_x = (D_x - q_x)/2$, $\delta_y = (D_y - q_y)/2$,
   $i_0 = \lceil \frac{x-\delta_x}{q_x} \rceil, \quad i_1 = \lceil \frac{x+\delta_x}{q_x} \rceil; \quad j_0 = \lceil \frac{y-\delta_y}{q_y} \rceil, \quad j_1 = \lceil \frac{y+\delta_y}{q_y} \rceil$
   $P(i, j) + 1 \rightarrow P(i, j), \forall i = i_0, \cdots, i_1, \, j = j_0, \cdots, j_1$
4. $(i_{\max}, \, j_{\max}) = \arg\max_{i,j}(P(i, j))$.
5. *return* $P(i_{\max}, j_{\max})$.

In algorithm 2, $(q_x, \, q_y)^T$ may differ to $\triangle_{xy}$. If $(q_x, \, q_y)^T = \triangle_{xy}$, it will be the most common matching algorithm; if $(q_x, \, q_y)^T = \triangle_{xy}/2$ as we used here, it is the more accurate matching algorithm.

Algorithm 2 is used for the rough rotation precision to speed up the matching process. For the last fine rotation precision case in algorithm 1 algorithm 3 is used. The exact number of matched pair between two fingerprint is impossible to obtained but the following theorem will tell us when the exact number of matched pair can be obtained.

Theorem 2 The Duplicate Count of the Matched Pair
Suppose two minutiae v_j *and* v_k *match to* u_i *simultaneously with the error bound* $\triangle_{xy}$ *then* $\|v_j - v_k\| \leq 2\|\triangle_{xy}\|$

The Theorem 2 shows that if $\|u_j - u_k\| \leq 2\|\triangle_{xy}\|$, and $\|v_j - v_k\| \leq 2\|\triangle_{xy}\|, \forall j, \, k$, then there is no duplicate count of the matched pair, in other words, no minutiae is matched to more than one minutiae in the other set.

Algorithm 3 *Fine Matching Algorithms of* $H_2(Q, D, \beta, n)$

1. *Quantizing* $[x_{\min}, \, x_{\max}] \times [y_{\min}, \, y_{\max}]$ *into an accumulator* $P(i, j) = 0$,
   $i = \lceil \frac{x_{\min}}{q_x} \rceil, \cdots, \lceil \frac{x_{\max}}{q_x} \rceil, j = \lceil \frac{y_{\min}}{q_y} \rceil, \cdots, \lceil \frac{y_{\max}}{q_y} \rceil$.
2. $V' = \{v' | v' = R_\beta v, \, v \in V\}$, $Q' = \{(x, y) | (x, y) = u_i - v'_j, \, (i, j) \in Q\}$
3. *For each point* $(x, y) \in Q'$,
   $i_0 = \lceil \frac{x-(D_x-q_x)/2}{q_x} \rceil, \, i_1 = \lceil \frac{x+(D_x-q_x)/2}{q_x} \rceil; \; j_0 = \lceil \frac{y-(D_y-q_y)/2}{q_y} \rceil, \, j_1 = \lceil \frac{y+(D_y-q_y)/2}{q_y} \rceil$
   $P(i, j) + 1 \rightarrow P(i, j), \forall i = i_0, \cdots, i_1, \, j = j_0, \cdots, j_1$
4. $(i_{\max}, \, j_{\max}) = \arg\max_{i,j}(P(i, j))$ *and* $\delta_x = (i_{\max} + \frac{1}{2})q_x, \delta_y = (j_{\max} + \frac{1}{2})q_y$,
5. *if* $P(i_{\max}, \, j_{\max}) \leq n$, *return 0;*
6. *Generating the matching matrix* $M(U, V') = \{m_{ij}\}_{ij}$, $i = 1, \cdots, M$, $j = 1, \cdots, N$, *where* $m_{ij} = 0$, *if* $|u_i - v'_j - (\delta_x, \, \delta_y)^T| \leq \triangle_{xy}$, *otherwise* $m_{ij} = 1$.
7. *return* $m = \min(\sum_i (1 - \prod_j m_{ij}), \, \sum_j (1 - \prod_i m_{ij}))$.

Remarks

1. The actual number of matched pairs m_0 satisfies $rank(M) \leq m_0 \leq m$.
2. When the application prefers the strict criteria for the matched fingerprint, $m = rank(M)$ could be used in last step.
3. The theorem 2 shows that if $\|u_j - u_k\| \leq 2\|\triangle_{xy}\|$, and $\|v_j - v_k\| \leq 2\|\triangle_{xy}\|, \forall j$, k, then $rank(M) = m_0 = m$. That also implies $m_0 = P(i_{\max}, j_{\max})$, therefore if the two fingerprints U, V satisfies the condition in theorem 2, we will use algorithm 2 for all rotation angle precisions.

The three algorithms, Hough Transform for orientation angle, coarse matching algorithm and fine matching algorithm in HHT provide a solution to overcome the problems caused by two kind of difficulties of GHT.

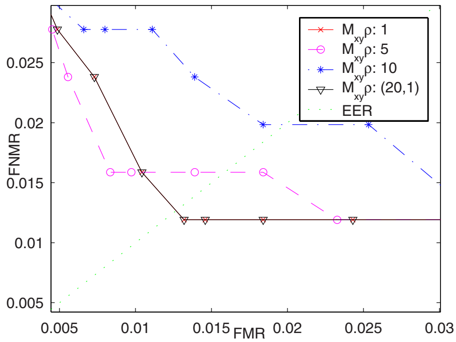

Fig. 1. The matching accuracy for different $M_{xy}\rho$ when $(q_x, q_y)^T = \triangle_{xy}/2$

4 Experimental Results

The performance of the HHT can be convinced by the comparison with the GHT. The test set DB1 of Fingerprint Verification Competition [14] is used for the simulation. The image resolution is 300×300, which means the $M_{xy} = 150$ and maximum rotation is in the range $[-15°, 15°]$, $\triangle_x = \triangle_y = 7$, $\triangle_\alpha = 15°$.

We conduct the simulation result for different rotation precisions $(\rho_1, \cdots, \rho_K)$ for algorithm 1 and different $(q_x, q_y)^T$ in algorithm 2. Obviously, for the larger ρ, the speed is faster, and if $K = 1$, HHT will be the conventional GHT. Instead of using ρ, we list $M_{xy}\rho$ as $M_{xy,\frac{\rho}{2}} \approx \frac{1}{2}M_{xy}\rho$, that means when $M_{xy}\rho = 1$, corresponding possible distortion of the error bound using in algorithm 1 will be less than 0.5 and can be negligible. For $K = 1$, $M_{xy}\rho$ ranges from 1 to 10 and for $K = 2$, $M_{xy}(\rho_1, \rho_2) = (20, 1)$ is the result of HHT.

It shows that the accuracy of HHT equals to the accuracy of the finest precision case for conventional GHT and the speed of matching is around $6-7$ times faster. The EER (equal error rate, defined in [14]) row shows that if we do not consider the problems encountered by GHT, the matching accuracy will degrade a lot. Also the use of $\triangle_{xy}/2$ in algorithm 2 improves the accuracy range from 10% to 50%. The result is listed in table 1 and figure 1 shows FNMR (False Non-Match Rate) and FMR (False Match Rate), also defined in [14], the intersection points of $y = x$ to the curves for different $M_{xy}\rho$ are the EER points.

Table 1. Comparison between HHT and GHT

$M_{xy}\rho$	(20, 1)		1		5		10	
$(q_x, q_y)^T$	$\frac{\triangle_{xy}}{2}$	$\triangle_x$	$\frac{\triangle_x}{2}$	$\triangle_x$	$\frac{\triangle_x}{2}$	$\triangle_x$	$\frac{\triangle_x}{2}$	$\triangle_x$
EER	**1.29%**	1.98%	1.29%	1.98%	1.59%	2.20%	1.98%	2.13%
time(ms)	**4.86**	4.00	30.37	28.26	6.39	5.90	3.37	3.10

5 Conclusions

In this paper we analysis two kinds of errors in fingerprint matching, namely missing matched pair and duplicate matching problem and proposed a new algorithm to overcome these difficulties encountered by conventional GHT algorithm. The proposed algorithm can achieve the accurate matching result in an efficient way. Experiment results show that the error will degrade the matching performance and our hierarchical Hough transform will overcome this problem in the efficient way.

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