1 Belief Update w/ HMM Observation Model

1.1 Belief Update

Computing $b' : S \to \mathbb{R}$ (the new belief state) given $b : S \to \mathbb{R}$ (the previous belief state), $\Lambda = \lambda_1\lambda_2...\lambda_n$ (an observation comprised of single words λ_i), and a the action performed to arrive at s' the new state. α is a normalizing constant.

$$b'(s') = \alpha P(\Lambda|s', a) \sum_s P(s'|s, a) b(s)$$

1.2 Probability of observation

We wish to compute $P(\Lambda|s, a)$ the probability of an observation Λ given a state and an action taken to arrive at that state. We will make the assumption $P(\Lambda|s, a) = P(\Lambda|s)$, that is, the action taken doesn't matter. We will also assume each word λ_i in the observation is independent of other words, and that their order does not matter. β is a normalizing constant.

$$P(\Lambda|s) = \beta \prod_i P(\lambda_i|s)$$

We now wish to calculate $P(\lambda_i|s)$ through word-counting. Define $W_s \in W^*$ (where W is the set of all words, W^* is the Kleene closure of this set) to be the list $w_1w_2...w_n$ of words observed in state s. In addition, for concise notation define the function $C : W^* \times W \to \mathbb{N}$ such that

$$C(W_s, \lambda_i) = |\{w | w \in W_s \wedge w = \lambda_i\}|$$

I.e. C counts the number of occurrences of a word λ_i in a list W_s of words. Given this, we can now simply define

$$P(\lambda_i|s) = \frac{C(W_s, \lambda_i)}{|W_s|}$$

1.3 New Belief Update

Putting these together yields

$$b'(s') = \alpha \left(\beta \prod_i P(\lambda_i|s') \right) \sum_s P(s'|s, a) b(s)$$

Or, expanded and with single normalization:

$$b'(s') = \alpha \prod_i \frac{C(W_{s'}, \lambda_i)}{|W_{s'}|} \sum_s P(s'|s, a) b(s)$$