NYU-ViscoMOD Software Manual

Extraction of Elastic Modulus from Dynamic Mechanical Analysis for Solid Materials

Software Introduction

In Dynamic Mechanical Analysis (DMA), the master curve is a fundamental concept used to predict the viscoelastic behavior of a material over a broad range of frequencies or time scales by employing the time-temperature superposition principle (TTS). Since direct experimental measurements at very high or low frequencies are often impractical, master curves are constructed by shifting isothermal frequency-dependent storage modulus along the logarithmic frequency axis. This process aligns data obtained at different temperatures to form a single, continuous curve representing the material's response over an extended frequency range. The shift factors used in this process are typically modeled using the Williams-Landel-Ferry (WLF) equation:

$$log(\alpha_T) = \frac{-C_1(T - T_r)}{C_2 + (T - T_r)} \tag{1}$$

where T is the temperature, T_r is a reference temperature chosen to construct the compliance master curve and C_1, C_2 are material-specific constants adjusted to fit the values of the shift factor a_T. However, for composite material, the constants are often unknown. In this software, free shifting method is utilized, which provides a more general approach to acquire a_T. With master curve composed, sigmoidal function is implemented

$$E'(\omega) = A\, tanh(B\, log(\omega) + C) + D \tag{2}$$

where E' is storage modulus, ω is frequency, *A*, *B*, *C*, and *D* are fitting coefficients. Once these coefficients are acquired, the equation 2 is used to represent the master curve throughout entire frequency range for the material. Using only $E'(\omega)$, the time domain relaxation modulus $E(t)$ can be determined by

$$E(t) = \frac{2}{\pi} \int_0^{\infty} \frac{E'(\omega)}{\omega} sin(\omega t)\, d\omega \tag{3}$$

where ω is the angular frequency and *t* is time. Equation 3 is integrated numerically to obtain the relaxation function $E(t)$ to acquire the stress response in the time-domain. To use a time-domain relaxation function to determine the stress history in a material based on its strain history

$$\sigma(t) = E(t) * d\varepsilon = \int_{-\infty}^{t} E(t - \tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau \tag{4}$$

Here, σ, ε, and τ represent stress, strain and the time variable used in the integration process, respectively. For situations involving constant strain rate deformation with a strain rate of $\dot{\varepsilon}$ starting from time *t* = 0, the convolution integral simplifies to

$$\sigma(t) = \dot{\varepsilon} \int_0^{\tau} E(\tau) d\tau \tag{5}$$

where $\dot{\varepsilon}$ is strain rate. For Quasi-static mechanical property, $\dot{\varepsilon}$ is often selected between 10^{-2} to 10^{-5}s^{-1}.

User Manual[1]

The user needs to upload a comma-separated values (CSV) file with a format shown as Table 1**,** where frequency is in Hz and storage modulus is in MPa (in default), and temperature in degree Celsius.

Table 1. CSV dataset formation for software

Temperature	Frequency	Storage Modulus
30	0.1	34.09
30	0.127	34.178
30	0.162	34.285
	⋮	
150	6.17	19.946
150	7.87	19.791
150	10	19.29

The dataset will be plotted on a Cartesian coordinate system with frequency in X-axis and storage modulus in Y-axis for visualization. Free shifting method is applied to automatically compose the master curve and shift factors are provided.

Equation 2 is then used to fit the master curve to determine the coefficient of *A*, *B*, *C* and *D*. Here, coefficient A and D can be tuned to increase accuracy and a real-time fitting curve will be plotted for user to make the adjustment.

Then, master curves for all reference temperatures with fitting lines will be presented for checking the fitting status at each temperatures. If the fitting line is incorrect, that means the coefficient A and D need to be adjusted.

Once all the fitting lines are correct, the transformation can be proceeded to estimate the elastic modulus at strain rate from 10^{-2} to $10^{-5}s^{-1}$.

At each graph, the user can download the graph and the calculated data as .jpg and .csv file respectively. And it is recommended user follow the steps to complete the transforming calculation.