Tutorials and Problems for Discrete-Time Signals and Systems

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Contents

Introduction

This document contains links to video tutorials, problem statements, and links to video-based solutions of the problems. Open the PDF of this document to conveniently launch the videos by clicking the cyan-highlighted links; click the red-highlighted entries in the table of contents to jump to the desired tutorial or problem.

The problems contain links to specific time points in the videos represented by these symbols: ★ = overall approach to the problem, $\surd$ = final answer, and Ⓢ = full problem solution.

Tutorials

Discrete-time sinusoids

`http://youtu.be/yLezP5ziz0U` (11:41) – Study the behavior of sinusoidal signals in discrete-time.

Linearity system property

`http://youtu.be/hWPJ67Z8dkc` (9:01) – Development of a formal proof method to establish whether or not a system is linear. Includes two examples at 2:38 and 5:35.

Time invariance (shift invariance) system property

`http://youtu.be/cKv_dlDAaIE` (7:44) – Development of a formal proof method to establish whether or not a system is time invariant, also known as shift invariant. Includes two examples at 3:19 and 5:01.

Causality system property

`http://youtu.be/JP5xAnmJl4s` (7:44) – Development of a formal proof method to establish whether or not a system is causal. Includes two examples at 2:06 and 4:31.

Stability system property

`http://youtu.be/crX0BxKxHGw` (6:04) – Development of a formal proof method to establish whether or not a system is stable in a bounded-input bounded-output (BIBO) sense. Includes two examples at 2:18 and 4:33.

Convolution sum

`http://youtu.be/U3BwStzKvs0` (9:16) – Discrete-time convolution sum of a linear time-invariant (LTI) system with numerical example.

Geometric series closed-form equation

`http://youtu.be/JJZ-shHiayU` (3:48) – Derivation of the closed-form equation of a geometric series.

Nth roots of unity

`http://youtu.be/1BjDKRrR8-E` (6:34) – Derivation and visualization of the equations for the Nth roots of unity and of -1.

Group delay

`http://youtu.be/k5x9s6bMZ5s` (7:33) – Develop the equation for group delay of a system and apply the equation with two examples at 2:18 and 3:55.

Direct Form I flow graph

`http://youtu.be/PS94WZFqiiY` (8:52) – Development of the Direct Form I flow graph from a system function.

Direct Form II flow graph

`http://youtu.be/yW6zD36GDcs` (2:56) – Development of the Direct Form II flow graph from a system function.

Bilinear transform

`http://youtu.be/OXhW-h8Tkao` (17:34) – A study of the bilinear transform, especially through visualization of the mapping between the continuous-time s-plane and the discrete-time z-plane. Includes development of the prewarping equations for discrete-time filter design.

System Properties

Linearity #1

★

Formally prove whether or not each system is linear.

[a] $T_1\{x[n]\} = 5\,x[n-10]$ √ Ⓢ

[b] $T_2\{x[n]\} = x[n]^2$ √ Ⓢ

[c] $T_3\{x[n]\} = x[n]/n$ √ Ⓢ

`http://youtu.be/jwDYSBsChUA` (5:42)

Linearity #2

★

Formally prove whether or not each system is linear.

[a] $T_4\{x[n]\} = x[-n]$ √ Ⓢ

[b] $T_5\{x[n]\} = n\,x[n]$ √ Ⓢ

[c] $T_6\{x[n]\} = \sum_{k=0}^{n} x[k]$ √ Ⓢ

`http://youtu.be/GwLdxBf5pqI` (5:37)

Time Invariance #1

★

Formally prove whether or not each system is time invariant.

[a] $T_1\{x[n]\} = 5\,x[n-10]$ √ Ⓢ

[b] $T_2\{x[n]\} = x[n]^2$ √ Ⓢ

[c] $T_3\{x[n]\} = x[n]/n$ √ Ⓢ

`http://youtu.be/zMkXxI63_Og` (4:55)

Time Invariance #2

★

Formally prove whether or not each system is time invariant. NOTE: The video solution to Part [c] includes a second solution technique based on *proof by counterexample*.

[a] $T_4\{x[n]\} = x[-n]$ √ Ⓢ

[b] $T_5\{x[n]\} = n\,x[n]$ √ Ⓢ

[c] $T_6\{x[n]\} = \sum_{k=0}^{n} x[k]$ √ Ⓢ

√ Ⓢ

`http://youtu.be/zT3xot8b3Ls` (9:30)

Causality #1

Formally prove whether or not each system is causal. ★

[a] $T_1\{x[n]\} = 5\,x[n-10]$ √ Ⓢ

[b] $T_2\{x[n]\} = x[n]^2$ √ Ⓢ

[c] $T_3\{x[n]\} = x[n]/n$ √ Ⓢ

`http://youtu.be/eXZwvrX47V0` (4:15)

Causality #2

Formally prove whether or not each system is causal. ★

[a] $T_4\{x[n]\} = x[-n]$ √ Ⓢ

[b] $T_5\{x[n]\} = n\,x[n]$ √ Ⓢ

[c] $T_6\{x[n]\} = \sum_{k=0}^{n} x[k]$ √ Ⓢ

`http://youtu.be/n2CyTrAOUbg` (7:40)

Stability #1

Formally prove whether or not each system is BIBO stable. ★

[a] $T_1\{x[n]\} = 5\,x[n-10]$ √ Ⓢ

[b] $T_2\{x[n]\} = x[n]^2$ √ Ⓢ

[c] $T_3\{x[n]\} = x[n]/n$ √ Ⓢ

`http://youtu.be/taaisxUnE1E` (3:46)

Stability #2

Formally prove whether or not each system is BIBO stable. ★

[a] $T_4\{x[n]\} = x[-n]$ √ Ⓢ

[b] $T_5\{x[n]\} = n\,x[n]$ √ Ⓢ

[c] $T_6\{x[n]\} = \sum_{k=0}^{n} x[k]$ √ Ⓢ

`http://youtu.be/1MeunVzResU` (5:00)

Convolution

Convolution #1

★

An LTI system has the impulse response $h[n] = \{1, \underline{2}, 0, -3\}$; the underline locates the $n = 0$ value. For each input sequence below, find the output sequence $y[n] = x[n] * h[n]$ expressed both as a list (underline the $n = 0$ value) and as a stem plot.

[a] $x_1[n] = \delta[n]$ √ Ⓢ

[b] $x_2[n] = \delta[n+1] + \delta[n-2]$ √ Ⓢ

[c] $x_3[n] = \{\underline{1}, 1, 1\}$ √ Ⓢ

[d] $x_4[n] = \{2, 1, \underline{-1}, -2, -3\}$ √ Ⓢ

`http://youtu.be/_RsMMkuQVUE` (12:42)

Convolution #2

★

An LTI system has the impulse response $h[n] = \alpha^n u[n]$ with $|\alpha| < 1$. The input to the system is $x[n] = \beta^n (u[n] - u[n-5])$ with no restriction on the value of β.

[a] Find the general closed-form equation for the system output $y[n]$. √ Ⓢ

[b] Evaluate $y[n]$ at n = 0, 2, and 10 for $\alpha = 0.6$ and $\beta = 0.8$. √ Ⓢ

[c] Create stem plots of $x[n]$, $h[n]$, and $y[n]$ over the time range $0 \leq n \leq 10$ for $\alpha = 0.6$ and $\beta = 0.8$. √ Ⓢ

[d] Repeat Part [c] for $\alpha = 0.6$ and $\beta = -0.8$. √ Ⓢ

`http://youtu.be/ps38QdEGL24` (13:36)

z-Transform

z-Transform #1

★

Given $x[n] = (1/3)^n u[n] + 3^n u[-n-1]$.

[a] Plot $x[n]$. √ Ⓢ

[b] Find $X(z)$ written as a ratio of two polynomials in z^{-1}. State the region of convergence (ROC). √ Ⓢ

[c] Plot the pole-zero diagram of $X(z)$ and indicate the ROC. √ Ⓢ

`http://youtu.be/IdnH4LDs6BA` (9:28)

z-Transform #2

★

A system has an impulse response $h[n]$ defined as

$$h[n] = \begin{cases} b^n, & 0 \le n \le N-1, \\ 0, & n \ge N, \end{cases}$$

where $N = 8$.

[a] Tabulate the values of $h[n]$ to four significant digits from $n = 0$ to 10 for $b = 0.8$ and for $b = 1.25$. Visualize each version of $h[n]$ as a stem plot. √ Ⓢ

[b] Find the system function $H(z)$ in terms of z (not z^{-1}) and written in closed form, i.e., with no summation symbol. State the region of convergence (ROC). √ Ⓢ

[c] Plot the pole-zero diagram of $H(z)$ for $b = 0.8$ and for $b = 1.25$. √ Ⓢ

`http://youtu.be/Cwit5qvAcRU` (10:30)

Inverse z-Transform #1

★

Given the following system function:

$$H(z) = \frac{1 + 0.25z^{-1}}{1 + 0.8z^{-1} - 0.84z^{-2}}$$

.

[a] Plot the pole-zero diagram of $H(z)$. √ Ⓢ

[b] Find a stable impulse response $h[n]$. √ Ⓢ

[c] Find a causal impulse response $h[n]$. √ Ⓢ

[d] Does an impulse response exist that is both stable and causal? Explain your answer. √ Ⓢ

`http://youtu.be/oEqq3S0itnQ` (13:35)

Frequency Response

Frequency Response #1

★

An LTI system with input $x[n]$ and output $y[n]$ has impulse response $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$. Find the system's

[a] Frequency response $H(e^{j\omega})$, and √ Ⓢ

[b] Steady-state output $y[n]$ when $x[n] = -2 + 3\cos((\pi/4)n + \pi/3) + 10\cos((3\pi/4)n - \pi/5)$. √ Ⓢ

`http://youtu.be/8eILsUt8PKw` (11:53)

Frequency Response #2

★

An LTI system has impulse response $h[n] = 2\delta[n] + \delta[n-1] + 2\delta[n-2]$.

[a] Determine the system's frequency response $H(e^{j\omega})$ written in the form $A(e^{j\omega})\phi(e^{j\omega})$, i.e., isolate the real-valued amplitude function $A(e^{j\omega})$ and the phase function $\phi(e^{j\omega})$. √ Ⓢ

[b] Plot the frequency response magnitude $|H(e^{j\omega})|$ and phase $\angle H(e^{j\omega})$. Label all significant magnitude values, angles, and frequencies. √ Ⓢ

[c] Use a suitable computer tool to plot the frequency response magnitude and phase plots directly from the impulse response coefficients as a check on your work.

`http://youtu.be/Bj-Wf2Rpn2A` (13:23)

Frequency Response #3

★

An LTI system has impulse response $h[n] = 3\delta[n] + 3\delta[n-3]$.

[a] Determine the system's frequency response $H(e^{j\omega})$ written in the form $A(e^{j\omega})\phi(e^{j\omega})$, i.e., isolate the real-valued amplitude function $A(e^{j\omega})$ and the phase function $\phi(e^{j\omega})$. √ Ⓢ

[b] Plot the frequency response magnitude $|H(e^{j\omega})|$ and phase $\angle H(e^{j\omega})$. Label all significant magnitude values, angles, and frequencies. √ Ⓢ

[c] Use a suitable computer tool to plot the frequency response magnitude and phase plots directly from the impulse response coefficients as a check on your work.

`http://youtu.be/d7AQ5-lL6QU` (11:30)

Flow Graphs

Flow Graphs #1

★

Consider the system function

$$H(z) = \frac{8 + 10z^{-1} - 12z^{-2}}{2 - 4z^{-1} + 6z^{-2}}.$$

Implement the system function as a flow graph using the following forms:

[a] Direct Form I, √ Ⓢ

[b] Direct Form II, and

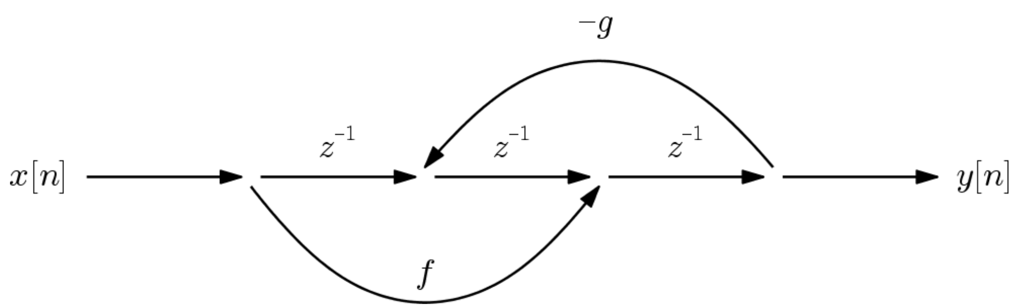
[c] Direct Form II transposed.

http://youtu.be/PpGeSGgQulg (3:58)

Flow Graphs #2

★

Given the flow graph below with $f = 3$ and $g = 1/2$:



[a] Find the system function $H(z)$. √ Ⓢ

[b] Write a single difference equation for the system by writing $y[n]$ as a function of $x[n]$. √ Ⓢ

[c] Draw the transpose of the given flow graph. √ Ⓢ

[d] Determine the system function $H_T(z)$ of the transposed flow graph. √ Ⓢ

[e] Is $H_T(z)$ equal to $H(z)$? Explain your results. √ Ⓢ

http://youtu.be/ovBQX2DPDVw (13:24)

Flow Graphs #3

★

Three system functions are defined as follows:

$$H_1(z) = \frac{2 + 4z^{-1}}{1 - z^{-1}}$$

$$H_2(z) = \frac{-3 + 5z^{-2}}{1 + 3z^{-1} + 4z^{-2}}$$

$$H_3(z) = \frac{3 + 7z^{-1} + 3z^{-2}}{1 + 2z^{-1} - 4z^{-2}}$$

The composite system function $H(z)$ is the cascade of the three defined system functions.

[a] Implement $H(z)$ as a cascade of three subsystems, each implemented as a Direct Form II flow graph. $\surd$ Ⓢ

[b] Write $H(z)$ as a ratio of two polynomials. $\surd$ Ⓢ

[c] Implement $H(z)$ as a single Direct Form II flow graph. $\surd$ Ⓢ

[d] Discuss the relative advantages and disadvantages of the two implementations. $\surd$ Ⓢ

`http://youtu.be/NdEL2-UPxj0` (11:46)

Filter Design

Filters #1

★

Design and evaluate a discrete-time Butterworth lowpass filter for an audio noise reduction application that meets the following specifications:

- 44.1 kHz system sampling frequency,
- 5.0 kHz passband edge with 3 dB maximum passband ripple, and
- 10.0 kHz stopband edge with a minimum 25 dB stopband loss.

Follow the design procedure based on converting a continuous-time filter with the bilinear transform.

[a] Determine the difference equation of the lowpass filter. √ Ⓢ

[b] Plot the frequency response magnitude in decibels of your design as a function of cyclic frequency f in Hz; mark up the plot to show that it meets specifications. √ Ⓢ

[c] Confirm the accuracy of your difference equation coefficient calculations by using a discrete-time filter design tool to create the filter coefficients directly from the design specifications. √ Ⓢ

[d] Plot the pole-zero diagram of the filter using a suitable computer tool. Discuss the relationship between the pole/zero positions and the frequency response magnitude. NOTE: A correct plot depends on high-precision coefficients; use the coefficients from the filter-design tool of the previous step. √ Ⓢ

`http://youtu.be/XlKxeRsSxPo` (16:54)