Aventuras no Project Euler:

Ou como eu ganhei uma tatuagem de graça

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31 de Maio de 2016

Quem Sou

- Cursando o 7º semestre(4-8 Fase) do BCC
- Speedcuber
- Origamista
- IC na área de Otimização Combinatorial

Project Euler

O que é?

- Começou com Colin Hughes (a.k.a euler) no `http://mathschallenge.net/` (RIP)
- Tornou-se um projeto própio em 2006.
- É uma série de problemas envolvendo matemática e computação.
- Todos os problemas podem ser resolvidos em menos de 1 minuto.
- 1 problema novo toda semana.

Project Euler

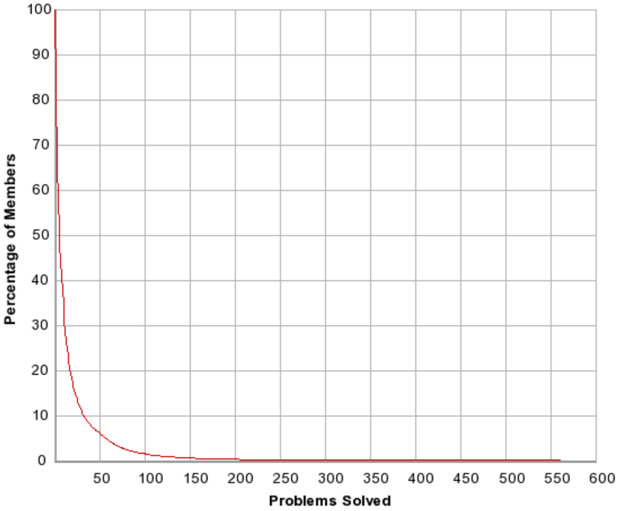
"Project Euler exists to encourage, challenge, and develop the skills and enjoyment of anyone with an interest in the fascinating world of mathematics."

Números

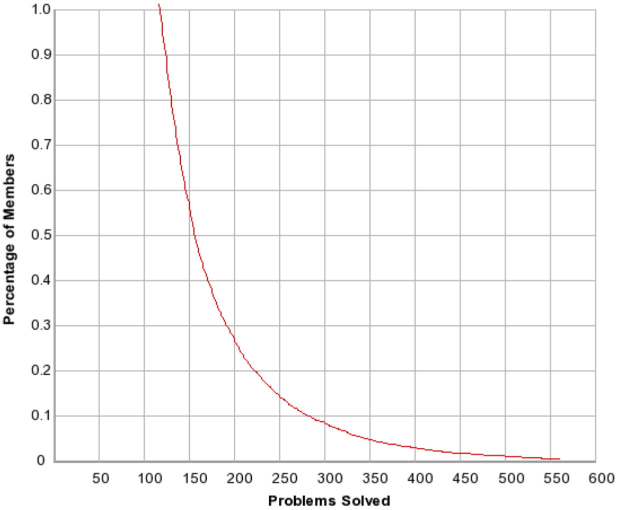
- 600000 membros inscritos
- 7950000 Problemas resolvidos
- Média de 13.3 por membro
- 82000 membros resolveram pelo menos 25 problemas (13.7%)
- 60 membros resolveram 500 problemas ou mais (561 problemas ao todo)

Linguagens mais usadas: Python, C/C++, Java, C#, Haskell

Mais números



Mais números



Top 100

Top 100 de alguns países:

País	Mínimo para o top 100	Membros ativos
USA	256	37527
Holanda	106	3508
França	148	4419
Russia	119	4419
Brazil	35	3110
República Tcheca	37	1087

Ranking Brasileiro

	Username	Country	Solved	Level	Language
1	gustavotcabral		317	12	Delphi
2	RicardoB		262	10	C#
3	Marcscol		221	8	C/C++
4	andregrw		217	8	Java
5	marcoskwkm		192	7	Python
6	bobby_eletrica		191	7	C/C++
7	starkad		180	7	Python
8	kunigami		168	6	Haskell
9	fidelis		157	6	Lua
10	*Davi M J Barbosa*		150	6	
11	Guircs		149	5	C/C++
12	daniel.assuncao		144	5	Python
13	bernardofpc		141	5	
14	vinilima		130	5	Java
15	firer		128	5	Python
16	ruilov		126	5	Java
17	tiago.melo84		125	5	Java
18	*Pedro Cayres*		117	4	Python
19	*RenanUnleashed*		110	4	Haskell
20	haphaeu		103	4	Python

Problemas

Fáceis	Difíceis
1	$75 \rightarrow 139$
5	108
$18 \rightarrow 67$	142
125	104

Problema 1

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Find the sum of all the multiples of 3 or 5 below 1000.

Soluçao:

```
sum [ x|x<-[2..999], x ‘mod‘ 3 == 0 || x ‘mod‘5 == 0 ]
```

Problema 5

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

Soluçao:

$$\prod_{i=1}^{20} i \tag{1}$$

Problema 5

```python
#!/usr/bin/python3
from timeit import timeit
from itertools import combinations

def magic():

    z = [ x for x in range(2,20)]

    for w in range(1,20):
        for i in combinations(z,w):
            n = 1
            for x in i:
                n *= x
            flag = True
            for i in range(2,20):
                if n % i != 0:
                    flag = False
                    break
            if flag:
                print(n)
                return

print(timeit(magic,number=1))
```

Figura: Solução vergonhosa

Problema 5

Solução de gente:

$$2^4 * 3^2 * 5 * 7 * 11 * 13 * 17 * 19$$

Problema 18

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3
7 4
2 **4** 6
8 5 **9** 3

That is, $3 + 7 + 4 + 9 = 23$. Find the maximum total from top to bottom of the triangle below: NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, Problem 67, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

Problema 18

```
                            75
                          95 64
                        17 47 82
                      18 35 87 10
                    20 04 82 47 65
                  19 01 23 75 03 34
                88 02 77 73 07 63 67
              99 65 04 28 06 16 70 92
            41 41 26 56 83 40 80 70 33
          41 48 72 33 47 32 37 16 94 29
        53 71 44 65 25 43 91 52 97 51 14
      70 11 33 28 77 73 17 78 39 68 17 57
    91 71 52 38 17 14 91 43 58 50 27 29 48
  63 66 04 68 89 53 67 30 73 16 69 87 40 31
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23
```

Problema 18

```python
def magic(i,j):

    if i==15:
        return 0;

    a=magic(i+1,j)
    b=magic(i+1,j+1)

    if a > b:
        return a + v[i][j]
    else:
        return b + v[i][j]

n = magic(0,0)

print(n)
```

Figura: Solução de força bruta

Problema 67

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3
7 4
2 **4** 6
8 5 **9** 3

That is, 3 + 7 + 4 + 9 = 23. Find the maximum total from top to bottom in triangle.txt (right click and 'Save Link/Target As...'), a 15K text file containing a triangle with one-hundred rows.

NOTE: This is a much more difficult version of Problem 18. It is not possible to try every route to solve this problem, as there are 299 altogether! If you could check one trillion (1012) routes every second it would take over twenty billion years to check them all. There is an efficient algorithm to solve it. ;o)

Problema 67

```python
def blackMotherFuckingMagic():

    for i in range(98, -1, -1):
        for j in range(0, i+1):
            v[i][j] += max(v[i+1][j+1], v[i+1][j])

    return v[0][0]

n = blackMotherFuckingMagic()

print(n)
```

Figura: Solução Uliziando PD

Problema 75

It turns out that 12 cm is the smallest length of wire that can be bent to form an integer sided right angle triangle in exactly one way, but there are many more examples.

12 cm: (3,4,5)	24 cm: (6,8,10)	30 cm: (5,12,13)
36 cm: (9,12,15)	40 cm: (8,15,17)	48 cm: (12,16,20)

In contrast, some lengths of wire, like 20 cm, cannot be bent to form an integer sided right angle triangle, and other lengths allow more than one solution to be found; for example, using 120 cm it is possible to form exactly three different integer sided right angle triangles.

120 cm: (30,40,50), (20,48,52), (24,45,51)

Given that L is the length of the wire, for how many values of $L \leq$ 1,500,000 can exactly one integer sided right angle triangle be formed?

Problema 75

Solução: Formula de Euclides (Elementos)

$$a = m^2 - n^2 \tag{2}$$
$$b = 2 * m * n \tag{3}$$
$$c = m^2 + n^2 \tag{4}$$

onde m e n são coprimos, $m + n$ é ímpar e $n < m$.
Para cara (m, n) obtem-se uma tripla pitagorica.

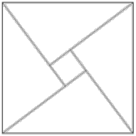
Problema 75

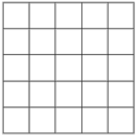
```python
#!/usr/bin/python3

from fractions import gcd
from math import *
import timeit

def magic():
    L = int(1.5 * 10**6)

    ls = [ 0 for x in range(1, L+2) ]

    for m in range(2, int(sqrt(L))):
        for n in range(1, m):
            if (n+m) % 2 == 1 and gcd(m, n) == 1:
                a = m**2 - n**2
                b = 2 * m * n
                c = m**2 + n**2
                p = a + b + c
                while p <= L:
                    ls[p] += 1
                    p += a + b + c

    c = 0
    for i in ls:
        if i == 1:
            c += 1

    print(c)

print(timeit.timeit(magic,number=1))
```

Figura: Solução do problema 75

Problema 139

Let (a, b, c) represent the three sides of a right angle triangle with integral length sides. It is possible to place four such triangles together to form a square with length c.
For example, (3, 4, 5) triangles can be placed together to form a 5 by 5 square with a 1 by 1 hole in the middle and it can be seen that the 5 by 5 square can be tiled with twenty-five 1 by 1 squares.

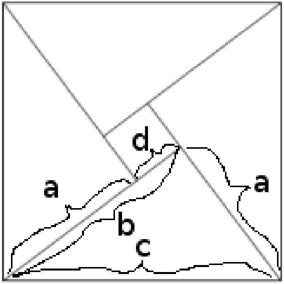




Problema 139

However, if (5, 12, 13) triangles were used then the hole would measure 7 by 7 and these could not be used to tile the 13 by 13 square.
Given that the perimeter of the right triangle is less than one-hundred million, how many Pythagorean triangles would allow such a tiling to take place?

Problema 139



$$d = b - a$$
$$c \equiv 0 \bmod d$$

Problema 139

3	4	5
21	20	29
119	120	169
697	696	985
4059	4060	5741
23661	23660	33461
137903	137904	195025
803761	803760	1136689
4684659	4684660	6625109
27304197	27304196	38613965

Problema 108

In the following equation x, y, and n are positive integers.

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{n} \tag{5}$$

For n = 4 there are exactly three distinct solutions:

$$\frac{1}{5} + \frac{1}{20} = \frac{1}{4} \tag{6}$$

$$\frac{1}{6} + \frac{1}{12} = \frac{1}{4} \tag{7}$$

$$\frac{1}{8} + \frac{1}{8} = \frac{1}{4} \tag{8}$$

What is the least value of n for which the number of distinct solutions exceeds one-thousand?

Problema 108

Soluçao:

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{n} \tag{9}$$

seja $x = n + a$ e $y = n + b$

$$\frac{1}{(n+a)} + \frac{1}{(n+b)} = \frac{1}{n} \tag{10}$$

$$\frac{n+a+n+b}{(n+a)(n+b)} = \frac{1}{n} \tag{11}$$

$$n * (n+a+n+b) = (n+a)(n+b) \tag{12}$$

$$n^2 + na + n^2 + nb = n^2 + nb + na + ab \tag{13}$$

$$n^2 = ab \tag{14}$$

Problema 142

Find the smallest $x + y + z$ *with integers* $x > y > z > 0$ *such that* $x + y$, $x - y$, $x + z$, $x - z$, $y + z$, $y - z$ *are all perfect squares.*

Problema 142

Soluçao:

$$a = x + y \quad (15)$$
$$b = x - y \quad (16)$$
$$c = x + z \quad (17)$$
$$d = x - z \quad (18)$$
$$e = y + z \quad (19)$$
$$f = y - z \quad (20)$$

$$a - c = f \quad (21)$$
$$a - d = e \quad (22)$$
$$a - e = d \quad (23)$$
$$b + f = d \quad (24)$$
$$c + f = a \quad (25)$$
$$c - e = b \quad (26)$$
$$d - b = f \quad (27)$$
$$d - f = c \quad (28)$$

Problema 142

$$a - c = f \tag{29}$$

$$a - e = d \tag{30}$$

$$(x + y) - (x + z) = f \tag{31}$$

$$(x + y) - (y + z) = d \tag{32}$$

$$2x + 2y - x - y - 2z = f + d \tag{33}$$

$$x + y - 2z = f + d \tag{34}$$

$$a - 2z = f + d \tag{35}$$

$$-2z = f + d - a \tag{36}$$

$$-2z = f - e \tag{37}$$

$$z = \frac{e - f}{2} \tag{38}$$

Problema 142

Soluçao:

- Calcula-se todos os quadrados perfeitos dentro de um intervalo $(x^2, x \in [1, 1000])$.
- Iterando a, c, d sobre os quadrados perfeitos obtem-se b, e, f utlizando as equações $f = a - c$, $e = a - d$ e $b = c - e$.
- Verifica-se se $a < c < d$ e se $b, e, f > 0$.
- Verifica-se se e, f possuem a mesma paridade.
- Verifica-se se b, e, f são quadrados perfeitos.
- Sabendo que $z = \frac{e-f}{2}$, $x = c - z$ e $y = a - x$, obtem-se x, y, z.

Problema 125

The palindromic number 595 is interesting because it can be written as the sum of consecutive squares:
$6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 + 12^2$.
There are exactly eleven palindromes below one-thousand that can be written as consecutive square sums, and the sum of these palindromes is 4164. Note that $1 = 0^2 + 1^2$ *has not been included as this problem is concerned with the squares of positive integers. Find the sum of all the numbers less than 108 that are both palindromic and can be written as the sum of consecutive squares.*

Problema 125

```python
#!/usr/bin/env python
# -*- coding: utf-8 -*-

top      = 10**8
limit    = 10**4
sol      = 0
repeated = set()

for i in range(1, limit+1):
    n = i**2
    for j in range(i+1, limit+1):
        n += j**2

        if n > top:
            break

        if (lambda x: x == x[::-1])(str(n)) and not n in repeated:
            repeated.add(n)
            sol += n

print(sol)
# WAT
```

Problema 104

The Fibonacci sequence is defined by the recurrence relation:
$F_n = F_n - 1 + F_n - 2$, *where* $F_1 = 1$ *and* $F_2 = 1$.
It turns out that F_{541}, *which contains 113 digits, is the first Fibonacci number for which the last nine digits are 1-9 pandigital (contain all the digits 1 to 9, but not necessarily in order). And* F_{2749}, *which contains 575 digits, is the first Fibonacci number for which the first nine digits are 1-9 pandigital.*
Given that F_k *is the first Fibonacci number for which the first nine digits AND the last nine digits are 1-9 pandigital, find k.*

Problema 104

Solução simples:

```python
#!/usr/bin/env python
# -*- coding: utf-8 -*-

def is_pandigital(nr, n=9):
    beg=set(nr[0:n])
    end=set(nr[-n:])
    return beg==end and beg==set(map(str, range(1, n + 1)))


def magic(n):
    return is_pandigital(str(n)[0:9]) and is_pandigital(str(n)[-9:])

i, a, b = 1, 1, 1
while True:
    i += 1
    a, b = b, a + b
    if i % 10**4 == 0:
        print(i, " ", len(str(b)))
    if magic(b):
        print(i+1)
        break
```

Problema 104

Solução decente:

$$F_n = \frac{\phi^n}{\sqrt{5}},\ \phi = \frac{1+\sqrt{5}}{2} \tag{39}$$

$$log_{10}(F_n) = log_{10}(\frac{\phi^n}{\sqrt{5}}) = \tag{40}$$

$$log_{10}(\phi^n) - log_{10}(\sqrt{5}) = \tag{41}$$

$$nlog_{10}(\phi) - log_{10}(\sqrt{5}) = t \tag{42}$$

$$10^{t-\lfloor r \rfloor+8} \tag{43}$$

$$\tag{44}$$

Duvidas?

Figura: Cubo supremo