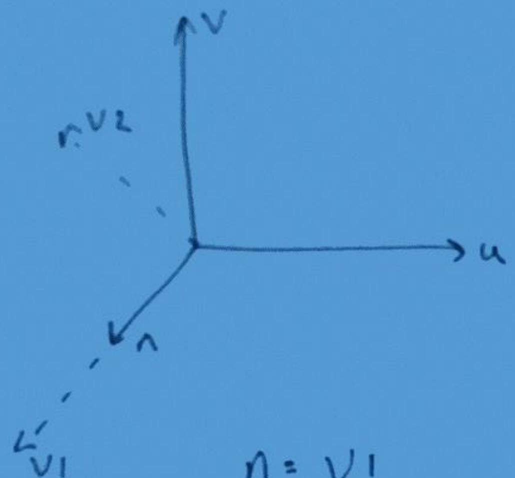
Trevor M. Tomesh - CS 805
Assignment 1



$$V1 = (V1_x, V1_y, V1_z)$$
$$V2 = (V2_x, V2_y, V2_z)$$
$$u = (u_x, u_y, u_z)$$
$$V = (V_x, V_y, V_z)$$
$$n = (n_x, n_y, n_z)$$

$$n = \frac{V1}{|V1|}$$

$$n_x = \frac{V1_x}{|V1|} = \frac{V1_x}{\sqrt{(V1_x^2 + V1_y^2 + V1_z^2)}}$$
$$n_y = \frac{V1_y}{|V1|} = \frac{V1_y}{\sqrt{(V1_x^2 + V1_y^2 + V1_z^2)}}$$
$$n_z = \frac{V1_z}{|V1|} = \frac{V1_z}{\sqrt{(V1_x^2 + V1_y^2 + V1_z^2)}}$$

$$u = \frac{V2 \times V1}{|V2 \times V1|}, \quad V2 \times V1 = (-V2_z \cdot V1_y + V2_y V1_z,\ V2_z \cdot V1_x - V2_x \cdot V1_z,\ -V2_y \cdot V1_x + V2_x \cdot V1_y)$$

$$u_x = \frac{-V2_z \cdot V1_y + V2_y \cdot V1_z}{|V2 \times V1|} \qquad u_z = \frac{-V2_y \cdot V1_x + V2_x \cdot V1_y}{|V2 \times V1|}$$

$$u_y = \frac{V2_z \cdot V1_x - V2_x \cdot V1_z}{|V2 \times V1|}$$

$$V = n \times u$$
$$= \frac{V1}{|V1|} \times \frac{V2 \times V1}{|V2 \times V1|}$$

... plug n and u in here! (It gets messy so i'll stick with this answer)

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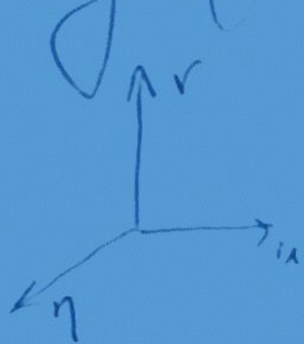
3. a)

$$T^{-1} \cdot T = \begin{bmatrix} 1 & 0 & 0 & -VRP_x \\ 0 & 1 & 0 & -VRP_y \\ 0 & 0 & 1 & -VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & VRP_x \\ 0 & 1 & 0 & VRP_y \\ 0 & 0 & 1 & VRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0+0 & 0+0+0+0 & 0+0+0+0 & VRP_x+0+0+-VRP_x \\ 0+0+0+0 & 0+1+0+0 & 0+0+0+0 & 0+VRP_y+0+-VRP_y \\ 0+0+0+0 & 0+0+0+0 & 0+0+1+0 & 0+0+VRP_z+-VRP_z \\ 0+0+0+0 & 0+0+0+0 & 0+0+0+0 & 0+0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

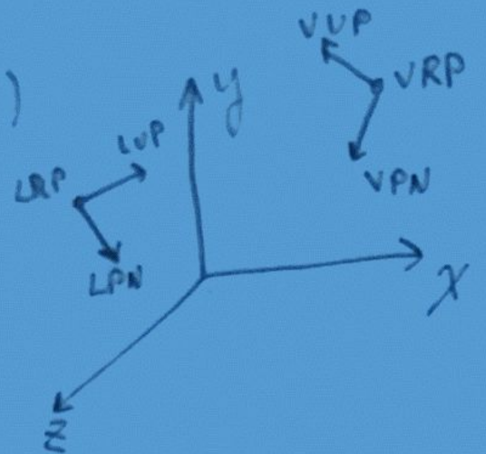
b) An orthogonal matrix is a square matrix with orthogonal vectors as rows & collumns



$$\begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_x & v_x & n_x & 0 \\ u_y & v_y & n_y & 0 \\ u_z & v_z & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} \overset{1}{\cancel{u_x \cdot u_x}}, & \overset{0}{\cancel{u_y \cdot v_x}}, & \overset{0}{\cancel{n_x u_z}}, & 0 \\ \overset{0}{\cancel{u_y \cdot v_x}}, & \overset{1}{\cancel{v_y v_y}}, & \overset{0}{\cancel{n_y v_z}}, & 0 \\ \overset{0}{\cancel{n_x u_z}}, & \overset{0}{\cancel{n_y v_z}}, & \overset{1}{\cancel{n_z n_z}}, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} = \begin{bmatrix} 1, & 0, & 0, & 0 \\ 0, & 1, & 0, & 0 \\ 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix} = I$$

C)



$$n = \frac{(VPN - VRP)}{|VPN - VRP|}$$

$$u = \frac{(VUP - VRP) \times (VPN - VRP)}{|(VUP - VRP) \times (VPN - VRP)|}$$

$$V = \frac{(VPN - VRP)}{|VPN - VRP|} \times \frac{(VUP - VRP) \times (VPN - VRP)}{|(VUP - VRP) \times (VPN - VRP)|}$$

$$M_{wc} = \overset{R}{\begin{bmatrix} u_x & u_y & u_z & \emptyset \\ V_x & V_y & V_z & \emptyset \\ n_x & n_y & n_z & \emptyset \\ \emptyset & \emptyset & \emptyset & 1 \end{bmatrix}} * \begin{bmatrix} 1 & \emptyset & \emptyset & -VRP_x \\ \emptyset & 1 & \emptyset & -VRP_y \\ \emptyset & \emptyset & 1 & -VRP_z \\ \emptyset & \emptyset & \emptyset & 1 \end{bmatrix}$$

$$= \begin{bmatrix} u_x & u_y & u_z & -(u_x \cdot VRP_x + u_y VRP_y + u_z VRP_z) \\ V_x & V_y & V_z & (V_x VRP_x + V_y VRP_y + V_z VRP_z) \\ n_x & n_y & n_z & (n_x VRP_x + n_y VRP_y + n_z \cdot VRP_z) \\ \emptyset & \emptyset & \emptyset & 1 \end{bmatrix}$$

$$M_{cw} = M_{wc}^{-1} = T^{-1} R^{-1}$$

$$= \begin{bmatrix} 1 & \emptyset & \emptyset & VRP_x \\ \emptyset & 1 & \emptyset & VRP_y \\ \emptyset & \emptyset & 1 & VRP_z \\ \emptyset & \emptyset & \emptyset & 1 \end{bmatrix} \cdot \begin{bmatrix} u_x & V_x & n_x & \emptyset \\ u_y & V_y & n_y & \emptyset \\ u_z & V_z & n_z & \emptyset \\ \emptyset & \emptyset & \emptyset & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} u_x & v_x & n_x & VRP_x \\ u_y & v_y & n_y & VRP_y \\ u_z & v_z & n_z & VRP_z \\ \phi & \phi & \phi & 1 \end{bmatrix}$$

d) ~~A=(LRP-LPN)~~

$$n = \frac{LPN - LRP}{|LPN - LRP|}$$

$$u = \frac{(LUP - LRP) \times (LPN - LRP)}{|(LUP - LRP) \times (LPN - LRP)|}$$

$$V = \frac{LPN - LRP}{|LPN - LRP|} \times \frac{(LUP - LRP) \times (LPN - LRP)}{|(LUP - LRP) \times (LPN - LRP)|}$$

$$M_{wl} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ V_x & V_y & V_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & -LRP_x \\ 0 & 1 & 0 & -LRP_y \\ 0 & 0 & 1 & -LRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} u_x & u_y & u_z & u_x LRP_x + u_y LRP_y + u_z LRP_z \\ V_x & V_y & V_z & V_x LRP_x + V_y LRP_y + V_z LRP_z \\ \eta_x & \eta_y & \eta_z & \eta_x LRP_x + \eta_y LRP_y + \eta_z LRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{lw} = M_{wl}^{-1} = \begin{bmatrix} u_x & V_x & \eta_x & LRP_x \\ u_y & V_y & n_y & LRP_y \\ u_z & V_z & n_z & LRP_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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e) $M_{cl} = M_{wl} \cdot M_{cw}$ 11

$$M_{lc} = M_{cl}^{-1}$$
$$= (M_{wl} \cdot M_{cw})^{-1}$$
$$= M_{cw}^{-1} \cdot M_{wl}^{-1}$$
$$= M_{wc} \cdot M_{lw}$$

$M_{cl} =$

$$\begin{bmatrix} u_{xc} \cdot u_{xl}, & u_{yl} \cdot v_{xc}, & n_{xc} \cdot u_{zl}, & VRP_x (LRP_x \cdot u_{xl} + LRP_y \cdot u_{yl} + LRP_z \cdot u_{zl}) \\ u_{yc} \cdot v_{xl}, & v_{yc} \cdot v_{yl}, & n_{yc} \cdot v_{zl}, & VRP_y (LRP_x \cdot v_{xl} + LRP_y \cdot v_{yl} + LRP_z \cdot v_{zl}) \\ n_{xl} \cdot u_{zc}, & n_{yl} \cdot v_{zc}, & n_{zc} \cdot n_{zl}, & VRP_z (LRP_x \cdot n_{xl} + LRP_y \cdot n_{yl} + LRP_z \cdot n_{zl}) \\ \phi, & \phi, & \phi, & 1 \end{bmatrix}$$

$M_{lc} =$

$$\begin{bmatrix} u_{xc} \cdot u_{xl}, & u_{yc} \cdot v_{xl}, & n_{xl} \cdot u_{zc}, & LRP_x (VRP_x \cdot u_{xc} + VRP_y \cdot u_{yc} + VRP_z \cdot u_{zc}) \\ u_{yl} \cdot v_{xc}, & v_{yc} \cdot v_{yl}, & n_{yl} \cdot v_{zc}, & LRP_y (VRP_x \cdot v_{xc} + VRP_y \cdot v_{yc} + VRP_z \cdot v_{zc}) \\ n_{xc} \cdot u_{zl}, & n_{yc} \cdot v_{zl}, & n_{zc} \cdot n_{zl}, & LRP_z (VRP_x \cdot n_{xc} + VRP_y \cdot n_{yc} + VRP_z \cdot n_{zc}) \\ \phi, & \phi, & \phi, & 1 \end{bmatrix}$$