

Puzzle#1 : Binary Digits

Insert 5 commas between the digits of 21218453415461109221 and obtain 6 numbers, so that no two consecutive digits will be the same in the binary representation of these numbers. What is the sum of these 6 numbers?

If the problem was asked for 542 with one comma, then the answer would be 47 ($5+42=47$). The

binary representation of 5 is 101, the binary representation of 42 is 101010, and no two consecutive digits are the same in these numbers.

Reference : Puzzle Up Puzzle #10 Year 2014

Solution : 38591 ($21+21845 + 341 + 5461 + 10922 + 1$)

Approach :

Numbers in binary representation having alternative 1 and 0 are :
1, 10, 101, 1010, 10101, 101010, 1010101,... etc

Decimal conversion of above binary numbers are
1,2,5,10,21,42,85...etc

If we see the numbers there is pattern in number series

Number on odd position is equals to one more than 2 times of previous number.

Number on even position is equal to 2 times of previous number.

Number on first place : 1

Number on second place : $2*1 = 2$ ($2x$)

Number on third place : $2*2+1 = 5$ ($2x+1$)

Number on fourth place : $2*5 = 10$ ($2x$) etc...

Now if we extend the logic we get the following numbers in series.

1

2

5

10

21

42

85

170

341
682
1365
2730
5461
10922
21845
43690

The number 21218453415461109221 is combination of numbers in above series. Lets try to put 5 commas in this number to get 6 numbers in above series.

21, 21845, 341, 5461, 10922, 1

Sum of these six numbers : $21 + 21845 + 341 + 5461 + 10922 + 1 = 38591$