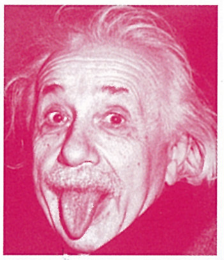
Algebra is the language through which we describe patterns.

A 40 × 40 white square is divided into 1 × 1 squares by lines parallel to its sides. Some of these 1 × 1 squares are coloured red so that each of the 1 × 1 squares, regardless of whether it is coloured red or not, shares a side with at most one red square (not counting itself). What is the largest possible number of red squares?

I have $10 in 10-cent coins, $10 in 20-cent coins and $10 in 50-cent coins. How many coins do I have?

It's not that I'm so smart, it's just that I stay with problems longer. Albert Einstein

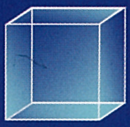


MONOMIAL BINOMIAL TRINOMIAL

Australian Mathematics Competition

2016 Solutions

problem solving



TEST YOURSELF

I have a $10 note and an ice-cream costs $2.20. What is the greatest number of ice-creams I can buy?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

MATHS CAN TAKE YOU ANY WHERE

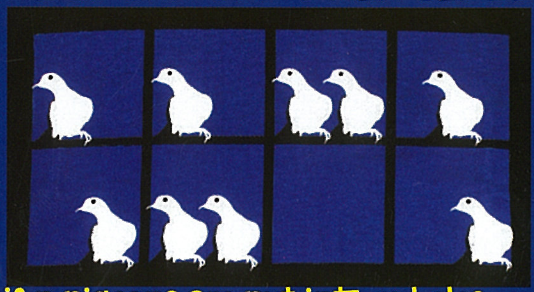
π 3.14159265358979323846264338327 9

How many integers in the set 100, 101, 102,…,999 do not contain the digits 1 or 2 or 3 or 4?

If $p = 11$ and $q = -4$, then $p^2 - q^2$ equals …

TIME DISTANCE SPEED

PIGEONHOLE PRINCIPLE



if n pigeons are put into m holes, with n > m, then at least one hole must contain more than one pigeon.

$$a^2 + b^2 = c^2$$

Pythagoras' theorem

parabola

Australian Mathematics Trust
www.amt.edu.au

Australian Mathematics Competition

2016 Solutions

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Australian Mathematics Trust

About the Australian Mathematics Competition

The Australian Mathematics Competition (AMC) was introduced in Australia in 1978 as the first Australia-wide mathematics competition for students. Since then it has served almost all Australian secondary schools and many primary schools, providing feedback and enrichment to schools and students. A truly international event, there are entries from more than 30 countries across South-East Asia, the Pacific, Europe, Africa and the Middle East. As of 2016, the AMC has attracted more than 14.75 million entries.

The AMC is for students of all standards. Students are asked to solve 30 problems in 60 minutes (Years 3–6) or 75 minutes (Years 7–12). The earliest problems are very easy. All students should be able to attempt them. The problems get progressively more difficult until the end, when they are challenging to the most gifted student. Students of all standards will make progress and find a point of challenge.

The AMC is a fun competition with many of the problems set in situations familiar to students and showing the relevance of mathematics in their everyday lives. The problems are also designed to stimulate discussion and can be used by teachers and students as springboards for investigation.

There are five papers: Middle Primary (Years 3–4), Upper Primary (Years 5–6), Junior (Years 7–8), Intermediate (Years 9–10) and Senior (Years 11–12). Questions 1–10 are worth 3 marks each, questions 11–20 are worth 4 marks, questions 21–25 are worth 5 marks, while questions 26–30 are valued at 6–10 marks, for a total of 135 marks.

Questions – Middle Primary Division

1. What is the value of $20 + 16$?

(A) 24 (B) 26 (C) 36 (D) 9 (E) 216

2. Which of these numbers is the smallest?

(A) 655 (B) 566 (C) 565 (D) 555 (E) 556

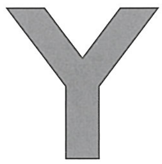
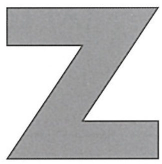
3. In the number 83 014, the digit 3 represents

(A) three (B) thirty (C) three hundred (D) three thousand (E) thirty thousand

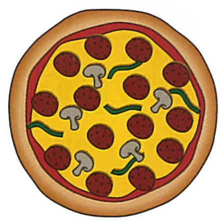
4. My sister is 6 years old and I am twice her age. Adding our ages gives

(A) 14 (B) 15 (C) 18 (D) 20 (E) 21

5. Four of these shapes have one or more lines of symmetry. Which one does not?

(A) 
(B) 
(C) 
(D) 
(E) 

6. Two pizzas are sliced into quarters. How many slices will there be?



(A) 2 (B) 10 (C) 6 (D) 8 (E) 16

7. Will has a 45-minute music lesson every Tuesday afternoon after school. If it begins at 4:30 pm, at what time does it finish?

(A) 4:45 pm (B) 4:55 pm (C) 4:75 pm (D) 5:00 pm (E) 5:15 pm

8. In our garage there are 4 bicycles, 2 tricycles and one quad bike. How many wheels are there altogether?

(A) 3 (B) 6 (C) 7 (D) 14 (E) 18

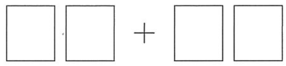
9. Ten chairs are equally spaced around a round table. They are numbered 1 to 10 in order. Which chair is opposite chair 9?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

10. Lee's favourite chocolates are 80c each. He has five dollars to spend. How many of these chocolates can he buy?

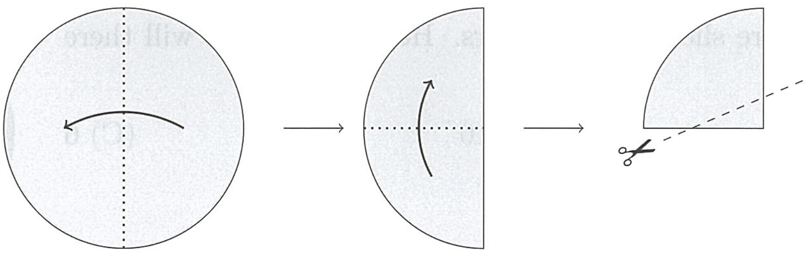
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

11. The four digits 2, 3, 8 and 9 are placed in the boxes so that when both two-digit numbers are added, the sum is as large as possible. What is this sum?

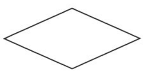
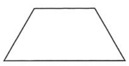
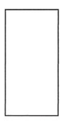
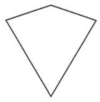
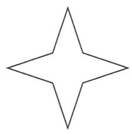


(A) 175 (B) 67 (C) 156 (D) 179 (E) 121

12. A circular piece of paper is folded in half twice and then a cut is made as shown.



When the piece of paper is unfolded, what shape is the hole in the centre?

(A)  (B)  (C)  (D)  (E) 

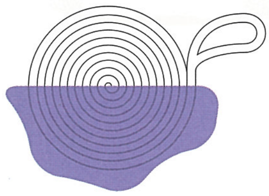
13. Phoebe put her hand in her pocket and pulled out 60 cents. How many different ways could this amount be made using 10c, 20c and 50c coins?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

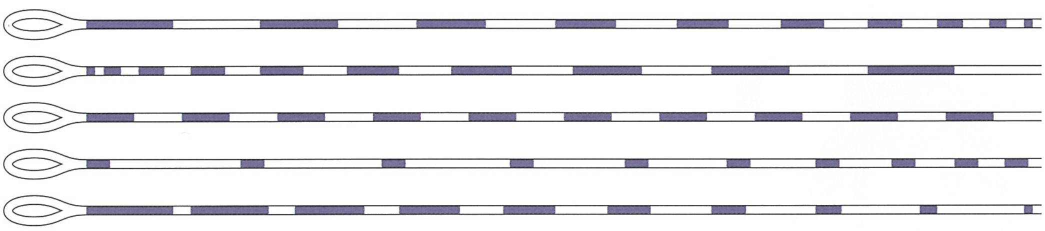
14. There are 5 red, 5 green and 5 yellow jelly beans in a jar. How many would you need to take out of the jar without looking to make sure that you have removed at least two of the same colour?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

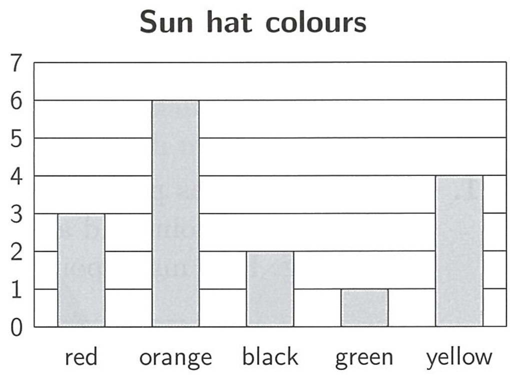
15. A sailor coiled a rope on his ship's deck, and some paint was spilled across half of it. What did the rope look like when it was uncoiled?



(A)
(B)
(C)
(D)
(E)



16. The students in Mr Day's class were asked the colour of their sun hat. The results are shown in the graph.
Mr Day chooses two colours which include the hat colours of exactly half of the class. Which two colours does he choose?

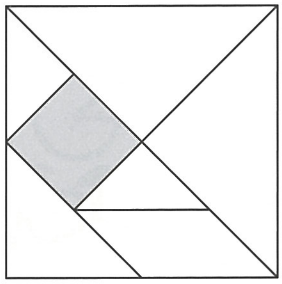


(A) orange and black
(B) green and yellow
(C) black and yellow
(D) red and orange
(E) red and yellow

17. The sum of the seven digits in Mario's telephone number is 34. The first five digits are 73903. How many possibilities are there for the last two digits?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

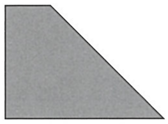
18. If the area of the tangram shown is 64 square centimetres, what is the area in square centimetres of the small square?

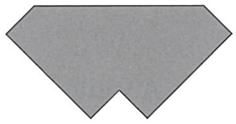


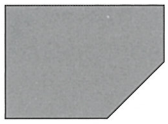
(A) 32 (B) 24 (C) 16 (D) 8 (E) 4

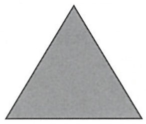
19. By making just one fold on a rectangular piece of paper, which of the following shapes is NOT possible?

(A) 

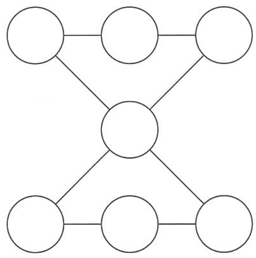
(B) 

(C) 

(D) 

(E) 

20. In this diagram there are four lines with three circles each. Place the numbers from 1 to 7 into the circles, so that each line adds up to 12. Which number must go into the circle at the centre of the diagram?



(A) 7 (B) 6 (C) 5 (D) 4 (E) 2

21. Four hockey teams play each of the other three teams once. A win scores 3 points, a draw scores 1 point and a loss scores 0 points. Some figures in the following table are missing. How many points did the Hawks get?

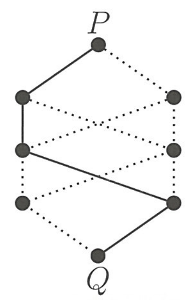
	Played	Win	Draw	Loss	Points
Eagles	3	3			9
Hawks	3				
Falcons	3	0	1		
Condors	3	0		2	1

(A) 1 (B) 4 (C) 6 (D) 7 (E) 10

22. In this grid you can only move downward, going from point to point along the lines shown.

One route from P to Q is drawn in.

How many different routes are there from P to Q?



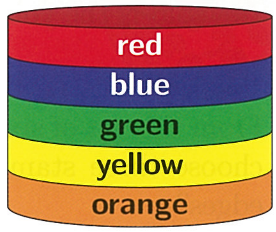
(A) 2 (B) 4 (C) 6 (D) 8 (E) 12

23. I have five coloured discs in a pile as shown.

I take the top two discs and put them on the bottom (with the red disc still on top of the blue disc).

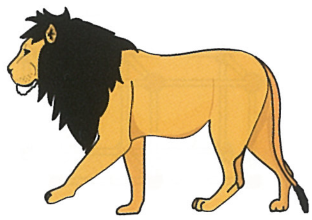
Then I again take the top two discs and put them on the bottom.

If I do this until I have made a total of 21 moves, which disc will be on the bottom?



(A) red (B) blue (C) green (D) yellow (E) orange

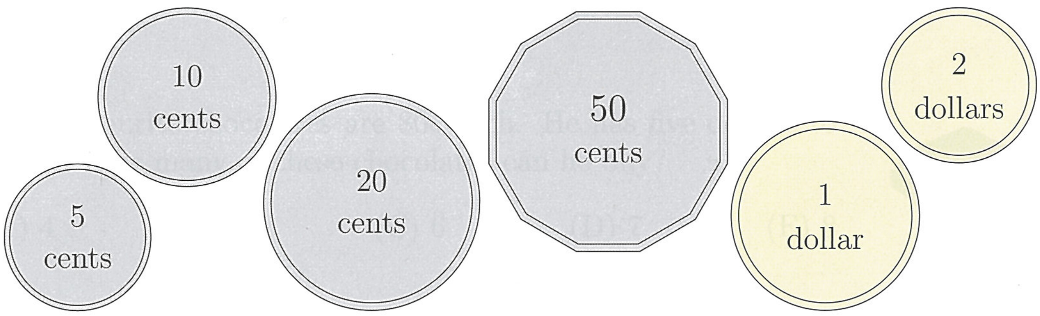
24. A zoo keeper weighed some of the animals at Melbourne Zoo. He found that the lion weighs 90 kg more than the leopard, and the tiger weighs 50 kg less than the lion. Altogether the three animals weigh 310 kg. How much does the lion weigh?



(A) 180 kg (B) 150 kg (C) 140 kg (D) 130 kg (E) 100 kg

25. Jane and Tom each have \$3.85 in coins, one of each Australian coin. They each give some coins to Angus so that Tom has exactly twice as much money as Jane.

What is the smallest number of coins given to Angus?



(A) 2 (B) 3 (C) 4 (D) 6 (E) 8

26. With some 3-digit numbers, the third digit is the sum of the first two digits. For example, with the number 213 we can add 1 and 2 to get 3, so the third digit is the sum of the first two digits.

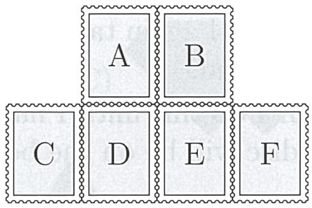
How many 3-digit numbers are there where the third digit is the sum of the first two digits?

27. In a family with two sons and two daughters, the sum of the children's ages is 55.

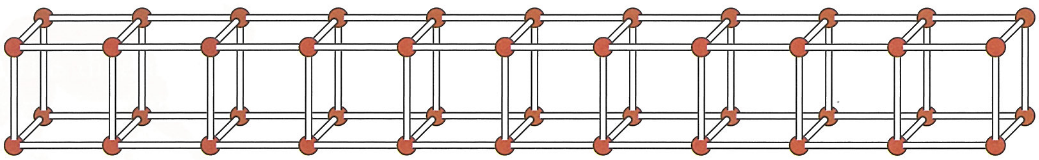
The two sons were born three years apart, and the two daughters were born two years apart. The younger son is twice the age of the older daughter.

How old is the youngest child?

28. From this set of six stamps, how many ways could you choose three stamps that are connected along their edges?



29. A class has 2016 matchsticks. Using blobs of modelling clay to join the matches together, they make a long row of cubes. This is how their row starts.



They keep adding cubes to the end of the row until they don't have enough matches left for another cube. How many cubes will they make?

30. Mary has four children of different ages, all under 10, and the product of their ages is 2016. What is the sum of their ages?

Questions – Upper Primary Division

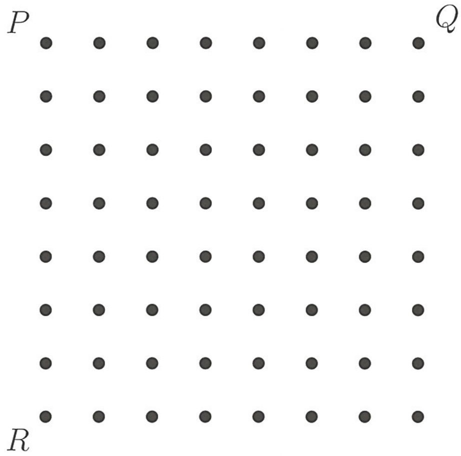
1. Which of these numbers is the smallest?

(A) 655 (B) 566 (C) 565 (D) 555 (E) 556

2. Two pizzas are sliced into quarters. How many slices will there be?

(A) 2 (B) 10 (C) 6 (D) 8 (E) 16

3. Join the dots P, Q, R to form the triangle PQR.



How many dots lie *inside* the triangle PQR?

(A) 13 (B) 14 (C) 15 (D) 17 (E) 18

4. $0.3 + 0.4$ is

(A) 0.07 (B) 0.7 (C) 0.12 (D) 0.1 (E) 7

5. Lee's favourite chocolates are 80c each. He has five dollars to spend. How many of these chocolates can he buy?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

6. Ten chairs are equally spaced around a round table. They are numbered 1 to 10 in order. Which chair is opposite chair 9?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

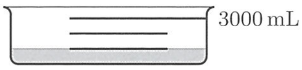
7. In a piece of music, a note like ♩ is worth one beat, ♪ is worth half a beat, 𝅗𝅥 is worth 2 beats and 𝅝 is worth 4 beats. How many beats are in the following piece of music?

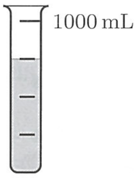
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

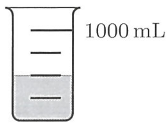
8. Phoebe put her hand in her pocket and pulled out 60 cents. How many different ways could this amount be made using 10c, 20c and 50c coins?

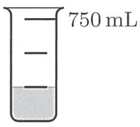
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

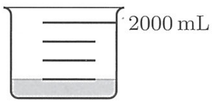
9. Which of these containers is currently holding the most water?

(A) 

(B) 

(C) 

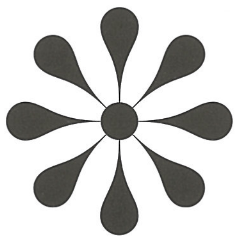
(D) 

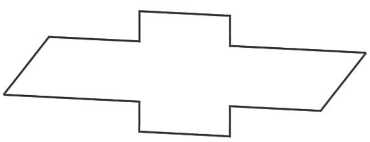
(E) 

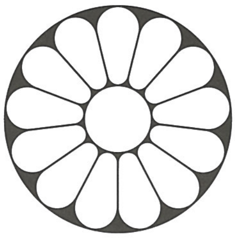
10. Which of these shapes has the most axes of symmetry (mirror lines)?

(A) 

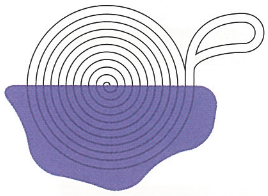
(B) 

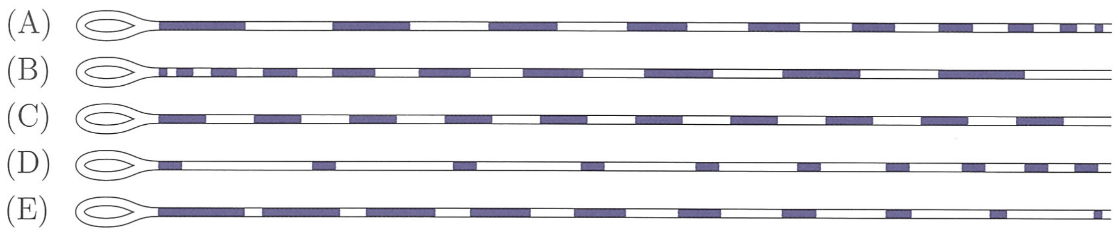
(C) 

(D) 

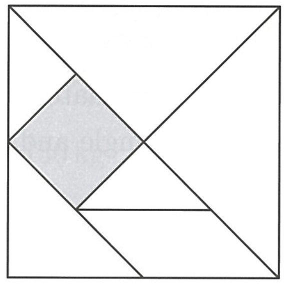
(E) 

11. A sailor coiled a rope on his ship's deck, and some paint was spilled across half of it. What did the rope look like when it was uncoiled?





12. If the area of the tangram shown is 64 square centimetres, what is the area in square centimetres of the small square?



(A) 32 (B) 24 (C) 16 (D) 8 (E) 4

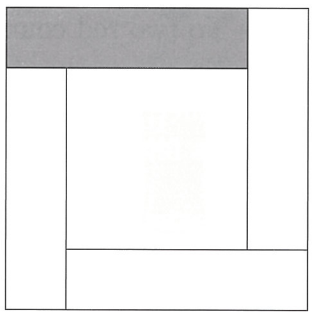
13. For each batch of 25 biscuits, Jack uses $2\frac{1}{2}$ packets of chocolate chips. How many packets does he need if he wants to bake 200 biscuits?

(A) 20 (B) 8 (C) 80 (D) 10 (E) 50

14. Which one of the following is correct?

(A) Two even numbers add to an odd number.
(B) An odd number minus an odd number is always odd.
(C) Adding 2 odd numbers and an even number is always odd.
(D) Adding 3 odd numbers is always odd.
(E) An odd number multiplied by an odd number always equals an even number.

15. The perimeter of the outer square is 36 cm, and the perimeter of the inner square is 20 cm.
If the four rectangles are all identical, what is the perimeter of the shaded rectangle in centimetres?

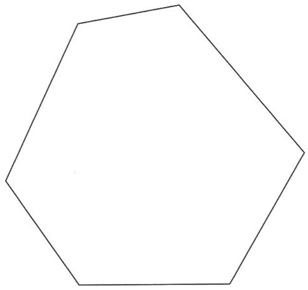


(A) 12 (B) 14 (C) 24 (D) 20 (E) 18

16. George has a new lock that opens if the four numbers 1, 2, 3 and 4 are pressed once each in the correct order.
If the first number must be larger than the second number, how many combinations are possible?

(A) 10 (B) 12 (C) 15 (D) 18 (E) 20

17. A straight cut is made through the hexagon shown to create two new shapes. Which of the following could **not** be made?



(A) one triangle and one hexagon
(B) two pentagons
(C) two quadrilaterals
(D) one quadrilateral and one pentagon
(E) one triangle and one quadrilateral

18. The numbers 3, 9, 15, 18, 24 and 29 are divided into two groups of 3 numbers and each group is added. The difference between the two sums (totals) of 3 numbers is as small as possible. What is the smallest difference?

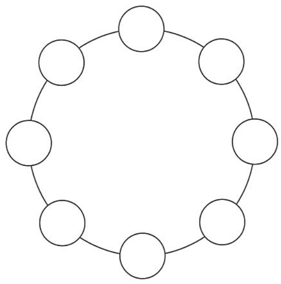
(A) 0 (B) 1 (C) 2 (D) 5 (E) 8

19. Benny built a magic square using the numbers from 1 to 16, where the numbers in each row, each column and each diagonal add up to the same total.
What number does he place at the X?

X			13
5		10	
	7		12
4			1

(A) 16 (B) 15 (C) 17 (D) 11 (E) 14

20. Andy has a number of red, green and blue counters.
He places eight counters equally spaced around a circle according to the following rules:



- No two red counters will be next to each other.
- No two green counters will be diagonally opposite each other.
- As few blue counters as possible will be used.

How many blue counters will Andy need to use?

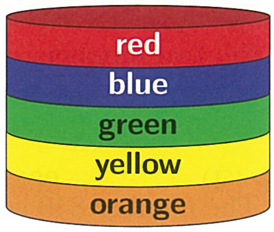
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

21. I have five coloured discs in a pile as shown.

I take the top two discs and put them on the bottom (with the red disc still on top of the blue disc).

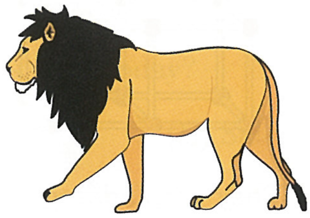
Then I again take the top two discs and put them on the bottom.

If I do this until I have made a total of 21 moves, which disc will be on the bottom?



(A) red (B) blue (C) green (D) yellow (E) orange

22. A zoo keeper weighed some of the animals at Melbourne Zoo. He found that the lion weighs 90 kg more than the leopard, and the tiger weighs 50 kg less than the lion. Altogether the three animals weigh 310 kg. How much does the lion weigh?



(A) 180 kg (B) 150 kg (C) 140 kg (D) 130 kg (E) 100 kg

23. Adrienne, Betty and Cathy were the only three competitors participating in a series of athletic events. In each event, the winner gets 3 points, second gets 2 points and third gets 1 point. After the events, Adrienne has 8 points, Betty has 11 points and Cathy has 5 points. In how many events did Adrienne come second?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

24. Jane and Tom are comparing their pocket money. Jane has as many 5c coins as Tom has 10c coins and as many 10c coins as Tom has 20c coins. However, Jane has as many 50c coins as Tom has 5c coins.

They have no other coins and they find that they each have the same amount of money.

What is the smallest number of coins they each can have?

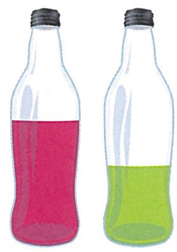
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

25. A tuckshop has two jars of cordial mixture.

Jar A is 30% cordial, while Jar B is 60% cordial.

Some of Jar A is mixed with some of Jar B to make 18 litres of 50% cordial.

How many litres from Jar A are used?

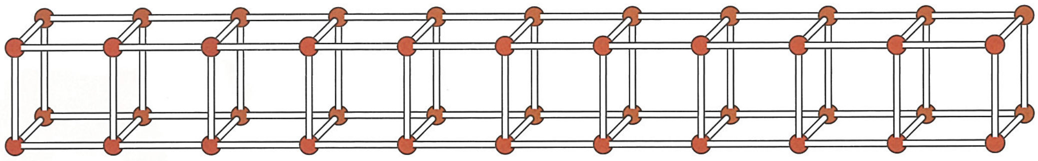


(A) 9 (B) 12 (C) 4 (D) 3 (E) 6

26. Qiang, Rory and Sophia are each wearing a hat with a number on it. Each adds the two numbers on the other two hats, giving totals of 11, 17 and 22. What is the largest number on a hat?

27. The number 840 is the 3-digit number with the most factors. How many factors does it have?

28. A class has 2016 matchsticks. Using blobs of modelling clay to join the matches together, they make a long row of cubes. This is how their row starts.



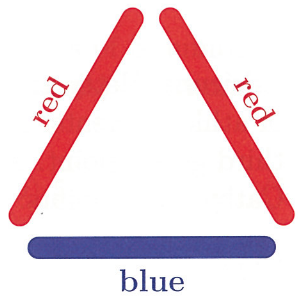
They keep adding cubes to the end of the row until they don't have enough matches left for another cube. How many cubes will they make?

29. You have an unlimited supply of five different coloured pop-sticks, and want to make as many different coloured equilateral triangles as possible, using three sticks.

One example is shown here.

Two triangles are not considered different if they are rotations or reflections of each other.

How many different triangles are possible?



30. Today my three cousins multiplied their ages together and it came to 2016. This day last year their ages multiplied to 1377.

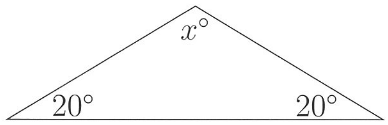
When they multiplied their ages together 2 years ago today, what was their answer?

Questions – Junior Division

1. The value of 2016×2 is

(A) 4026 (B) 4212 (C) 4022 (D) 432 (E) 4032

2. In the diagram, the value of x is



(A) 30 (B) 20 (C) 90 (D) 140 (E) 100

3. Today is Thursday. What day of the week will it be 30 days from today?

(A) Sunday (B) Monday (C) Tuesday (D) Friday (E) Saturday

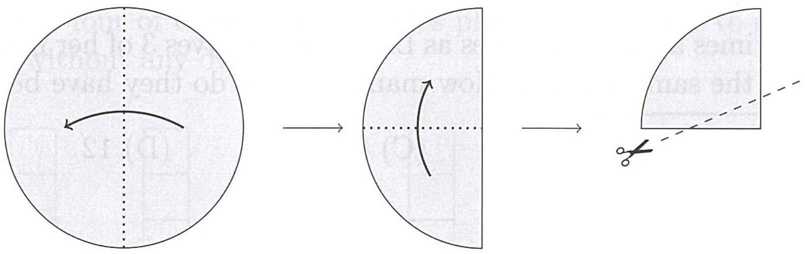
4. Today in Berracan, the minimum temperature was -5°C and the maximum was 8°C warmer than this. What was the maximum temperature?

(A) -3°C (B) 8°C (C) -13°C (D) 13°C (E) 3°C

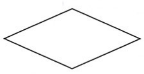
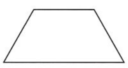
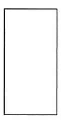
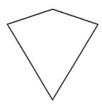
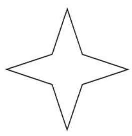
5. What is 25% of $\frac{1}{2}$?

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 2 (E) 1

6. A circular piece of paper is folded in half twice and then a cut is made as shown.



When the piece of paper is unfolded, what shape is the hole in the centre?

(A)  (B)  (C)  (D)  (E) 

7. I used a \$100 note to pay for a \$29 book, a \$16 calculator and a packet of pens for \$8.95. What change did I get?

(A) \$56.05 (B) \$45.05 (C) \$46.05 (D) \$37.05 (E) \$57.05

8. Which of the following numbers is between 0.08 and 0.4?

(A) 0.019 (B) 0.009 (C) 0.109 (D) 0.91 (E) 0.409

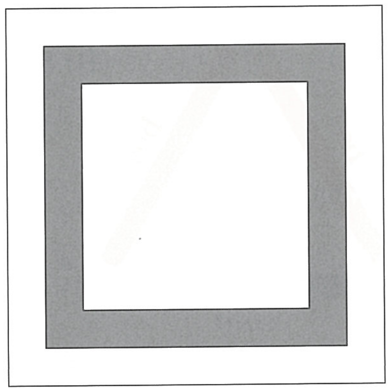
9. The cycling road race through the Adelaide Hills started at 11:50 am and the winner took 74 minutes. The winner crossed the finishing line at

(A) 1:24 pm (B) 12:54 pm (C) 12:04 pm (D) 1:04 pm (E) 12:24 pm

10. The fraction $\frac{720163}{2016}$ is

(A) between 0 and 1 (B) between 1 and 10 (C) between 10 and 100 (D) between 100 and 1000 (E) greater than 1000

11. The three squares shown have side lengths 3, 4 and 5. What percentage of the area of the largest square is shaded?

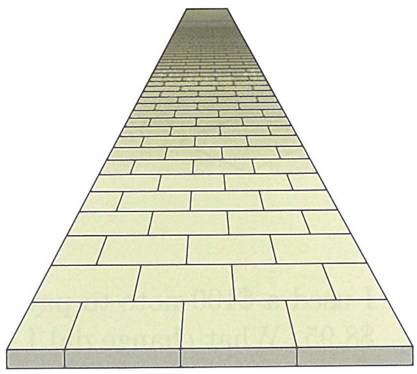


(A) 27% (B) 28% (C) 25% (D) 24% (E) 20%

12. Jan has three times as many marbles as Liana. If Jan gives 3 of her marbles to Liana, they will have the same number. How many marbles do they have between them?

(A) 18 (B) 6 (C) 8 (D) 12 (E) 16

13. One of the pedestrian walkways in Hyde Park is exactly $3\frac{1}{2}$ sandstone pavers wide. The pavers are arranged as shown.
The information sign says that 1750 pavers were used to make the walkway. How many pavers were cut in half in the construction of this walkway?



(A) 250 (B) 350 (C) 175 (D) 125 (E) 500

14. On Monday, I planted 10 apple trees in a row. On Tuesday, I planted orange trees along the same row and noticed at the end of the day that no apple tree was next to an apple tree. On Wednesday, I planted peach trees along the same row and noticed at the end of the day that no apple tree was next to an orange tree. What is the smallest number of trees that I could have planted?

(A) 28 (B) 43 (C) 37 (D) 40 (E) 36

15. Adrienne, Betty and Cathy were the only three competitors participating in a series of athletic events. In each event, the winner gets 3 points, second gets 2 points and third gets 1 point. After the events, Adrienne has 8 points, Betty has 11 points and Cathy has 5 points. In how many events did Adrienne come second?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

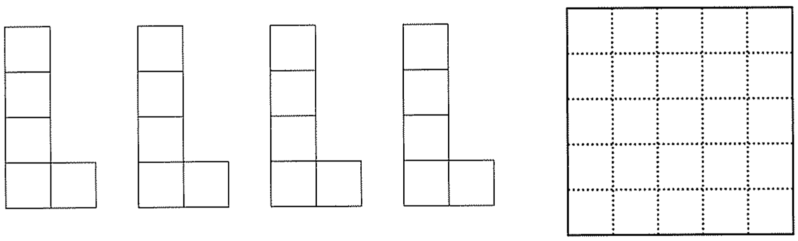
16. In the expression below, the letters A, B, C, D and E represent the numbers $1, 2, 3, 4$ and 5 in some order.

$$A \times B + C \times D + E$$

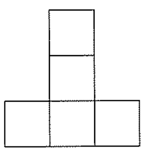
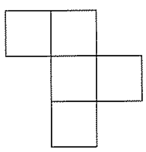
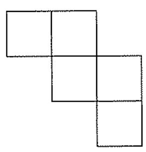
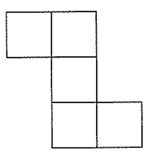
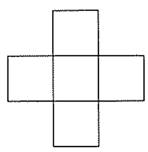
What is the largest possible value of the expression?

(A) 24 (B) 27 (C) 26 (D) 51 (E) 25

17. Llewellyn uses four of these L-shaped tiles plus one other tile to completely cover a 5 by 5 grid without any overlaps.

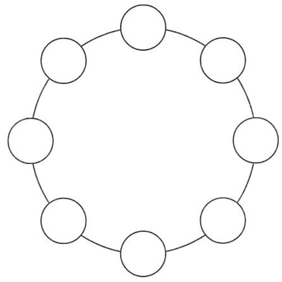


Which one of the following could be the other tile?

(A)  (B)  (C)  (D)  (E) 

18. Andy has a number of red, green and blue counters.

He places eight counters equally spaced around a circle according to the following rules:



- No two red counters will be next to each other.
- No two green counters will be diagonally opposite each other.
- As few blue counters as possible will be used.

How many blue counters will Andy need to use?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

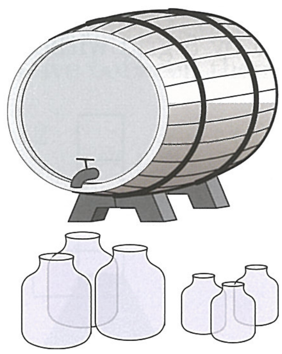
19. In a packet of spaghetti, one-third of the strands of spaghetti are intact, but the rest have each been snapped into two pieces. Of all the pieces of spaghetti from the packet (broken and whole), what is the largest fraction guaranteed to be at least as long as half an unbroken strand?

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

20. Mary has four children of different ages, all under 10, and the product of their ages is 2016. What is the sum of their ages?

(A) 30 (B) 34 (C) 28 (D) 29 (E) 32

21. Angelo has a 50 L barrel of water and two sizes of jug to fill, large and small. Each jug, when full, holds a whole number of litres.



He fills three large jugs, but does not have enough to fill a fourth. With the water remaining he then fills three small jugs, but does not have enough to fill a fourth.

In litres, what is the capacity of the small jug?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

22. How many 5-digit numbers contain all the digits 1, 2, 3, 4 and 5 and have the property that the difference between each pair of adjacent digits is at least 2?

(A) 24 (B) 14 (C) 18 (D) 20 (E) 10

23. A number of people are standing in a line in such a way that each person is standing next to exactly one person who is wearing a hat. Which of the following could *not* be the number of people standing in the line?

(A) 98 (B) 99 (C) 100 (D) 101 (E) 102

24. Josh, Ruth and Sam each begin with a pile of lollies. From his pile Josh gives Ruth and Sam as many as each began with. From her new pile, Ruth gives Josh and Sam as many lollies as each of them then has. Finally, Sam gives Josh and Ruth as many lollies as each then has.

If in the end each has 32 lollies, how many did Josh have at the beginning?

(A) 64 (B) 96 (C) 28 (D) 16 (E) 52

25. A poem can have any number of lines and each line may rhyme with any of the other lines.

For poems with only two lines, there are two different rhyming structures: either the lines rhyme or they do not.

For poems with three lines, there are five different rhyming structures: either all three lines rhyme, exactly one pair of lines rhyme (occurring in three ways), or none of the lines rhyme.

How many different rhyming structures are there for poems with four lines?

(A) 18 (B) 15 (C) 12 (D) 20 (E) 26

26. Digits a, b and c can be chosen to make the following multiplication work. What is the 3-digit number abc?

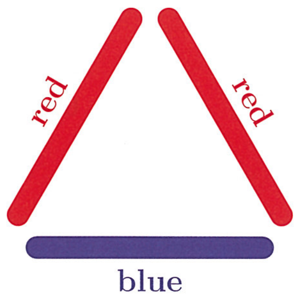
$$\begin{array}{rrrrr} & & a & b & c \\ \times & & & 2 & 4 \\ \hline 1 & c & b & a & 2 \\ \hline \end{array}$$

27. You have an unlimited supply of five different coloured pop-sticks, and want to make as many different coloured equilateral triangles as possible, using three sticks.

One example is shown here.

Two triangles are not considered different if they are rotations or reflections of each other.

How many different triangles are possible?

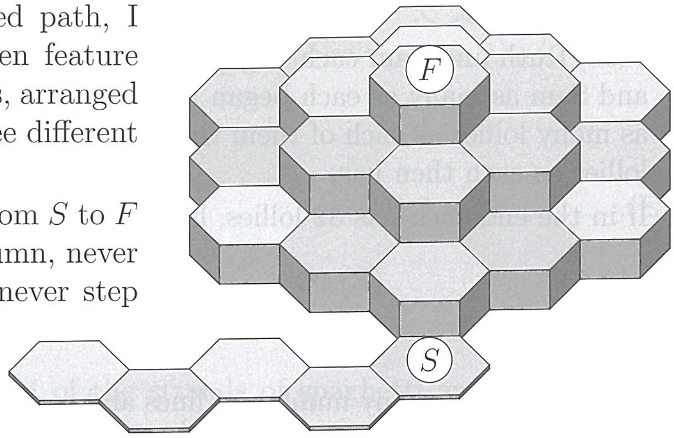


28. What is the largest 3-digit number that has all of its digits different and is equal to 37 times the sum of its digits?

29. Lucas invented the list of numbers 2, 1, 3, 4, 7, ... where each number after the first two is the sum of the previous two. He worked out the first 100 numbers by hand, but unfortunately he made one mistake in the 90th number, which was out by 1. How far out was the 100th number?

30. To match my hexagonally paved path, I built a *Giant's Causeway* garden feature from 19 hexagonal stone columns, arranged in a hexagonal pattern with three different levels, as shown.

In how many ways can I climb from S to F if I only step to an adjacent column, never step on any column twice and never step down a level?

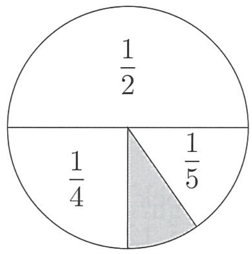


Questions – Intermediate Division

1. What is the value of 20×16?

(A) 320 (B) 140 (C) 2016 (D) 32 (E) 800

2. In the figure, the shaded region is what fraction of the circle?



(A) $\frac{1}{20}$ (B) $\frac{1}{10}$ (C) $\frac{1}{2}$ (D) $\frac{1}{60}$ (E) $\frac{1}{40}$

3. The cycling road race through the Adelaide Hills started at 11:15 am and the winner finished at 2:09 pm the same day. The winner's time in minutes was

(A) 135 (B) 174 (C) 164 (D) 294 (E) 186

4. The fraction $\dfrac{720163}{2016}$ is

(A) between 0 and 1 (B) between 1 and 10 (C) between 10 and 100 (D) between 100 and 1000 (E) greater than 1000

5. What is the value of $(1 \div 2) \div (3 \div 4)$?

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{3}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{24}$

6. 0.75% of a number is 6. The number is

(A) 800 (B) 300 (C) 1200 (D) 400 (E) 100

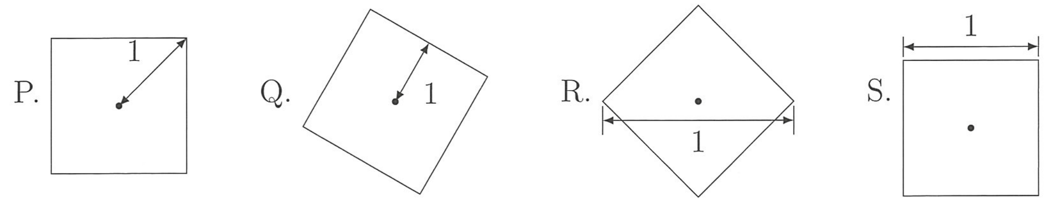
7. In the expression below, the letters A, B, C, D and E represent the numbers $1, 2, 3, 4$ and 5 in some order.

$$A \times B + C \times D + E$$

What is the largest possible value of the expression?

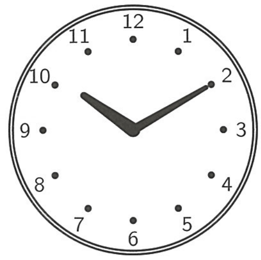
(A) 24 (B) 27 (C) 26 (D) 51 (E) 25

8. In each of these squares, the marked length is 1 unit. Which of the squares would have the greatest perimeter?



(A) P (B) Q (C) R (D) S (E) all are the same

9. On a clock face, a line is drawn between 9 and 3 and another between 12 and 8. What is the acute angle between these lines?

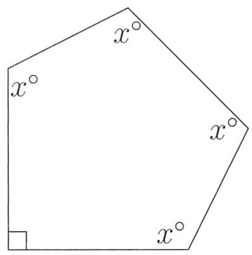


(A) 45° (B) 60° (C) 50° (D) 30° (E) 22.5°

10. There are 3 blue pens, 4 red pens and 5 yellow pens in a box. Without looking, I take pens from the box one by one. How many pens do I need to take from the box to be certain that I have at least one pen of each colour?

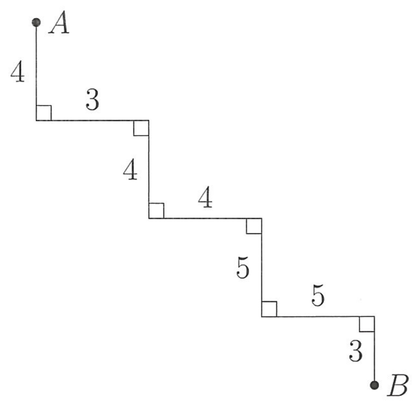
(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

11. In the diagram, the value of x is



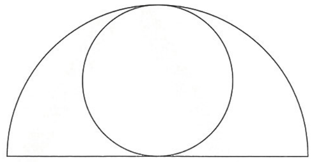
(A) 120 (B) 108 (C) 105 (D) 135 (E) 112.5

12. How far is it from A to B measured in a straight line?



(A) 20 (B) 28 (C) $10 + 9\sqrt{2}$ (D) $8 + 9\sqrt{2}$ (E) 16

13. A circle of radius 1 metre is inscribed inside a semicircle of radius 2 metres. What is the area in square metres of the semicircle *not* covered by the circle?



(A) 2π (B) $\pi - 1$ (C) 2 (D) $2\pi - 1$ (E) π

14. The value of n for which $4^{n+1} = 2^{10}$ is

(A) 9 (B) 8 (C) 4 (D) 10 (E) 2

15. Adrienne, Betty and Cathy were the only three competitors participating in a series of athletic events. In each event, the winner gets 3 points, second gets 2 points and third gets 1 point. After the events, Adrienne has 8 points, Betty has 11 points and Cathy has 5 points. In how many events did Adrienne come second?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

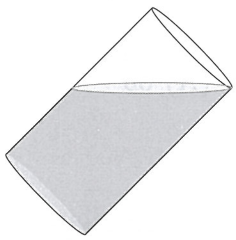
16. What is the smallest number N for which $\dfrac{2016}{N}$ is a perfect square?

(A) 14 (B) 2 (C) 56 (D) 12 (E) 7

17. Five people are sitting around a circle. Some always tell the truth, whilst the others always lie. Each person claims to be sitting between two liars. How many of them are telling the truth?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

18. A cylindrical glass of (inside) diameter 6 cm and height 11 cm is filled and then tilted to a 45° angle so that some water overflows. How much water is left in it?



(A) 48π mL (B) 45π mL (C) 66π mL (D) 72π mL (E) 63π mL

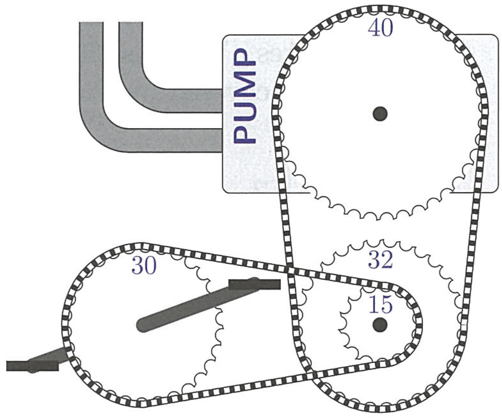
19. Ten students sit a test consisting of 20 questions. Two students get 8 questions correct and one student gets 9 questions correct. The remaining seven students all get at least 10 questions correct and the average number of questions answered correctly by these seven students is an integer. If the average number of questions answered correctly by all ten students is also an integer, then that integer is

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

20. This pedal-powered water pump is made from bicycle parts.

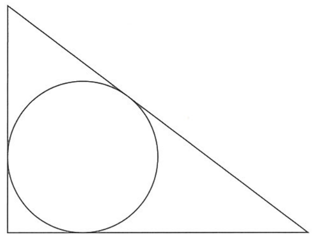
A 30-tooth gear on the pedals has a chain to a 15-tooth gear. On the same axle as the 15-tooth gear is a 32-tooth gear that drives a chain to a 40-tooth gear on the pump.

For every 100 complete revolutions of the pedals, how many times does the gear on the pump turn?



(A) 160 (B) 250 (C) 107 (D) 93 (E) $37\frac{1}{2}$

21. A gardener wishes to put a circular water feature (pool) in a right-angled triangular plot that has sides of 6 m and 8 m on its two smallest sides. What is the radius in metres of the largest pool that will fit?



(A) $2\sqrt{2}-1$ (B) 2 (C) 4 (D) 3 (E) $2\sqrt{2}$

22. A sequence of 10 letters is made according to the following rules.

- The letter P can only be followed by Q or R.
- The letter Q can only be followed by R or S.
- The letter R can only be followed by S or T.
- The letter S can only be followed by T or P.
- The letter T can only be followed by P or Q.

How many possible sequences are there where the first, fourth, and tenth letters are all Q?

(A) 63 (B) 39 (C) 32 (D) 45 (E) 36

23. Cynthia's afternoon train normally arrives at her station at 5:30 pm each day, where she is picked up by Alan and driven home.

One day she was on an earlier train which arrived at 5 pm, and she decided to walk in the direction Alan was coming from home. Alan had left in time to meet the 5:30 pm train, but this time he picked up Cynthia and they arrived home 10 minutes earlier than usual.

For how many minutes had Cynthia walked before Alan picked her up?

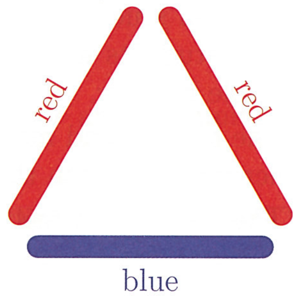
(A) 20 (B) 30 (C) 25 (D) 10 (E) 15

24. You have an unlimited supply of five different coloured pop-sticks, and want to make as many different coloured equilateral triangles as possible, using three sticks.

One example is shown here.

Two triangles are not considered different if they are rotations or reflections of each other.

How many different triangles are possible?



(A) 35 (B) 5 (C) 20 (D) 56 (E) 10

25. A *super-Fibonacci* sequence is a list of whole numbers with the property that, from the third term onwards, every term is the sum of all of the previous terms. For example,

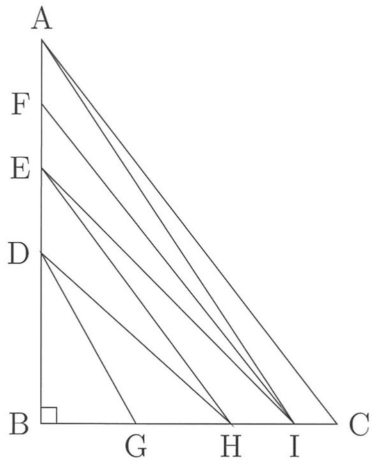
$$1,\ 4,\ 5,\ 10,\ \ldots$$

How many super-Fibonacci sequences starting with 1 involve the number 2016?

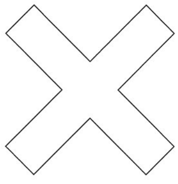
(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

26. The right-angled triangle ABC has area $2016\,\text{cm}^2$. Lines AI, IF, IE, EH, HD, DG divide the large triangle into seven smaller triangles of equal area.

If $\triangle BIE$ is an isosceles triangle, find the length of BG in centimetres.



27. A symmetrical cross with equal arms has an area of $2016\,\text{cm}^2$ and all sides of integer length in centimetres. What is the smallest perimeter the cross can have, in centimetres?



28. The ten students in Malcolm's maths class all took a test. The scores of the other nine students were 82, 83, 85, 89, 90, 92, 95, 97, and 98, and Malcolm's score was a whole number.

The teacher had made a mistake in calculating Malcolm's score. After she corrected her mistake, both the mean and the median of all the scores increased by 0.5.

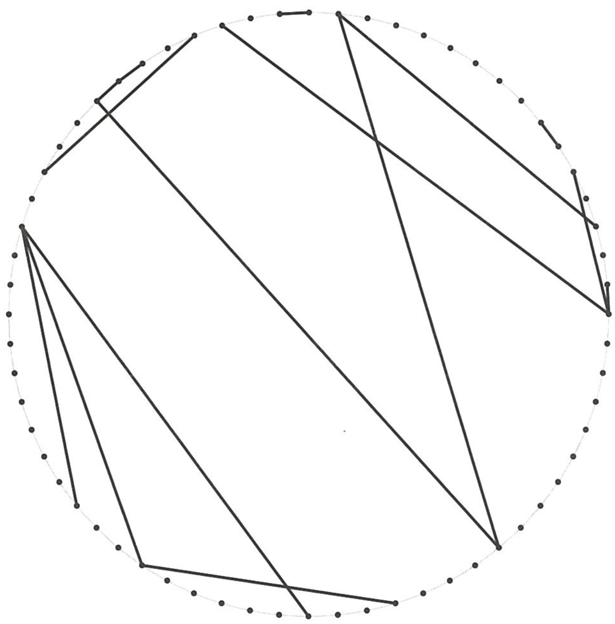
There are two possible correct scores that Malcolm could have had. What is the sum of these two scores?

29. A high school marching band can be arranged in a rectangular formation with exactly three boys in each row and exactly five girls in each column. There are several sizes of marching band for which this is possible. What is the sum of all such possible sizes?

30. Around a circle, I place 64 equally spaced points, so that there are $64 \times 63 \div 2 = 2016$ possible chords between these points.

I draw some of these chords, but each chord cannot cut across more than one other chord.

What is the maximum number of chords I can draw?



Questions – Senior Division

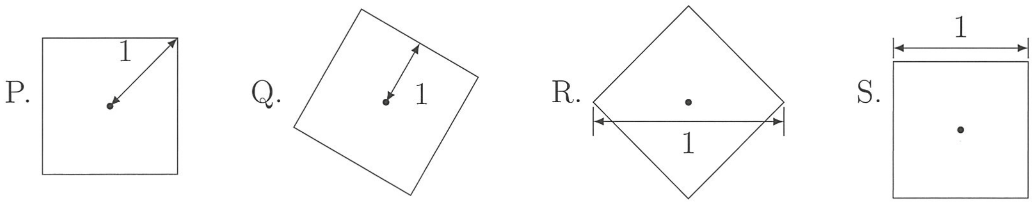
1. How many 3-digit numbers have the hundreds digit equal to 7 and the units digit equal to 8?

(A) 10 (B) 100 (C) 20 (D) 19 (E) 90

2. If $p = 7$ and $q = -4$, then $p^2 - 3q^2 =$

(A) 49 (B) 48 (C) 0 (D) 97 (E) 1

3. In each of these squares, the marked length is 1 unit. Which of the squares would have the greatest perimeter?



(A) P (B) Q (C) R (D) S (E) all are the same

4. Given that n is an integer and $7n + 6 \geq 200$, then n must be

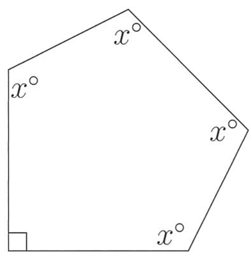
(A) even (B) odd (C) 28 or more (D) either 27 or 28 (E) 27 or less

5. King Arthur's round table has a radius of three metres. The area of the table top in square metres is closest to which of the following?

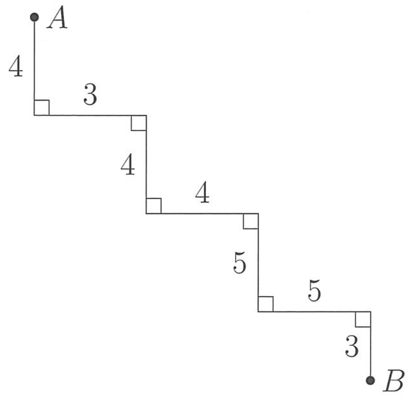
(A) 20 (B) 30 (C) 40 (D) 50 (E) 60

6. In the diagram, the value of x is

(A) 120 (B) 108 (C) 105 (D) 135 (E) 112.5



7. How far is it from A to B measured in a straight line?



(A) 20 (B) 28 (C) $10 + 9\sqrt{2}$ (D) $8 + 9\sqrt{2}$ (E) 16

8. The solution to the equation $\sqrt{x^2+1} = x + 2$ is

(A) $x = \frac{22}{7}$ (B) $x = -\frac{3}{4}$ (C) $x = -\frac{3}{2}$ (D) $x = 3$ (E) no number x satisfies the equation

9. A large tray in the bakery's display case has an equal number of three types of croissants: plain, chocolate and cheese. Norm rushes into the bakery and buys two croissants at random without looking. The probability that Norm does not have a chocolate croissant is closest to

(A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{1}{27}$

10. Which of these expressions is always a multiple of 3, whenever n is a whole number?

(A) n^3 (B) $n^3 + 2n$ (C) $3n^3 + 1$ (D) $n^3 + 3n^2$ (E) $n^2 + 2$

11. The value of $2^{2016} - 2^{2015}$ is

(A) 2 (B) $2^{\frac{2016}{2015}}$ (C) 2^{2015} (D) -2^{2016} (E) 0

12. Oaklands and Brighton are two busy train stations on the same train line. On one particular day:

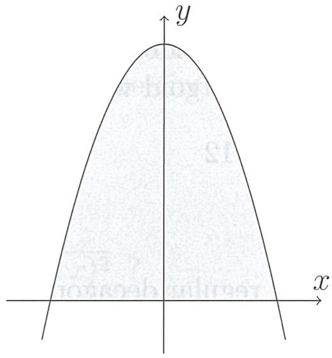
- One-fifth of the trains do not stop at Oaklands.
- 45 trains do not stop at Brighton.
- 60 trains stop at both Brighton and Oaklands.
- 60 trains stop at only Brighton or Oaklands (not both).

How many trains do not stop at either of these stations on that day?

(A) 60 (B) 20 (C) 45 (D) 5 (E) 40

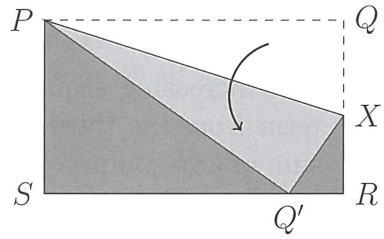
13. Consider the region bounded by the parabola $y = 6 - x^2$ and the x-axis.
How many points with integer coordinates lie within that region, but not on the boundary?

(A) 11 (B) 12 (C) 14 (D) 15 (E) 17



14. A rectangular sheet of paper $PQRS$ has width PQ twice the height QR. The paper is folded along a line PX such that Q lies on top of Q', a point on RS. Angle SPX is

(A) 72° (B) 45° (C) 60° (D) 67.5° (E) 75°

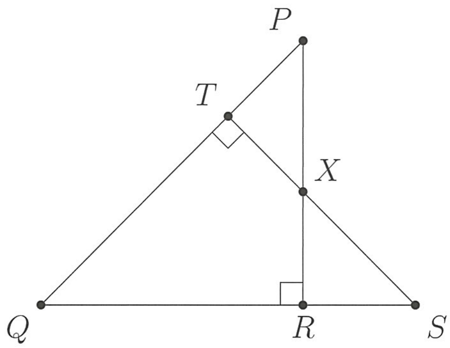


15. Ten students sit a test consisting of 20 questions. Two students get 8 questions correct and one student gets 9 questions correct. The remaining seven students all get at least 10 questions correct and the average number of questions answered correctly by these seven students is an integer. If the average number of questions answered correctly by all ten students is also an integer, then that integer is

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

16. In the diagram, $QR = QT = 4\,\text{cm}$ and PQR and QST are both isosceles right-angled triangles. In square centimetres, what is the area of the quadrilateral $QRXT$?

(A) $16(\sqrt{2} - 1)$ (B) $4\sqrt{2}$
(C) $16\sqrt{2} - 10$ (D) $8(\sqrt{2} - 1)$
(E) $8 - \sqrt{2}$



17. What is the smallest positive integer x for which the sum $x + 2x + 3x + 4x + \cdots + 100x$ is a perfect square?

(A) 202 (B) 5050 (C) 1010 (D) 100 (E) 101

18. Ten identical solid gold spheres will be melted down and recast into a number of smaller identical spheres whose diameter is 80% of the original ones. Ignoring any excess gold which may be left over, how many smaller spheres will be made?

(A) 12 (B) 20 (C) 8 (D) 15 (E) 19

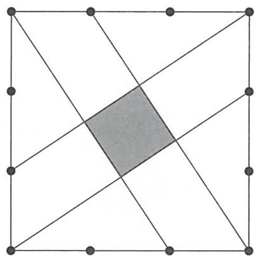
19. A regular decagon has 10 sides of length 1. Four vertices are chosen to make a convex quadrilateral that has all side lengths greater than 1. How many non-congruent quadrilaterals can be formed this way?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

20. The increasing sequence of positive integers $1, 2, 4, \ldots$ has the property that each term from the third onwards is the next positive integer which is not equal to the sum of any two previous terms of the sequence. How many terms of the sequence are less than 2016?

(A) 1008 (B) 63 (C) 11 (D) 15 (E) 673

21. A line is drawn from each corner of a large square to the point of trisection of the opposite side, as in the diagram.



If the shaded square has area 1, what is the area of the large square?

(A) 16 (B) 15 (C) 14 (D) 13 (E) 12

22. What is the sum of all positive integers n such that $n^2 + n + 34$ is a perfect square?

(A) 50 (B) 16 (C) 43 (D) 34 (E) 49

23. What is the radius of the largest sphere that will fit inside a hollow square pyramid, all of whose edges are of length 2?

(A) $\sqrt{2} - 1$ (B) $\frac{2 - \sqrt{2}}{2}$ (C) $\frac{\sqrt{6} - \sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{2}}{4}$

24. Ten positive integers are written on cards and the cards are placed around a circle. If a number is greater than the average of its two neighbours, the card is coloured green. What is the largest number of green cards there can be in the circle?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 9

25. What is the least value of $\sqrt{x^2+(1-x)^2}+\sqrt{(1-x)^2+(1+x)^2}$?

(A) 2 (B) $\dfrac{\sqrt{2}+\sqrt{10}}{2}$ (C) $\sqrt{5}$ (D) 0 (E) $1+\sqrt{2}$

26. A high school marching band can be arranged in a rectangular formation with exactly three boys in each row and exactly five girls in each column. There are several sizes of marching band for which this is possible. What is the sum of all such possible sizes?

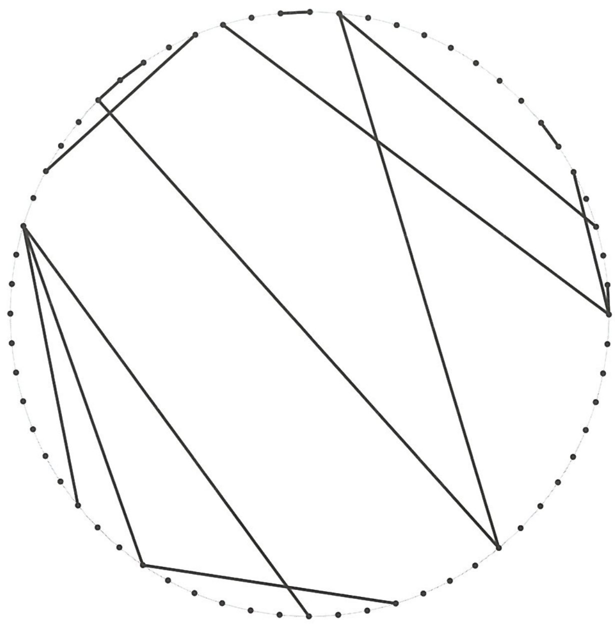
27. Let a, b, c, m, and n be integers such that $m < n$ and define the quadratic function $f(x) = ax^2 + bx + c$ where x is real. Then $f(x)$ has a graph that contains the points $(m, 0)$ and $(n, 2016^2)$. How many values of $n - m$ are possible?

28. If a and b are whole numbers from 1 to 100, how many pairs of numbers (a, b) are there which satisfy $a^{\sqrt{b}} = \sqrt{a^b}$?

29. Around a circle, I place 64 equally spaced points, so that there are $64 \times 63 \div 2 = 2016$ possible chords between these points.

I draw some of these chords, but each chord cannot cut across more than one other chord.

What is the maximum number of chords I can draw?



30. A function f defined on the set of positive integers has the properties that, for any positive integer n, $f(f(n)) = 2n$ and $f(4n+1) = 4n+3$. What are the last three digits of $f(2016)$?

Solutions – Middle Primary Division

1. $20 + 16 = 36$,

hence (C).

2. (Also UP1)
The numbers in order are 555, 556, 565, 566, 655,

hence (D).

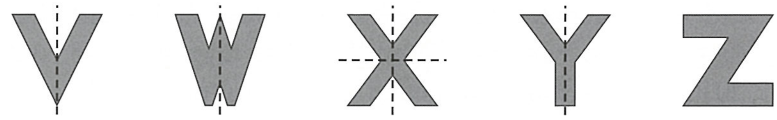
3. The digit 3 is in the thousands position, so it represents 3000,

hence (D).

4. I am 12 years old, so our combined ages are $6 + 12 = 18$,

hence (C).

5. Here are all possible lines of symmetry:



Although the Z shape has a point of symmetry, it does not have a line of symmetry,
hence (E).

6. (Also UP2)
One pizza will have 4 quarters, so two pizzas will have $2 \times 4 = 8$ quarters,
hence (D).

7. After 30 minutes it is 5 pm, and after another 15 minutes it is 5:15 pm,

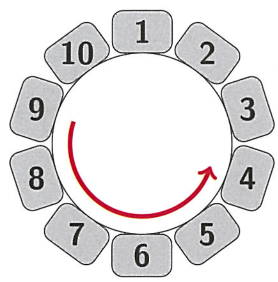
hence (E).

8. The number of wheels is $4 \times 2 + 2 \times 3 + 1 \times 4 = 18$,

hence (E).

9. (Also UP6)

The opposite chair is both 5 places forward and 5 places back.
Five places back from chair 9 is chair 4,



hence (D).

10. (Also UP5)
Alternative 1
In cents, $500 \div 80 = 6\text{r}20$ so that he buys 6 chocolates and has 20 cents left,
hence (C).

Alternative 2

Multiples of 80 are $80, 160, 240, 320, 400, 480, 560$. From this, he can afford 6 chocolates but not 7,

hence (C).

11. To make the total as large as possible, the large digits should have place value as large as possible. That is, 9 and 8 will be in the tens positions of the two numbers.

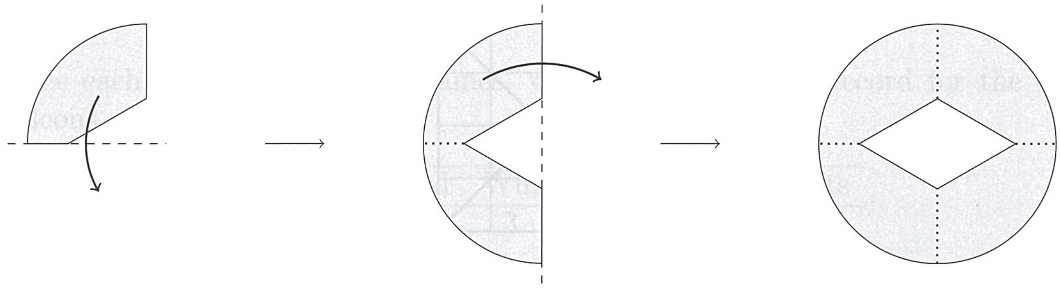
Then 3 and 2 will be in the units positions.

Then the sum is either $93 + 82 = 175$ or $92 + 83 = 175$, both with total 175,

hence (A).

12. (Also J6)

Each time the paper is unfolded, the two parts will be reflections of each other through the fold line.



hence (A).

13. (Also UP8)

She either has a 50c coin or not.

If she has a 50c coin, then she has one other 10c coin: $50 + 10 = 60$.

If she has no 50c coins, then she either has 0, 1, 2 or 3 20c coins:

$$20 + 20 + 20 = 60 \qquad 20 + 20 + 10 + 10 = 60$$

$$20 + 10 + 10 + 10 + 10 = 60 \qquad 10 + 10 + 10 + 10 + 10 + 10 = 60$$

In all, there are 5 possibilities,

hence (D).

14. Since there are 3 colours, you can't be sure that the first 3 beans include a pair.

However, with 4 beans, they can't all be different colours, so there must be a pair of the same colour.

So 4 beans (and no more) are needed to make sure you have a pair,

hence (B).

15. (Also UP11)

Starting from the outer end of the spiral (the loop on the rope) the dark and light sections are longest, and the light sections are of similar length to the dark sections.

As you move towards the other end of the rope, both dark and light sections get shorter. Only rope (A) shows this,

hence (A).

16. There are $3 + 6 + 2 + 1 + 4 = 16$ students, so half the class is 8 students.

The 6 with orange hats are in one-half of the class, and so the other two students in that half of the class have black hats.

So the only way to split the class into two equal groups is with orange and black in one group and red, green and yellow in the other,

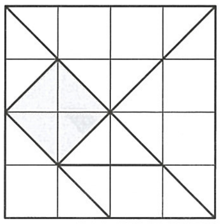
hence (A).

17. The sum of the first five digits is 22. Therefore the sum of the last two digits must be 12, as $34 - 22 = 12$. There are 7 possibilities for the last two digits: 39, 93, 48, 84, 57, 75, and 66,

hence (B).

18. (Also UP12)

The large square must have side 8 cm. Then all of the corners of the pieces in the tangram lie on a grid of $2\,\text{cm} \times 2\,\text{cm}$ squares.

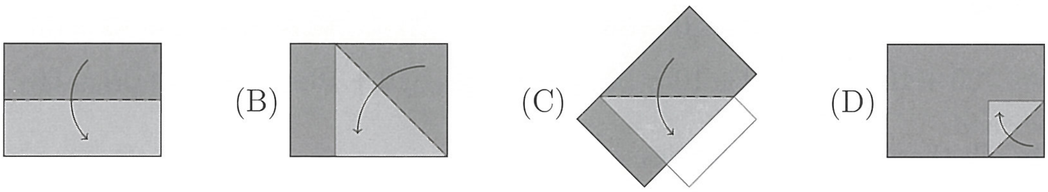


The shaded square has the same area as 2 of the grid squares, or $2 \times (2\,\text{cm} \times 2\,\text{cm}) = 8\,\text{cm}^2$,

hence (D).

19. No matter how a single fold is made, there will be one of the original right-angle corners that is on the boundary of the shape. Of the 5 shapes, (E) does not have any right angles, so it can't be made with a single fold. The other four figures can be made with a single fold:

(A) (B) (C) (D)



hence (E).

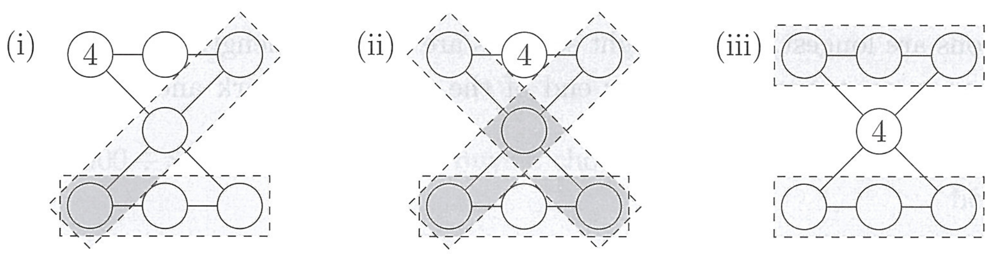
20. *Alternative 1*

There are 5 ways of making 12 from these numbers:

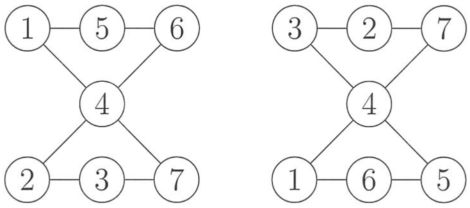
$7 + 4 + 1 = 12, \;\; 7 + 3 + 2 = 12, \;\; 6 + 5 + 1 = 12, \;\; 6 + 4 + 2 = 12, \;\; 5 + 4 + 3 = 12$

The number 4 is in 3 of these sums, and not in $1 + 5 + 6$ and $2 + 3 + 7$.

The number 4 could be either in a corner, a side or the centre:



However, the highlighted lines in each can only be $1+5+6$ and $2+3+7$ which don't have a number in common, so (i) and (ii) don't work. Placing 4 in the centre, there are several ways to arrange the other numbers:



hence (D).

Alternative 2

All seven circles add to $1+2+3+4+5+6+7=28$. The top and bottom rows together add to $2\times 12=24$. So the middle circle must be $28-24=4$,

hence (D).

21. Since each team played three games, we can complete the record for the Eagles, Falcons and Condors.

	Played	Win	Draw	Loss	Points
Eagles	3	3	0	0	9
Hawks	3				
Falcons	3	0	1	2	1
Condors	3	0	1	2	1

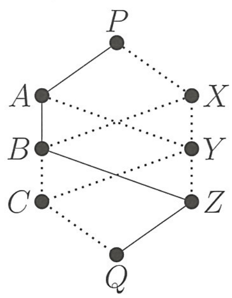
The Eagles won all their games, so the Hawks lost to the Eagles.

The Falcons and the Condors had no wins, so the game between them must have been drawn. Therefore both lost to the Hawks.

The Hawks' three games were 2 wins and 1 loss, for 6 points,

hence (C).

22. Label the points as shown and work down the diagram.



There is 1 route from P to A, and also 1 route from P to X.

Routes to B will come through either A or X, which gives only 2 routes. Likewise, there are 2 routes to Y.

Routes to C will come through either B or Y. There are 2 routes to B and 2 routes to Y, so there are 4 routes to C. Likewise, there are 4 routes to Z.

Routes to Q are the 4 routes that come through C plus the 4 routes that come through Z, making 8 routes,

hence (D).

23. (Also UP21)

The discs are back in their original positions after 5 moves.

r		g		o		b		y		r
b	1	y	2	r	3	g	4	o	5	b
g	$\longrightarrow$	o	$\longrightarrow$	b	$\longrightarrow$	y	$\longrightarrow$	r	$\longrightarrow$	g
y		r		g		o		b		y
o		b		y		r		g		o

They will be in their original positions again after 10, 15 and 20 moves. After 1 more move, blue will be on the bottom,

hence (B).

24. (Also UP22)

Alternative 1

Replacing the leopard by another lion (of the same weight as the lion) would add 90 kg, and replacing the tiger by another lion would add 50 kg. Then 3 lions weigh $310 + 90 + 50 = 450$ kg and 1 lion weighs $450 \div 3 = 150$ kg,

hence (B).

Alternative 2

If the lion weighs 100 kg, then the leopard weighs 10 kg and the tiger 50 kg for a total of 160 kg. This is 150 kg too light. Adding $150 \div 3 = 50$ kg to each weight keeps the differences in weight the same. So the lion weighs 150 kg,

hence (B).

25. Jane must have given away her \$2 coin, otherwise Tom would have \$4 or more. Tom must end up with an even number of cents, so he must have given away his 5c coin.

With just these two coins given to Angus, Jane has \$1.85 and Tom has \$3.80. So it can't be done with just 2 coins given to Angus.

It can be done with 3 coins: if Tom gives away his 5c and his 10c he has \$3.70, and if Jane gives away her \$2, she has \$1.85,

hence (B).

26. This table lists the numbers according to their first two digits.

First digit	0	1	2	3	4	5	6	7	8	Count
1	101	112	123	134	145	156	167	178	189	9
2	202	213	224	235	246	257	268	279		8
3	303	314	325	336	347	358	369			7
4	404	415	426	437	448	459				6
5	505	516	527	538	549					5
6	606	617	628	639						4
7	707	718	729							3
8	808	819								2
9	909									1
										45

(Second digit spans columns 0–8.)

The total number of possibilities is $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$,

hence (45).

27. Once the age of the younger daughter is known, the other four ages and the total can be calculated. Here are the first few possibilities:

Younger daughter	Older daughter	Younger son	Older son	Total
0	2	4	7	13
1	3	6	9	19
2	4	8	11	25

This pattern continues, with the total going up in 6s, since for each +1 on the younger daughter, there is +1 on the older daughter and +2 on both sons.

A total of 55 requires 30 more than in the last line above, which will happen 5 rows later. The final line shows:

Younger daughter	Older daughter	Younger son	Older son	Total
7	9	18	21	55

hence (7).

28. If A and B are both chosen, then the third stamp is either D or E. (2 possibilities)

If A but not B is chosen, then D must be chosen, and either C or E. (2 possibilities)

If B but not A is chosen, then E must be chosen, and either D or F. (2 possibilities)

If neither A nor B is chosen, then the 3 stamps must be in a row of three: CDE or DEF. (2 possibilities)

In all there are 8 possibilities,

hence (8).

29. (Also UP28)

The first cube uses 12 matches, then each subsequent cube uses 8 matches. Since $2016 - 12 = 2004$ and $2004 \div 8 = 250\text{r}4$, there are $1 + 250 = 251$ cubes made, with 4 matches left over,

hence (251).

30. (Also J20)

Consider the prime factorisation of $2016 = 2^5 \times 3^2 \times 7$. The factors of 2016 under 10 are 1, 2, 3, 4, 6, 7, 8 and 9.

Only 7 has prime factor 7, so this must be one of the ages.

The 3^2 in the prime factorisation will either be from $3 \times 6 = 2 \times 3^2$ or from $9 = 3^2$.

In the first case, the factorisation is $3 \times 6 \times 7 \times 16$, where 16 is too large.

In the second case, two of the ages are 7 and 9. Then the remaining two ages multiply to $2^5 = 32$, so they must be 4 and 8.

Hence the ages are 4, 7, 8 and 9, which add to 28,

hence (28).

Solutions – Upper Primary Division

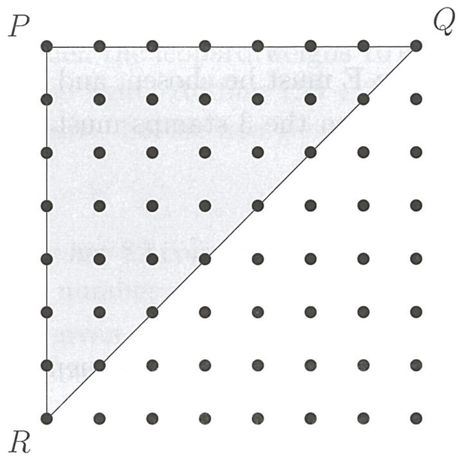
1. (Also MP2)
The numbers in order are 555, 556, 565, 566, 655,

hence (D).

2. (Also MP6)
One pizza will have 4 quarters, so two pizzas will have $2 \times 4 = 8$ quarters,

hence (D).

3.



The number of interior lattice points is $1 + 2 + 3 + 4 + 5 = 15$,

hence (C).

4. $0.3 + 0.4 = 0.7$,

hence (B).

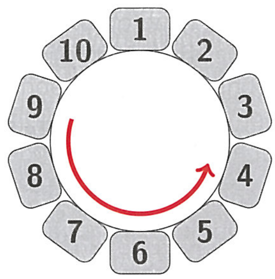
5. (Also MP10)
Alternative 1
In cents, $500 \div 80 = 6r20$ so that he buys 6 chocolates and has 20 cents left,

hence (C).

Alternative 2
Multiples of 80 are $80, 160, 240, 320, 400, 480, 560$. From this, he can afford 6 chocolates but not 7,

hence (C).

6. (Also MP9)
The opposite chair is both 5 places forward and 5 places back. Five places back from chair 9 is chair 4,



hence (D).

7. In order, 𝅝 𝅗𝅥 ♩ ♪ ♪ is worth $4+2+1+\frac{2}{2}=8$ beats,

hence (E).

8. (Also MP13)
She either has a 50c coin or not.
If she has a 50c coin, then she has one other 10c coin: $50+10=60$.
If she has no 50c coins, then she either has 0, 1, 2 or 3 20c coins:

$$20+20+20=60 \qquad 20+20+10+10=60$$
$$20+10+10+10+10=60 \qquad 10+10+10+10+10+10=60$$

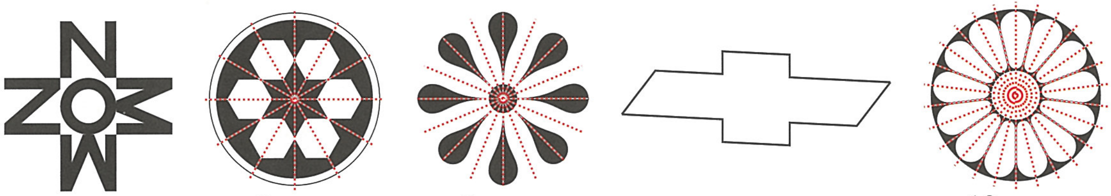
In all, there are 5 possibilities,

hence (D).

9. (A) holds $\frac{1}{3}$ of 3000 mL which is 1000 mL.
(B) holds $\frac{3}{4}$ of 1000 mL which is 750 mL.
(C) holds $\frac{1}{2}$ of 1000 mL which is 500 mL.
(D) holds $\frac{1}{3}$ of 750 mL which is 250 mL.
(E) holds $\frac{1}{4}$ of 2000 mL which is 500 mL.
Of these 1000 mL is the greatest,

hence (A).

10. Here are the axes of symmetry of each shape:



no axes | 6 axes | 8 axes | no axes | 13 axes

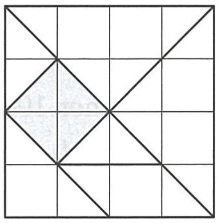
hence (E).

11. (Also MP15)
Starting from the outer end of the spiral (the loop on the rope) the dark and light sections are longest, and the light sections are of similar length to the dark sections. As you move towards the other end of the rope, both dark and light sections get shorter. Only rope (A) shows this,

hence (A).

12. (Also MP18)

The large square must have side 8 cm. Then all of the corners of the pieces in the tangram lie on a grid of 2 cm × 2 cm squares.



The shaded square has the same area as 2 of the grid squares, or $2 \times (2\,\text{cm} \times 2\,\text{cm}) = 8\,\text{cm}^2$,

hence (D).

13. *Alternative 1*

Since $8 \times 25 = 200$, he needs to make 8 batches. This requires $8 \times 2\frac{1}{2} = 20$ packets of chocolate chips,

hence (A).

Alternative 2

Each packet of chocolate chips is enough for 10 biscuits. So for 200 biscuits, 20 packets are required,

hence (A).

14. Two even numbers add to even. For example, $2 + 4 = 6$. So, not (A).

The difference between two odds is always even. For example, $5 - 1 = 4$. So, not (B).

The sum of two odd numbers is always even. For example, $5 + 1 = 6$. So, not (C).

Adding 3 odd numbers is the same as odd plus even, which is odd. For example, $3 + 5 + 9 = 8 + 9 = 17$. So (D) is true.

Two odd numbers multiplied is always odd. For example, $3 \times 7 = 21$. So, not (E),

hence (D).

15. The side of the outer square is $36 \div 4 = 9$ cm and the side of the inner square is $20 \div 4 = 5$ cm.

The difference is 4 cm, which is 2 cm on each side. So each rectangle is 7 cm × 2 cm, with perimeter 18 cm,

hence (E).

16. *Alternative 1*

The possibilities are

First number	Possibilities	Count
2	2134, 2143	2
3	3124, 3142, 3214, 3241	4
4	4123, 4132, 4213, 4231, 4312, 4321	6
		12

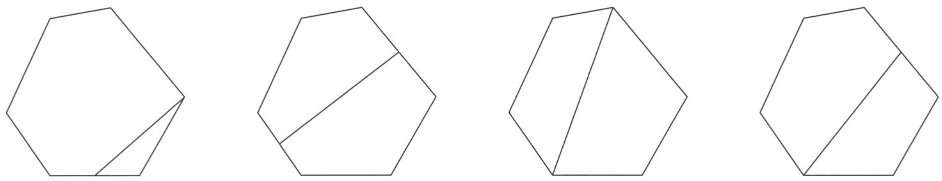
hence (B).

Alternative 2
Ignoring whether the first number is larger or smaller than the second, the number of combinations is $4 \times 3 \times 2 \times 1 = 24$.

Half of these will have the first number larger than the second, and half will be the other way around. So there are 12 combinations,

hence (B).

17. Each of (A), (B), (C) and (D) is possible:



However (E) is not possible. It has a total of 7 sides, whereas the hexagon has 6 sides and a cut increases the number of sides by at least 2, giving 8 or more sides,

hence (E).

18. As the sum $3 + 9 + 15 + 18 + 24 + 29 = 98$, we are looking for two groups of three numbers each with a sum close to 49. There are no three of the numbers adding to 49, so the difference must be 2 or more.

To get the difference as close as possible to 0, each sum will be as close to 49 as possible.

Noting that $3+18+29 = 50$ and $9+15+24 = 48$ gives $(3+18+29)-(9+15+24) = 2$,

hence (C).

19. Adding the upward-sloping diagonal, $4 + 7 + 10 + 13 = 34$ is the common total.

In the 4th column $13 + 12 + 1 = 26$ and $34 - 26 = 8$. In the 2nd row, $5 + 10 + 8 = 23$ and $34 - 23 = 11$. Place A and B in the third row as shown.

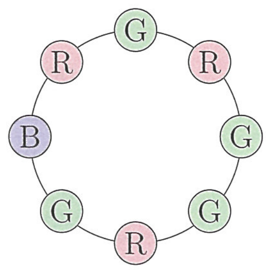
X			13
5	11	10	8
A	7	B	12
4			1

From the first column, $A + X = 34 - 9 = 25$. From the downward-sloping diagonal, $B + X = 34 - 12 = 22$. From the third row, $A + B = 34 - 19 = 15$.

Then $25 + 22 + 15 = 62$ counts each of A, B and X twice, so that $A + B + X = 31$ and $X = 31 - 15 = 16$,

hence (A).

20. (Also J18)



First try not to use a blue counter at all.

The counters can't be all red or all green, so start with a red counter at the bottom of the circle. By the first rule, the counters either side must be green.

Then, by the second rule, the counters opposite these green counters must be red. The top counter can now be coloured green.

The side counters cannot be red because they are adjacent to a red. However, only one of them can be green, so the other must be blue. This arrangement, as shown, satisfies the three rules.

Hence, the minimum number of blue counters is 1,

hence (B).

21. (Also MP23)
The discs are back in their original positions after 5 moves.

r		g		o		b		y		r
b	1	y	2	r	3	g	4	o	5	b
g	$\longrightarrow$	o	$\longrightarrow$	b	$\longrightarrow$	y	$\longrightarrow$	r	$\longrightarrow$	g
y		r		g		o		b		y
o		b		y		r		g		o

They will be in their original positions again after 10, 15 and 20 moves. After 1 more move, blue will be on the bottom,

hence (B).

22. (Also MP24)
Alternative 1
Replacing the leopard by another lion (of the same weight as the lion) would add 90 kg, and replacing the tiger by another lion would add 50 kg. Then 3 lions weigh $310 + 90 + 50 = 450$ kg and 1 lion weighs $450 \div 3 = 150$ kg,

hence (B).

Alternative 2
If the lion weighs 100 kg, then the leopard weighs 10 kg and the tiger 50 kg for a total of 160 kg. This is 150 kg too light. Adding $150 \div 3 = 50$ kg to each weight keeps the differences in weight the same. So the lion weighs 150 kg,

hence (B).

23. (Also J15, I15)
As the number of points per event is 6 and the total number of points gained is $8 + 11 + 5 = 24$, there must have been 4 events. As Betty has 11 points she must have 3 first places and one second place. Cathy could not have won an event, or her score would be 6 or more, so she must have just one second place. So Adrienne must have two second places,

hence (C).

24. Arrange the coins as follows:

	A	B	C
Jane	5c coins	10c coins	50c coins
Tom	10c coins	20c coins	5c coins

Suppose Jane has just one 50c coin, so that Tom has one 5c coin and Jane has 45c more than Tom in column C.

Tom and Jane have the same amount, so than in columns A and B, Tom has 45c more than Jane. But for each coin in columns A and B, Tom's coin is worth twice Jane's coin, so Tom has 90c and Jane has 45c. The fewest coins for this is 1 coin each in column A and 4 coins each in column B. Then they have 6 coins each.

If Jane has more than one 50c coin, the difference in columns A and B will be 90c or more, which requires more than 4 coins in these columns. Then they have more than 6 coins each.

So the smallest number of coins they can each have is 6, when Jane has $1 \times 50c + 4 \times 10c + 1 \times 5c$ and Tom has $1 \times 5c + 4 \times 20c + 1 \times 10c$,

hence (D).

25. *Alternative 1*

The final mixture will have 9 litres of cordial out of 18 litres.

The amount of the mixture from Jar A can vary from 0 litres to 18 litres, so we try increasing amounts from Jar A and calculate the amount of 100% cordial in the mixture.

Litres from Jar A	0	1	2	3	$\cdots$	
Litres from Jar B	18	17	16	15	$\cdots$	
Litres of cordial from Jar A	0	0.3	0.6	0.9		
Litres of cordial from Jar B	10.8	10.2	9.6	9.0		
Litres of cordial in mixture	10.8	10.5	10.2	9.9	$\cdots$	9.0

For every additional litre from Jar A there is 0.3 litres less cordial in the mixture, which is 0.6 additional litres from Jar B and 0.3 fewer litres from Jar A.

Following this pattern, there will be 9 litres of cordial when there are 6 litres from Jar A and 12 litres from Jar B,

hence (E).

Note: Checking this, Jar A contributes 1.8 litres of cordial and Jar B contributes 7.2 litres of cordial.

Alternative 2

Let the amount from Jar A be x, and the amount from Jar B be y. Then

$$\begin{array}{lllll} x + y = 18 & & & \text{so} & x = 18 - y \\ 0.3x + 0.6y = 0.5 \times 18 & \text{so} & 3x + 6y = 90 & \text{and} & x = 30 - 2y \end{array}$$

Therefore $18 - y = 30 - 2y$, which has solution $y = 12$, and then $x = 6$,

hence (E).

26. *Alternative 1*

Adding $11 + 17 + 22 = 50$ includes every hat twice. For example, Qiang's hat is in both Rory's and Sophia's totals.

Therefore the total of all three hats is 25. Then the person whose total is 11 has $25 - 11 = 14$ on their hat. The other two hats are $25 - 17 = 8$ and $25 - 22 = 3$.

So the three numbers are $3, 8$ and 14,

hence (14).

Alternative 2

Let the numbers be a, b and c. Then $a + b = 11$, $b + c = 17$ and $a + c = 22$. Then $2a + 2b + 2c = 50$, therefore $a + b + c = 25$. This gives $a = 8, b = 3, c = 14$ so the largest number is 14,

hence (14).

27. *Alternative 1*

We can test factors by division, finding each factor's partner until we see a factor that has occurred as a partner already.

Factor	1	2	3	4	5	6	7	8	9	10
Partner	840	420	280	210	168	140	120	105	–	84

Factor	11	12	13	14	15	16	17	18	19	20
Partner	—	70	—	60	56	—	—	—	—	42

Factor	21	22	23	24	25	26	27	28	29	30
Partner	40	—	—	35	—	—	—	30	—	28

From this, there are 16 pairs of factors, giving 32 factors,

hence (32).

Alternative 2

Factorised into primes, $840 = 2^3 \times 3 \times 5 \times 7$. Then every factor of 840 can be found by multiplying together:

- a factor of 8 (1, 2, 4 or 8)
- a factor of 3 (1 or 3)
- a factor of 5 (1 or 5)
- a factor of 7 (1 or 7)

This means there are $4 \times 2 \times 2 \times 2 = 32$ possible factors of 840,

hence (32).

28. (Also MP29)

The first cube uses 12 matches, then each subsequent cube uses 8 matches. Since $2016 - 12 = 2004$ and $2004 \div 8 = 250\text{r}4$, there are $1 + 250 = 251$ cubes made, with 4 matches left over,

hence (251).

29. (Also J27, I24)

There are three cases for how the triangles are coloured:

(i) All three sides are the same colour, with 5 possibilities.

(ii) Two sides are the same and one side is different, with $5 \times 4 = 20$ possibilities.

(iii) All three sides are different, with $\dfrac{5 \times 4 \times 3}{6} = 10$ possibilities.

So there are $5 + 20 + 10 = 35$ possibilities in all,

hence (35).

Note: The calculation in (iii) relies on the observation that if we are choosing the sides in order, there are 5 possibilities for the first side, then 4 for the second, then 3 for the third. However, in these $5 \times 4 \times 3 = 60$ possibilities, each selection xyz will appear 6 times: $xyz, xzy, yxz, yzx, zxy, zyx$. This idea appears in the general formula for $\binom{n}{m}$, the number of ways of choosing m objects from n objects.

30. If m is a cousin's age last year, it is a factor of 1377 and $m+1$ is a factor of 2016. When we factorise we get $2016 = 2^5 \times 3^2 \times 7$ and $1377 = 3^4 \times 17$.

Checking possible factors of 1377:

m	1	3	9	17	27	51	81	153	459	1377
$m+1$	2	4	10	18	28	52	82	154	460	1378
Factor of 2016?	✓	✓	×	✓	✓	×	×	×	×	×

So there are 4 possible ages.

To have three of these ages multiplying to 2016, we must include 28 (to get a factor of 7), 18 (to get a factor of 3^2) and then 4.

Checking, $2016 = 28 \times 18 \times 4$ and $1377 = 27 \times 17 \times 3$.

Then two years ago their ages multipled to $26 \times 16 \times 2 = 832$,

hence (832).

Solutions – Junior Division

1. $2016 \times 2 = 4032$,

hence (E).

2. The angles in the triangle add to 180°, so $x = 180 - 20 - 20 = 140$,

hence (D).

3. 30 days is 4 weeks and 2 days. So 30 days from today is the same as 2 days, which is a Saturday,

hence (E).

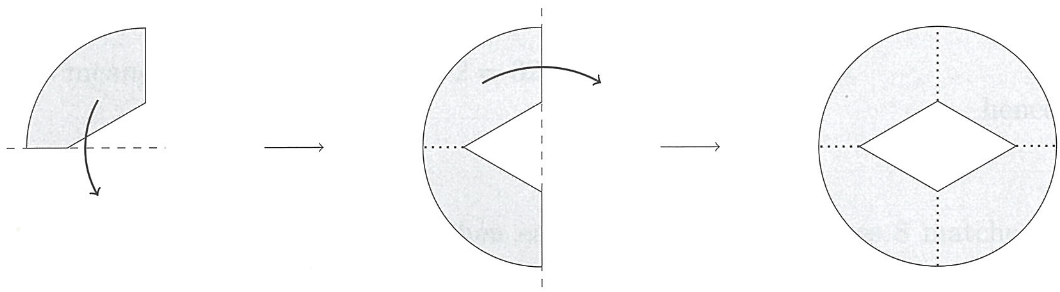
4. $-5 + 8 = 3$,

hence (E).

5. 25% is $\frac{1}{4}$, so 25% of $\frac{1}{2}$ is $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$,

hence (A).

6. (Also MP12)
Each time the paper is unfolded, the two parts will be reflections of each other through the fold line.



hence (A).

7. $100 - (29 + 16 + 8.95) = 100 - 53.95 = 46.05$,

hence (C).

8. In order, $0.009 < 0.019 < 0.08 < 0.109 < 0.4 < 0.409 < 0.91$,

hence (C).

9. 12 noon is 10 minutes after starting, 1 pm is 70 minutes, so 1:04 pm is 74 minutes,

hence (D).

10. (Also I4)
Estimating, $\frac{720163}{2016} \approx \frac{720000}{2000} = \frac{720}{2} = 360$. This suggests that $100 < \frac{720163}{2016} < 1000$.
Checking, $201600 < 720163 < 2016000$ and so $100 < \frac{720163}{2016} < 1000$,

hence (D).

11. The areas of the three squares, from smallest to largest, are 9, 16, and 25 square units. The shaded region has area $16 - 9 = 7$, so the portion of the largest square that is shaded is $100 \times \frac{7}{25} = 28$ percent,

hence (B).

12. *Alternative 1*

If Liana has m marbles, Jan has $3m$ marbles and $3m - 3 = m + 3$. Solving this, $2m = 6$ and $m = 3$. Between them, they have $4m = 12$ marbles,

hence (D).

Alternative 2

Jan has $\frac{3}{4}$ of the marbles and Liana has $\frac{1}{4}$. After transferring they each have $\frac{1}{2}$ of the marbles. So the 3 marbles transferred make up $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ of the marbles. Therefore they have 12 marbles between them,

hence (D).

13. In every two rows, there are 7 pavers, one of which will be cut in half. So the number of cut pavers is $1750 \div 7 = 250$,

hence (A).

14. On Monday, I planted 10 apple trees. On Tuesday, the smallest number of orange trees I could have planted is 9, one between each pair of neighbouring apple trees. On Wednesday, the smallest number of peach trees I could have planted is 18, one between each pair of neighbouring apple and orange trees. So the smallest number of trees that I planted altogether is $10 + 9 + 18 = 37$,

hence (C).

15. (Also UP23, I15)

As the number of points per event is 6 and the total number of points gained is $8 + 11 + 5 = 24$, there must have been 4 events. As Betty has 11 points she must have 3 first places and one second place. Cathy could not have won an event, or her score would be 6 or more, so she must have just one second place. So Adrienne must have two second places,

hence (C).

16. (Also I7)

If $A = 1$, then $A \times B + C \times D + E = B + E + C \times D$. On the other hand, if we swap A and E, we get $E \times B + C \times D + A = E \times B + 1 + C \times D$, which is larger, since both B and E are 2 or more. So the largest possible value can't have $A = 1$. Similarly, the largest possible value can't have any of B, C, or D equal to 1. Thus $E = 1$.

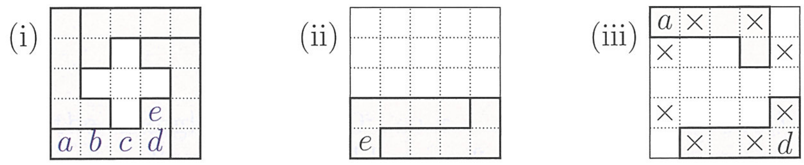
Then we only need to consider the following cases.

- $2 \times 3 + 4 \times 5 + 1 = 27$
- $2 \times 4 + 3 \times 5 + 1 = 24$
- $2 \times 5 + 3 \times 4 + 1 = 23$

Therefore, the largest possible value for the expression is 27,

hence (B).

17. By trial and error, the pattern in diagram (i) can be found, so that one possibility is that the other tile is (E).



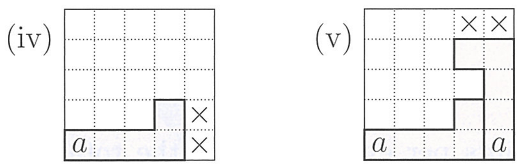
We need to confirm that this is the only possibility.

The grid can have at most one of its 4 corners occupied by the other tile, so either 3 or 4 of the L tiles will occupy a corner. Label the squares of each L a, b, c, d, e as shown in diagram (i), then the only squares of an L that can be in the corner of the 5×5 grid are a, d and e.

If e is in a corner, another L must fit in as in diagram (ii). For the remaining 5×3 rectangle to contain two Ls and leave the remaining area in one piece, the two Ls must make another 5×2 rectangle, leaving a 5×1 straight tile. This is not an option, so none of the Ls have square e on a corner.

So only a and d can be in a corner. Consider the 8 squares marked $\times$ in diagram (iii). An L with a in the corner will cover 2 of these, and an L with d in the corner will cover 3. Hence there can't be three Ls with d in the corner, so there must be at least one L with a in the corner.

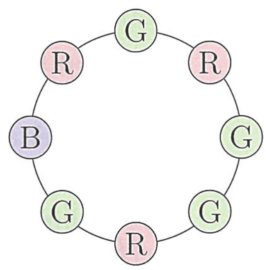
Place an L with a in a corner as in diagram (iv), then the squares marked $\times$ cannot be filled by any of the tiles (A)–(E), so they only be filled by another L with a in the corner, as in diagram (v).



Continuing like this leads to the solution already observed. So there is only one solution,

hence (E).

18. (Also UP20)



First try not to use a blue counter at all.

The counters can't be all red or all green, so start with a red counter at the bottom of the circle. By the first rule, the counters either side must be green.

Then, by the second rule, the counters opposite these green counters must be red. The top counter can now be coloured green.

The side counters cannot be red because they are adjacent to a red. However, only one of them can be green, so the other must be blue. This arrangement, as shown, satisfies the three rules.

Hence, the minimum number of blue counters is 1,

hence (B).

19. For every 3 strands in the original packet, there will be 5 pieces after breakage. Of these 5, 3 are guaranteed to be at least half a strand. So $\frac{3}{5}$ of the pieces are guaranteed to be at least as long as half an unbroken strand,

hence (B).

20. (Also MP30)

Consider the prime factorisation of $2016 = 2^5 \times 3^2 \times 7$. Factors of 2016 under 10 are 1, 2, 3, 4, 6, 7, 8 and 9.

Only 7 has prime factor 7, so this must be one of the ages.

The 3^2 in the prime factorisation will either be from $3 \times 6 = 2 \times 3^2$ or from $9 = 3^2$.

In the first case, the factorisation is $3 \times 6 \times 7 \times 16$, where 16 is too large.

In the second case, two of the ages are 7 and 9. Then the remaining two ages multiply to $2^5 = 32$, so they must be 4 and 8.

Hence the ages are 4, 7, 8 and 9, which add to 28,

hence (C).

21. *Alternative 1*

The large jug is between $50/4 = 12\frac{1}{2}$ L and $50/3 = 16\frac{2}{3}$ L. So its capacity x is either 13, 14, 15, or 16 litres. The amount left in the barrel is either 11, 8, 5 or 2 litres. Call this quantity $y = 50 - 3x$.

The small jug has capacity z between $y/4$ and $y/3$. The options are shown in the table:

x	y	$\frac{y}{4}$	$\frac{y}{3}$	possible z
13	11	$2\frac{3}{4}$	$3\frac{2}{3}$	3
14	8	2	$2\frac{2}{3}$	—
15	5	$1\frac{1}{4}$	$1\frac{2}{3}$	—
16	2	$\frac{1}{2}$	$\frac{2}{3}$	—

So the small jug has capacity 3 litres,

hence (C).

Alternative 2

Suppose one large jug holds x litres and one small jug holds y litres. If $x = 17$ or more, then 3 large jugs cannot be filled, and if $x = 12$ or less then more than 3 large jugs would be filled. So $x = 13, 14, 15$ or 16.

If $x = 16$, then 2 litres remain, and 3 small jugs can't be filled.

If $x = 15$, then 5 litres remain, and the small jug must hold 1 litre. But then 5 small jugs can be filled.

If $x = 14$, then 8 litres remain, and the small jug must hold 2 litres. But then 4 jugs can be filled.

Finally, if $x = 13$, then 11 litres remain. Then since $11 = 3 \times 3 + 2$, each small jug must hold 3 litres,

hence (C).

22. *Alternative 1*

If the middle digit is 1, then the number must either have the form $\boxed{2}\boxed{}\boxed{1}\boxed{}\boxed{}$ or $\boxed{}\boxed{}\boxed{1}\boxed{}\boxed{2}$. In each case the 4 cannot be adjacent to either the 3 or the 5, so it must go between the 1 and the 2, and there are then 2 ways to place the 3 and the 5. Therefore there are 4 such numbers whose middle digit is 1. By symmetry there are also 4 such numbers whose middle digit is 5.

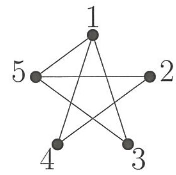
If the middle digit is 2, then the number must either have the form $\boxed{1}\boxed{}\boxed{2}\boxed{}\boxed{3}$ or $\boxed{3}\boxed{}\boxed{2}\boxed{}\boxed{1}$. In each case the 4 cannot be adjacent to the 3, so it must be between the 1 and the 2, and the position of the 5 is then determined. Therefore there are 2 such numbers whose middle digit is 2. By symmetry there are also 2 such numbers whose middle digit is 4.

If the middle digit is 3, then the number must either have the form $\boxed{2}\boxed{}\boxed{3}\boxed{}\boxed{4}$ or $\boxed{4}\boxed{}\boxed{3}\boxed{}\boxed{2}$. In each case the 1 must be between the 3 and the 4, and the position of the 5 is then determined. Therefore there are 2 such numbers whose middle digit is 3.

The total number of numbers with the required property is $4 + 4 + 2 + 2 + 2 = 14$,

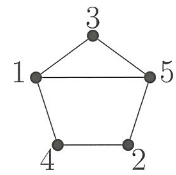
hence (B).

Alternative 2



In the first diagram, two digits are joined if they can be neighbouring digits in the number. The 5-digit numbers in the question correspond to paths that visit every digit exactly once.

The second diagram has the same edges, but is rearranged for clarity.



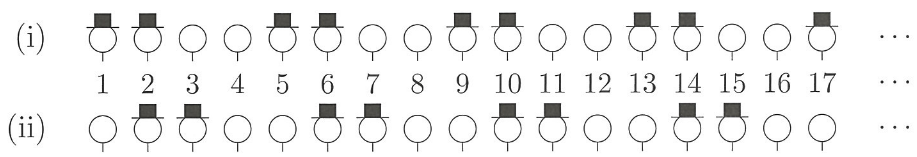
If edge 1–5 is not used, there are 10 possibilities, since there are 5 choices of starting digit, then 2 choices of second digit, and then all other digits follow.

If edge 1–5 is used, then there are 4 possibilities. These can be counted by choosing either edge 1–3 or edge 5–3, and then deciding whether the path will start or end at 3, since the rest of the path is determined by this. This gives four possibilities: 31524, 42513, 35142 and 24153.

In all there are $10 + 4 = 14$ possibilities,

hence (B).

23. Number the people $1, 2, 3, \dots$. Person 1 either (i) does wear a hat or (ii) does not. In either case, for persons 2, 3, 4, ..., we decide whether they have a hat by making sure that persons 1, 2, 3, ... have exactly one neighbour with a hat:



Each of these patterns repeats every 4 people, since if n is hatless then $n+2$ is hatted, and vice-versa.

From these patterns, persons $1, 3, 5, 7, \dots$ (odd numbers) may or may not have hats, persons $4, 8, 12, 16, \dots$ (multiples of 4) never have hats and persons $2, 6, 10, 14, \dots$ (even, not multiples of 4) always have hats.

Just as the second person must have a hat, so must the second-last. However, person 100 can't have a hat, so there can't be 101 people. The other answers 98, 99, 100 and 102 are all possible, since none of $97, 98, 99$ and 101 is a multiple of 4, so each will have a hat in either (i) or (ii) above,

hence (D).

24. *Alternative 1*

At each step, the person receiving lollies doubles their pile. Working backwards,

Josh	Ruth	Sam	
32	32	32	(Lollies at end)
16	16	64	(Josh and Ruth about to double to 32)
8	56	32	(Josh about to double to 16 and Sam to 64)
52	28	16	(Ruth about to double to 56 and Sam to 32)

hence (E).

Alternative 2

Suppose Josh starts with x lollies out of the 96 total.

Ruth and Sam together have $(96-x)$, so Josh gives away $(96-x)$ lollies, leaving $x-(96-x)=2x-96=2(x-48)$.

Ruth then gives Josh $2(x-48)$ more lollies so that Josh has $4(x-48)$.

Sam then gives Josh $4(x-48)$ more lollies so that Josh has $8(x-48)$ in the end.

Solving, $8(x-48)=32$, then $x-48=4$ and $x=52$,

hence (E).

25. *Alternative 1*

For convenience, represent the lines by symbols a, b, c or d so that lines use the same symbol if and only if they rhyme. Note that structures such as *abac* and *cdca* are not considered different since either one indicates that the first and third lines rhyme with each other, while neither of the second or fourth lines rhymes with any others. The following rules will generate a list of different rhyming structures:

1. The first letter must be a.
2. A letter can be used only if all of its predecessors in the alphabet have already been used.

The full list, in alphabetical order, is

aaaa, *aaab*, *aaba*, *aabb*, *aabc*, *abaa*, *abab*, *abac*, *abba*, *abbb*, *abbc*, *abca*, *abcb*, *abcc*, *abcd*

so there are 15 different rhyming structures in total. To count them more systematically, consider the five three-line poems:

$$aaa, aab, aba, abb, abc.$$

All four-line poems are constructed by appending a single letter to these, subject to rule 2 above. The 15 possibilities are

$$\begin{array}{lll} aaa & +a \text{ or } b & = 2 \text{ possibilities} \\ \left.\begin{array}{l} aab \\ aba \\ abb \end{array}\right\} & +a, b \text{ or } c & = 3\times 3 = 9 \text{ possibilities} \\ abc & +a, b, c \text{ or } d & = 4 \text{ possibilities} \end{array}$$

hence (B).

Alternative 2

Classify the rhyming patterns by the number of lines that rhyme:

(i) With all 4 lines rhyming, there is only one pattern 1

(ii) With 3 lines rhyming and one line not, the unrhymed line is line 1, 2, 3 or 4. So there are 4 patterns 4

(iii) With 2 pairs of 2 lines rhyming, the line that rhymes with line 1 is line 2, 3 or 4. So there are 3 patterns 3

(iv) With 2 lines rhyming and the other two not rhyming, there are 2 patterns for every pattern in (c). So there are 6 patterns 6

(v) If no lines rhyme, there is only 1 pattern 1

Then $1 + 4 + 3 + 6 + 1 = 15$,

hence (B).

26. Let N be the number abc. In the units column, $4c$ has units digit 2, so $c = 3$ or 8. If $c = 8$, then $18000 < 24N < 19000$. From the multiples of 24 listed below, we have $24 \times 700 = 16800$ and $24 \times 800 = 19200$ so that N must be in the 700s and $a = 7$. In the tens column, we must have $3 + 8 + 6 = 17$:

$$\begin{array}{cccccc}
 & & & 7 & b & 8 \\
\times & & & & 2 & 4 \\
\hline
 & & & & 3 & 2 \\
 & & & ? & ? & \\
 & & 2 & 8 & & \\
 & & & 1 & 6 & \\
 & & ? & ? & & \\
 & 1 & 4 & & & \\
\hline
 & 1 & 8 & b & 7 & 2 \\
\hline
\end{array}$$

n	$24n$	n	$24n$
1	24	6	144
2	48	7	168
3	72	8	192
4	96	9	216
5	120		

Then $4b$ ends in 8, so $b = 2$ or $b = 7$, but neither of these work.

If $c = 3$, then $13000 < 24N < 14000$. From the multiples of 24, $500 < N < 600$, so $a = 5$. Then in the tens column, $1 + 8 + 6 = 15$:

$$\begin{array}{cccccc}
 & & & 5 & b & 3 \\
\times & & & & 2 & 4 \\
\hline
 & & & & 1 & 2 \\
 & & & ? & ? & \\
 & & 2 & 0 & & \\
 & & & & 6 & \\
 & & ? & ? & & \\
 & 1 & 0 & & & \\
\hline
 & 1 & 3 & b & 5 & 2 \\
\hline
\end{array}$$

Again, $4b$ ends in 8 so that $b = 2$ or $b = 7$. Clearly $b = 2$ is too small, but $b = 7$ gives a solution: $573 \times 24 = 13752$. So $N = 573$ is the only solution,

hence (573).

27. (Also UP29, I24)
There are three cases for how the triangles are coloured:

(i) All three sides are the same colour, with 5 possibilities.
(ii) Two sides are the same and one side is different, with $5 \times 4 = 20$ possibilities.
(iii) All three sides are different, with $\dfrac{5 \times 4 \times 3}{6} = 10$ possibilities.

So there are $5 + 20 + 10 = 35$ possibilities in all,

hence (35).

Note: The calculation in (iii) relies on the observation that if we are choosing the sides in order, there are 5 possibilities for the first side, then 4 for the second, then 3 for the third. However, in these $5 \times 4 \times 3 = 60$ possibilities, each selection xyz will appear 6 times: $xyz, xzy, yxz, yzx, zxy, zyx$. This idea appears in the general formula for $\binom{n}{m}$, the number of ways of choosing m objects from n objects.

28. Let abc be the three-digit number. Then $100a + 10b + c = 37a + 37b + 37c$. This gives $63a = 27b + 36c$ and dividing by 9, $7a = 3b + 4c$. This gives many solutions where $a = b = c$, but it is a requirement that a, b and c are all different. Hence we are looking for $3b + 4c$ to be a multiple of 7 where $b \neq c$.

As we are looking for the largest, we try $a = 9$. Then $3b + 4c = 63$, which has solutions $(1, 15)$, $(5, 12)$, $(9, 9)$, $(13, 6)$, $(17, 3)$ and $(21, 0)$. None of these work.

Trying $a = 8$, then $3b + 4c = 56$, giving $(0, 14)$, $(4, 11)$, $(8, 8)$, $(12, 5)$ and $(16, 2)$. None of these work.

Trying $a = 7$, then $3b + 4c = 49$ giving $(3, 10)$, $(7, 7)$, $(11, 4)$ and $(15, 1)$. None of these work.

Trying $a = 6$, then $3b + 4c = 42$ giving $(2, 9)$, $(6, 6)$, $(10, 3)$ and $(14, 0)$. So the largest solution is 629,

hence (629).

Note: If you know or observe that $3 \times 37 = 111$, this gives an efficient way to search for solutions without the above algebra.

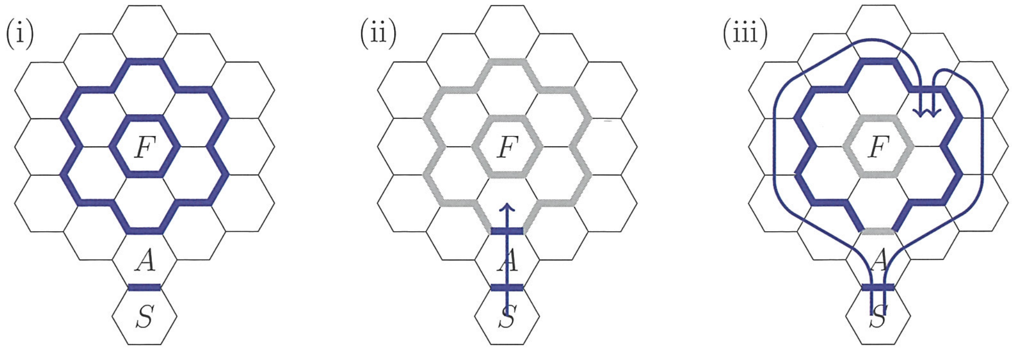
29. Since each term is the sum of the previous two, the amount of error follows the same pattern. Starting from the 89th term which is correct and the 90th term which has an error of 1, we have the following:

Term	89	90	91	92	93	94	95	96	97	98	99	100
Error	0	1	1	2	3	5	8	13	21	34	55	89

Note that we don't actually need to know whether the first error was above or below the correct value,

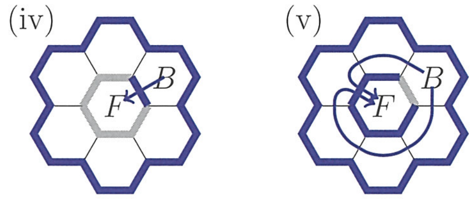
hence (89).

30. Call the levels 0 (paver S) up to 3 (column F). Diagram (i) shows the possible edges where I can step up.



There are 18 edges between level 1 and level 2. For the top edge of column A, diagram (ii) shows the only path from S that finishes by crossing that edge. Diagram (iii) shows that for the other 17 edges there are two paths. So there are 35 paths to level 2 that end by stepping across one of these edges.

Now if B is the level-2 column first stepped on, then of the 6 edges on column F, one has one path from B to F directly crossing that edge, and the other 5 have two paths, as shown in diagrams (iv) and (v) respectively. So there are 11 paths from B to F.



There are no restrictions on combining the path from S to B with a path from B to F, so the total number of paths is $35 \times 11 = 385$,

hence (385).

Solutions – Intermediate Division

1. $20 \times 16 = 320$,

hence (A).

2.

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{10 + 5 + 4}{20} = \frac{19}{20}$$

so that $\frac{1}{20}$ is shaded,

hence (A).

3. Between 11:15 am and 2:09 pm, there are $45 + 120 + 9 = 174$ minutes,

hence (B).

4. (Also J10)
Estimating, $\frac{720163}{2016} \approx \frac{720000}{2000} = \frac{720}{2} = 360$. This suggests that $100 < \frac{720163}{2016} < 1000$.
Checking, $201600 < 720163 < 2016000$ and so $100 < \frac{720163}{2016} < 1000$,

hence (D).

5. $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$,

hence (A).

6. *Alternative 1*

Let the number be x. Then $\frac{3}{4} \times \frac{1}{100} \times x = 6$, so $\frac{x}{400} = 2$ and $x = 800$,

hence (A).

Alternative 2

0.25% of the number is 2 (one-third of the given amount), so 1% of the number is 8 and 100% of the number is 800,

hence (A).

7. (Also J16)
If $A = 1$, then $A \times B + C \times D + E = B + E + C \times D$. On the other hand, if we swap A and E, we get $E \times B + C \times D + A = E \times B + 1 + C \times D$, which is larger, since both B and E are 2 or more. So the largest possible value can't have $A = 1$. Similarly, the largest possible value can't have any of B, C, or D equal to 1. Thus $E = 1$.
Then we only need to consider the following cases.

- $2 \times 3 + 4 \times 5 + 1 = 27$
- $2 \times 4 + 3 \times 5 + 1 = 24$
- $2 \times 5 + 3 \times 4 + 1 = 23$

Therefore, the largest possible value for the expression is 27,

hence (B).

8. (Also S3)
The perimeters of P, Q, R and S are $4\sqrt{2}$, 8, $2\sqrt{2}$ and 4 respectively,

hence (B).

9. *Alternative 1*
The equilateral triangle with corners 12, 4 and 8 has three angles of 60°, so the side from 12 to 8 is 60° from horizontal. The line from 9 to 3 is also horizontal, so the angle between the two lines is 60°,

hence (B).

Alternative 2
The line 1–7 is parallel to 12–8, and so makes the same angle with the line 3–9, due to the corresponding angle rule. Since both 1–7 and 3–9 pass through the centre of the clock, the angle between them is $\frac{2}{12} \times 360° = 60°$,

hence (B).

10. The maximum number of pens I can take without having at least one pen of each colour is $4 + 5 = 9$. This occurs if I take the 4 red pens and the 5 yellow pens. So I need to take 10 pens to be certain that I have at least one pen of each colour,

hence (C).

11. (Also S6)
Alternative 1
The sum of the exterior angles of the pentagon is $360 = 90 + 4 \times (180 - x)$, so that $180 - x = 270/4 = 67.5$ and $x = 180 - 67.5 = 112.5$,

hence (E).

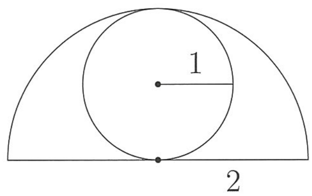
Alternative 2
The sum of the interior angles of the pentagon is $3 \times 180 = 90 + 4x$, so that $x = 450 \div 4 = 112.5$,

hence (E).

12. (Also S7)
If C is the point directly below A and to the left of B, then the right triangle ABC has sides 16 and 12, and hypotenuse x. Then $x^2 = 16^2 + 12^2 = 400$ and $x = 20$,

hence (A).

13. The area of the semicircle is $\frac{1}{2}\pi\, 2^2 = 2\pi$
The area of the circle is π.
Therefore the area not covered is $2\pi - \pi = \pi$,



hence (E).

14. We know $2^{10} = (2^2)^5 = 4^5$, so $4^{n+1} = 4^5$ and $n + 1 = 5$. Then $n = 4$,

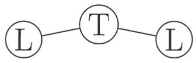
hence (C).

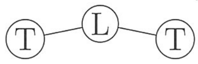
15. (Also UP23, J15)
As the number of points per event is 6 and the total number of points gained is $8 + 11 + 5 = 24$, there must have been 4 events. As Betty has 11 points she must have 3 first places and one second place. Cathy could not have won an event, or her score would be 6 or more, so she must have just one second place. So Adrienne must have two second places,

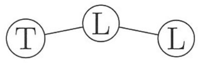
hence (C).

16. The number $\dfrac{2016}{N}$ will be the largest square factor of 2016. Factorising into primes, $2016 = 2^5 \times 3^2 \times 7 = 2 \times 7 \times (2^2 \times 3)^2$, so the largest square factor of 2016 is 12^2 and then $N = 14$,

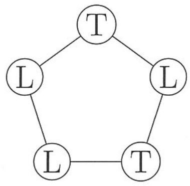
hence (A).

17. Anyone telling the truth is between two liars: 

Anyone lying is not between two liars—they are either between two truth-tellers or one liar and one truth-teller: 

or 

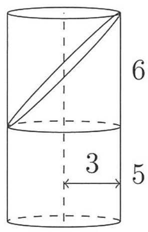
So there can't be two truth-tellers in a row nor three liars in a row.



Consequently, at least one person is telling the truth, and is between two liars. The remaining two people can't both be liars nor both truth-tellers, since then there would be four liars or two truth-tellers in a row. So they must be one liar and one truth-teller.
In all, there are three liars and two truth-tellers,

hence (C).

18. The liquid in the glass is equal to a cylinder of height 5 cm and half of a cylinder of height 6 cm.



Thus

$$V = \pi \times 3^2 \times 5 + \frac{1}{2} \times \pi \times 3^2 \times 6 = 45\pi + 27\pi = 72\pi \text{ cm}^3$$

Since $1\,\text{cm}^3 = 1\,\text{mL}$, the amount of water is 72π mL,

hence (D).

19. (Also S15)
Suppose that m is the average number of correct answers by the seven students whose marks weren't listed. Then we know that m is an integer and $10 \le m \le 20$. The average number of correct answers by all ten students is

$$\frac{8 + 8 + 9 + 7m}{10} = \frac{25 + 7m}{10}$$

So $25 + 7m$ is divisible by 10, which is possible only if m is an odd multiple of 5. However, $10 \le m \le 20$, so that $m = 15$.
Therefore, the average number of correct answers by all ten students is

$$\frac{25 + 7 \times 15}{10} = 13$$

hence (D).

20. In 100 revolutions of the pedals, the length of the first chain that passes over the gears is $100 \times 30 = 3000$ links, and so the 15-tooth gear revolves $3000 \div 15 = 200$ times.

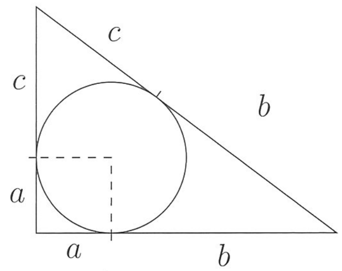
Then $200 \times 32 = 6400$ links of the second chain pass over the gears, and so the pump gear rotates $6400 \div 40 = 160$ times,

hence (A).

21. *Alternative 1*

Two tangents from a point to a circle have equal lengths, so

$$\begin{aligned} a+b &= 8 \\ a+c &= 6 \\ b+c &= \sqrt{6^2+8^2} = 10 \\ \\ 2a+2b+2c &= 24 \\ a+b+c &= 12 \end{aligned}$$

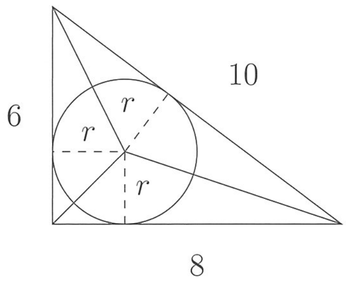


Then $a = 2$, $b = 6$ and $c = 4$. Due to the right angle, the radius of the circle is 2,

hence (B).

Alternative 2

The triangle has sides 6, 8 and 10 and area $\frac{1}{2} \times 6 \times 8 = 24$. It can be cut into three triangles of areas $4r$, $3r$ and $5r$ as shown.



Then $12r = 24$ and $r = 2$,

hence (B).

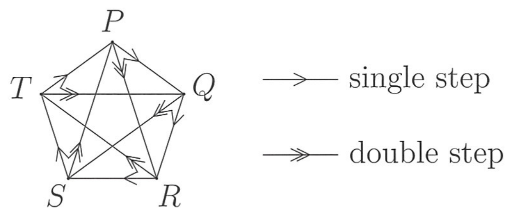
22. Here is the sequence:

Q			Q						Q

Moving from one letter to the next can only be done as in the diagram.

Counting the single steps as $+1$ and the double steps as $+2$, any circuit must add to a multiple of 5.

So to get from Q to Q in three steps (for the first 4 letters in the sequence) requires one single step and two double steps to add to 5.



There are three ways of doing this, $1+2+2$, $2+1+2$ and $2+2+1$, which are the triangles QRT, QST, QSP.

To get from Q to Q in 6 steps (for the remaining letters) the multiple of 5 must be 10, as the total is at least 6 and at most 12. Then there must be 2 steps that are $+1$ and 4 steps that are $+2$. These can be listed:

112222	121222	122122	122212	122221
211222	212122	212212	212221	221122
221212	221221	222112	222121	222211

So there are 15 ways of completing the second part of the sequence.
Alternatively, the combinatoric formula $\binom{6}{2} = 15$ can be used.
In all, the number of possible sequences is $3 \times 15 = 45$,

hence (D).

23. Since they arrived home 10 minutes earlier than usual, this has taken 5 minutes off each direction travelled by Alan. So Alan must have met Cynthia at 5:25 pm instead of 5:30 pm. As Cynthia started walking at 5:00 pm, she has walked for 25 minutes,

hence (C).

24. (Also UP29, J27)
There are three cases for how the triangles are coloured:

(i) All three sides are the same colour, with 5 possibilities.
(ii) Two sides are the same and one side is different, with $5 \times 4 = 20$ possibilities.
(iii) All three sides are different, with $\dfrac{5 \times 4 \times 3}{6} = 10$ possibilities.

So there are $5 + 20 + 10 = 35$ possibilities in all,

hence (A).

Note: The calculation in (iii) relies on the observation that if we are choosing the sides in order, there are 5 possibilities for the first side, then 4 for the second, then 3 for the third. However, in these $5 \times 4 \times 3 = 60$ possibilities, each selection xyz will appear 6 times: $xyz, xzy, yxz, yzx, zxy, zyx$. This idea appears in the general formula for $\binom{n}{m}$, the number of ways of choosing m objects from n objects.

25. A super-Fibonacci sequence is determined entirely by its first two terms, 1 and x, say. The sequence then proceeds as follows:

$$1,\ x,\ (1+x),\ 2(1+x),\ 4(1+x),\ 8(1+x),\ \ldots$$

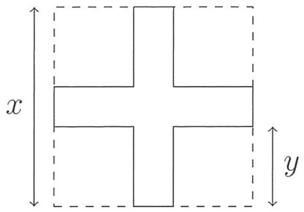
Since adding all previous terms amounts to doubling the last term, all terms from the third onwards are of the form $2^k(1+x)$, for some $k \geq 0$. If $2016 = 2^5 \times 63$ were one of these terms, then k will be one of the 6 values $k = 0, \ldots, 5$, giving 6 possible values for x. These are $x = 62, 125, 251, 503, 1007, 2015$. There is also the possibility that $x = 2016$, so there are 7 such sequences in total,

hence (D).

26. Each of the seven smaller triangles has area $a = \dfrac{2016}{7} = 288\,\text{cm}^2 = 2 \times 12^2$. The right isosceles triangle $\triangle BIE$ is made up of 4 of these, so its area is $4a = 8 \times 12^2 = \frac{1}{2} \times 48^2$ and its two equal sides are $BE = BI = 48\,\text{cm}$.
The area of $\triangle BHD$ is $\dfrac{2}{3}$ the area of $\triangle BHE$, so that $BD = \dfrac{2}{3}BE = 32\,\text{cm}$.
Then in $\triangle BDG$, let $x = BG$, then the area is $288 = \dfrac{1}{2} \times 32x$, so that $x = \dfrac{288}{16} = 18\,\text{cm}$,

hence (18).

27. The cross can be thought of as a large square with four equal small squares removed from its corners. Let the large square have side x and the smaller squares side y.



$$\begin{aligned} \text{Area} &= x^2 - 4y^2 = (x+2y)(x-2y) = 2016 \\ \text{Perimeter} &= 4x \end{aligned}$$

So $x + 2y$ and $x - 2y$ are factors of 2016 that differ by $4y$.

We want x to be as small as possible. This will occur when y is as small as possible, so the two factors of 2016 are as close together as possible.

The closest two are 42 and 48, but this does not give an integer value for y. Next best is 36 and 56, which gives $x + 2y = 56, x - 2y = 36$, so that $2x = 92$ and the perimeter is 184,

hence (184).

28. Because the mean of the 10 scores increased by 0.5 after the mistake was corrected, their sum increased by 5. No other score changed, so Malcolm's score increased by 5. The median of the 10 scores would not have changed if Malcolm's revised score had been 89 or lower or if his original score had been 92 or higher, so his original score must have been between 85 and 91, inclusive.

The possibilities can be summarised as follows:

original score	revised score	original median	revised median
85	90	89.5	90
86, 87, 88, 89	91, 92, 93, 94	89.5	more than 90
90	95	90	91
91	96	90.5	91

Only in the first and last cases does the median increase by 0.5. The sum of Malcolm's two possible correct scores is $90 + 96 = 186$,

hence (186).

29. (Also S26)

Let the formation have r rows and c columns, so the size of the band is rc.

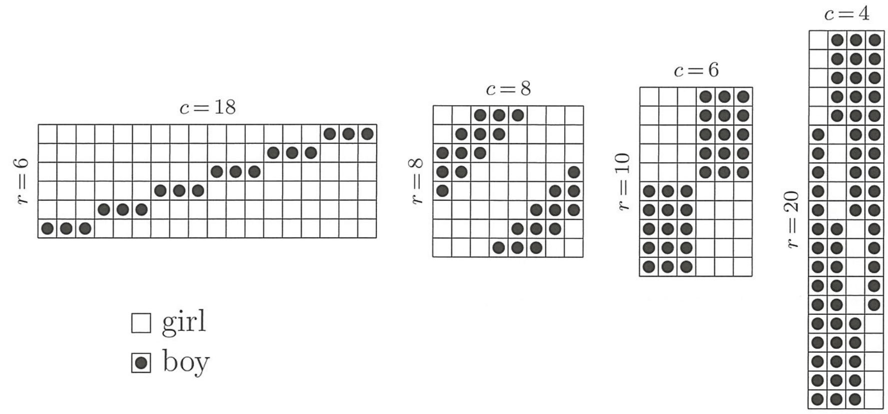
The band contains $3r$ boys and $5c$ girls, so the size of the band is also $3r + 5c$.

Then $rc - 3r - 5c = 0$, or equivalently, $(r - 5)(c - 3) = 15$.

The positive integer solutions for the ordered pair $(r - 5, c - 3)$ are $(1, 15)$, $(3, 5)$, $(5, 3)$, and $(15, 1)$. There are negative integer factorisations of 15, but these have $r \leq 0$ or $c \leq 0$.

The corresponding solutions for (r, c) are $(6, 18)$, $(8, 8)$, $(10, 6)$, and $(20, 4)$, and the corresponding values of rc are 108, 64, 60, and 80.

For each of these sizes, there is at least one arrangement of boys and girls:

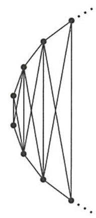


Therefore the sum of all possible band sizes is $108 + 64 + 60 + 80 = 312$,

hence (312).

30. (Also S29)

Alternative 1

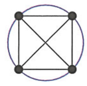


Draw the 64-gon and all 30 diagonals parallel to a fixed side, dividing it into 31 trapeziums. In each trapezium, draw both diagonals. This requires $64 + 30 + 2 \times 31 = 156$ chords. So the maximum number of chords is 156 or more.

In fact, the maximum is 156, as is shown below.

Firstly, for n points on a circle, where n is even, the same argument tells us that if M_n is the maximum number of chords, then $M_n \geq n + \frac{n}{2} - 2 + 2 \times (\frac{n}{2} - 1) = \frac{5}{2}n - 4$.

Claim: For all even values of n, $M_n = \frac{5}{2}n - 4$.

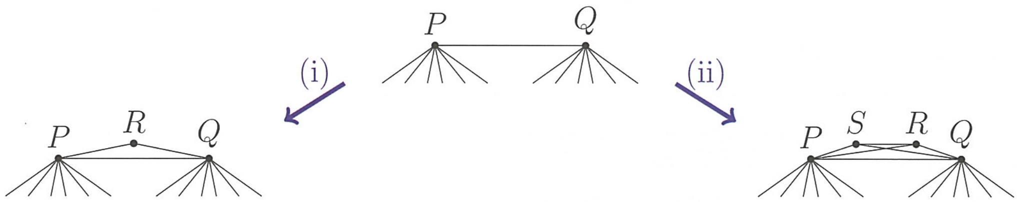


Clearly when $n = 4$, $M_n = 6 = \frac{5}{2} \times 4 - 4$, so the claim is true.

For a diagram with a chord PQ on the boundary, consider the following two possible steps:

(i) Between P and Q, add a point R and two chords PR and QR.

(ii) Between P and Q, add two points R, S and five chords PR, PS, QR, QS, RS.

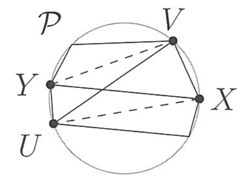


Step (ii) gives more chords per point, so a construction that starts with the 4-point diagram above and builds up using steps (i) and (ii) will have the greatest number of chords if it uses step (ii) as much as possible. For an even number of points, it will only use step (ii). Such a diagram will have $6 + 5 \times \frac{1}{2}(n - 4) = \frac{5}{2}n - 4$ chords.

The only uncertainty is whether a diagram with the maximum number of chords can be built from the $n = 4$ diagram using only steps (i) and (ii).

Suppose the maximum number of chords are drawn. Any chord in the diagram is either an edge of the outer n-gon, a crossed chord or an uncrossed chord. The uncrossed chords divide the n-gon into smaller polygons. If these were triangles and quadrilaterals then the n-gon could be built up using steps (i) and (ii), adding one polygon at a time.

Consider such a polygon $\mathcal{P}$ with each side an uncrossed chord and with 5 or more sides. Some diagonal XY must be a chord in the diagram, since the diagram has the maximum number of chords. Then XY must be crossed by another chord UV, or else the uncrossed chord XY would have split $\mathcal{P}$ into smaller polygons. Any chord passing through any edge of quadrilateral $XUYV$ would cross XY or UV, which would then be double-crossed, which is forbidden. So every edge of $XUYV$ crosses no chords, so it must be in the diagram (due to maximality) where it will be an uncrossed chord.



Now, since $\mathcal{P}$ has 5 or more sides, $XUYV \neq \mathcal{P}$ so at least one side of $XUYV$, say XU, is not a side of $\mathcal{P}$. Then XU is an uncrossed chord that splits $\mathcal{P}$ into smaller polygons, which cannot happen. Hence such a polygon $\mathcal{P}$ must have 4 or fewer edges.

In conclusion, any diagram with the maximum number of chords can be built from the 4-point diagram using steps (i) and (ii). When n is even, the most chords are obtained using only step (ii), which gives $M_n = \frac{5}{2}n - 4$. Consequently $M_{64} = 160 - 4 = 156$,

hence (156).

Alternative 2

As in the first solution, 156 chords are possible.

To see that this is the most, we first note a well-known result, known as *triangulation of a polygon*:

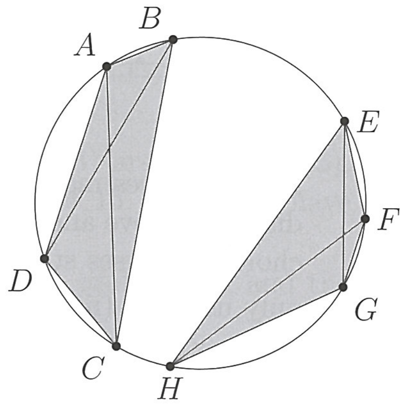
> When a polygon with n sides (n-gon) is cut into triangles, where each triangle's vertices are vertices of the original n-gon, there are $n-2$ triangles. Also, there are $n-3$ cuts, each along a diagonal of the n-gon.

A first consequence of triangulation is that the maximum number of non-intersecting diagonals that can be drawn inside an n-gon is $n - 3$.

Secondly, for triangles whose vertices are vertices of the original n-gon, the maximum number of non-overlapping triangles that can be drawn inside the n-gon is $n-2$. This is because we can add more triangles to get a triangulation.

Thirdly, for quadrilaterals whose vertices are vertices of the original n-gon, the maximum number of non-overlapping quadrilaterals that can be drawn inside the n-gon is $\dfrac{n-2}{2} = \dfrac{n}{2} - 1$. This is because each quadrilateral can be split into two non-overlapping triangles.

Returning to the question, for every pair of crossing chords AC and BD, shade in the quadrilateral $ABCD$. For two shaded quadrilaterals $ABCD$ and $EFGH$, neither diagonal AC or BD intersects EG or FH, so $ABCD$ and $EFGH$ do not overlap.



That is, the shaded quadrilaterals are non-overlapping, and so there are at most $\dfrac{n}{2} - 1 = 31$ of them. Thus there are at most 31 pairs of crossing chords.

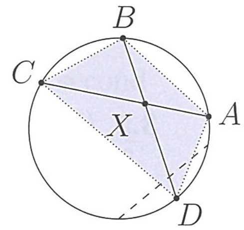
For each pair of crossing chords, remove one chord. There are at most 31 removed chords. The chords remaining have no crossings, and are either sides of the 64-gon (at most 64 of these) or diagonals of the 64-gon (at most 61 of these). Consequently the number of chords originally was at most $64 + 61 + 31 = 156$,

hence (156).

Alternative 3

As in the first solution, 156 chords can be drawn.

To see that 156 is the maximum, suppose the maximum number of chords are drawn—no more can be added. In particular, any possible chord that intersects no other drawn chord must be drawn. This includes all edges of the regular 64-gon. It also means that the only polygons that have all vertices on the circle and no chords inside are triangles, since otherwise a diagonal could be drawn.



When two of the chords AC and BD intersect at an interior point X, there are no other chords intersecting AC and BD, so no other chord will pass inside the quadrilateral $ABCD$. Consequently, the chords AB, BC, CD and DA do not intersect any other chords, so they must be included. Then $ABCD$ appears in the diagram as a *crossed quadrilateral*: all sides and both diagonals are drawn.

So that we can use Euler's formula $f + v = e + 2$, we consider the figure as a planar graph where the vertices include the 64 original points and the intersection points, and the edges include the chords that aren't cut by another chord and the two parts of the chords that are cut by another chord.

There are three types of faces: the exterior of the 64-gon, triangles that are part of a crossed quadrilateral, and triangles that have all vertices on the circle. Suppose there are q crossed quadrilaterals and t triangles. Then

- The number of vertices is $v = 64 + q$, the 64 initial vertices plus one for each crossed quadrilateral.
- The number of faces is $f = 1 + t + 4q$, the outside of the 64-gon, the t triangles, and 4 triangles for each crossed quadrilateral.
- The number of vertices is e where $2e = 64 + 3\,(t + 4q)$. This total is from adding the number of edges on each face, which counts each edge twice.
- The number of chords is $c = e - 2q = 32 + \frac{3}{2}t + 4q$, since for each crossed quadrilateral the number of edges is 2 more than the number of chords.

Then in Euler's formula $f + v = e + 2$:

$$\begin{aligned}
0 = (f + v) - (e + 2) &= (65 + t + 5q) - (34 + \frac{3}{2}t + 6q) \\
&= 31 - \frac{1}{2}t - q \\
c &= 32 + \frac{3}{2}t + 4(31 - \frac{1}{2}t) \\
&= 156 - \frac{1}{2}t
\end{aligned}$$

Hence the maximum number of chords drawn is 156, attained when $q = 31$ and $t = 0$, hence (156).

Solutions – Senior Division

1. They are $708, 718, 728, 738, 748, 758, 768, 778, 788, 798$,

hence (A).

2. $p^2 - 3q^2 = 7^2 - 3 \times (-4)^2 = 49 - 3 \times 16 = 1$,

hence (E).

3. (Also I8)
The perimeters of P, Q, R and S are $4\sqrt{2}$, 8, $2\sqrt{2}$ and 4 respectively,

hence (B).

4. Solving, $7n \geq 194$, and so $n \geq \dfrac{194}{7} = 27\frac{5}{7}$. So the smallest n can be is 28, and any value $n \geq 28$ is also a solution,

hence (C).

5. The table top is a circle with radius 3 metres, so its area in square metres is $\pi(3)^2 = 9\pi$. Because π is slightly greater than 3, 9π is slightly greater than 27. Of the given choices, 30 is closest,

hence (B).

6. (Also I11)
Alternative 1
The sum of the exterior angles of the pentagon is $360 = 90 + 4 \times (180 - x)$, so that $180 - x = 270/4 = 67.5$ and $x = 180 - 67.5 = 112.5$,

hence (E).

Alternative 2
The sum of the interior angles of the pentagon is $3 \times 180 = 90 + 4x$, so that $x = 450 \div 4 = 112.5$,

hence (E).

7. (Also I12)
If C is the point directly below A and to the left of B, then the right triangle ABC has sides 16 and 12, and hypotenuse x. Then $x^2 = 16^2 + 12^2 = 400$ and $x = 20$,

hence (A).

8. Square both sides of the equation to obtain:

$$\sqrt{x^2+1} = x + 2 \quad\Longrightarrow\quad x^2 + 1 = x^2 + 4x + 4 \quad\Longrightarrow\quad 4x = -3.$$

Hence, the only possible solution is $x = -\frac{3}{4}$. We substitute this value into the original equation and verify that both sides are indeed equal to $\frac{5}{4}$,

hence (B).

9. The probability that the first croissant is not chocolate is $\frac{2}{3}$. When the first croissant is not chocolate, the probability that the second croissant is not chocolate is slightly less than $\frac{2}{3}$. Consequently the probability that both croissants are not chocolate is slightly less than $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$,

hence (B).

Note: We can check that being *slightly less than* does not make the probability closer to the next smaller option, which is $\frac{1}{3}$. With 2 croissants of each type, the second probability is $\frac{3}{5}$, and the overall probability is $\frac{2}{5}$, which is closer to $\frac{4}{9}$ than it is to $\frac{1}{3}$. With more than 2 croissants of each type, the second probability will be even closer to $\frac{2}{3}$ and so the overall probability will be closest to $\frac{4}{9}$.

10. *Alternative 1*

When $n = 1$, the expressions evaluate to $A = 1$, $B = 3$, $C = 4$, $D = 4$ and $E = 3$. Only $B = n^3 + 2n$ and $E = n^2 + 2$ are possible. When $n = 2$, $B = 12$ and $E = 6$, and when $n = 3$, $B = 33$ and $E = 11$. So only B remains possible.

To see that B is always a multiple of 3, write $B = n(n^2-1)+3n = n(n-1)(n+1)+3n$. No matter what n is, one of n, $n-1$ or $n+1$ will be a multiple of 3, so that B is a multiple of 3,

hence (B).

Alternative 2

Options A, C, D and E can be eliminated as above.

To see that $B_n = n^3 + 2n$ is always a multiple of 3, note that

$$B_{n+1} - B_n = (n+1)^3 + 2(n+1) - n^3 - 2n = 3n^2 + 3n + 3$$

so once we know that $B_1 = 3$ is a multiple of 3, then so are all subsequent values,

hence (B).

11. $2^{2016} - 2^{2015} = 2 \times 2^{2015} - 2^{2015} = 2^{2015}$,

hence (C).

12. Suppose n trains stop at neither Oaklands (O) nor Brighton (B).

Then $45 - n$ stop at Oaklands but not Brighton.

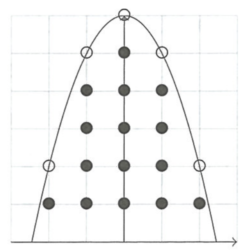
Then $60 - (45 - n) = 15 + n$ stop at Brighton but not Oaklands.

So $60 + 45 - n = 105 - n$ stop at Oaklands and $15 + n + n = 15 + 2n$ do not stop at Oaklands. The number of trains is $120 + n$ of which $24 + \frac{n}{5}$ stop at Oaklands. Hence $15 + 2n = 24 + \frac{n}{5}$, so $75 + 10n = 120 + n$ and $9n = 45$. Then $n = 5$,

	O	Not O
B	60	$15+n$
Not B	$45-n$	n
	$105-n$	$15+2n$

hence (D).

13. The parabola passes through grid points $(0, 6)$, $(\pm 1, 5)$ and $(\pm 2, 2)$ as shown. The number of grid points can be counted in columns, giving a total of $1+4+5+4+1 = 15$ grid points inside the shaded area,



hence (D).

14. In $\triangle SPQ'$, $\angle PSQ' = 90°$ and $PQ' = 2PS$, so that $\triangle SPQ'$ is a 30–60–90 degree triangle. Then $\angle SPQ = 60°$, $\angle QPQ' = 30°$, $\angle QPX = 15°$ and $\angle SPX = 75°$,

hence (E).

15. (Also I19)
Suppose that m is the average number of correct answers by the seven students whose marks weren't listed. Then we know that m is an integer and $10 \leq m \leq 20$. The average number of correct answers by all ten students is

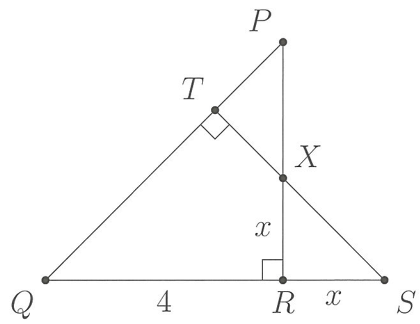
$$\frac{8+8+9+7m}{10} = \frac{25+7m}{10}$$

So $25 + 7m$ is divisible by 10, which is possible only if m is an odd multiple of 5. However, $10 \leq m \leq 20$, so that $m = 15$.
Therefore, the average number of correct answers by all ten students is

$$\frac{25 + 7 \times 15}{10} = 13$$

hence (D).

16. With all measurements in centimetres, we have $QR = PR = QT = TS = 4$, so that with $x = RS = RX$ as in the diagram, the area of $QRXT$ is $\frac{1}{2} \times 4^2 - \frac{1}{2}x^2 = 8 - \frac{1}{2}x^2$.
By Pythagoras' theorem, $PQ^2 = 4^2 + 4^2 = 32$ and $PQ = QS = 4\sqrt{2}$.
Then $x = 4\sqrt{2} - 4 = 4(\sqrt{2} - 1)$ so the area of $QRXT$ is



$$8 - \frac{1}{2}x^2 = 8 - 8(\sqrt{2}-1)^2 = 16(\sqrt{2}-1)$$

hence (A).

17. Adding the arithmetic progression,

$$S = x + 2x + \cdots + 100x = x(1 + 2 + \cdots + 100) = 5050x = 2 \times 5^2 \times 101x.$$

If S is a square, only even powers can occur in the prime factorisation, so that the smallest x can be is $2 \times 101 = 202$,

hence (A).

18. *Alternative 1*

If the radius of the original spheres is r, then the radius of the new ones is $0.8r$. The ratio of smaller to larger volumes is therefore

$$\frac{\frac{4}{3}\pi(0.8r)^3}{\frac{4}{3}\pi r^3} = (0.8)^3 = 0.512$$

If the ratio were exactly 0.5, there would be exactly 20 smaller spheres. Since the ratio is slightly more than 0.5, there will be 19 smaller spheres with some leftover gold.

Checking, $19 \times 0.512 = 9.728$, so that 19 of the smaller spheres, can be cast from the original 10 spheres,

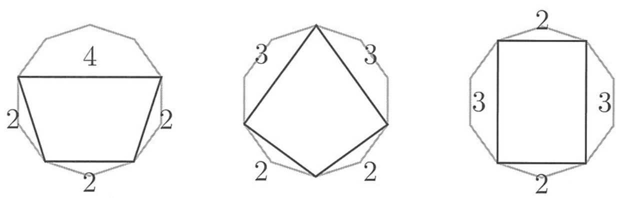
hence (E).

Alternative 2

Due to the way volume changes when all linear dimensions are scaled, the volume of a smaller sphere is $\left(\frac{4}{5}\right)^3 = \frac{64}{125}$ that of an original sphere. That is, each larger sphere is $\frac{125}{64}$ smaller spheres, and 10 larger spheres is $\frac{1250}{64} = \frac{625}{32} = 19\frac{17}{32}$ smaller spheres. So 19 smaller spheres can be made,

hence (E).

19. Each edge of the quadrilateral spans across 2 or more edges of the decagon. Going around the quadrilateral, label each edge with this number of decagon edges, giving 4 numbers (h, i, j, k), each 2 or more, with $h+i+j+k = 10$. By choosing a starting edge and direction, we can assume that h is the largest and that $i \geq k$. The possibilities are $(4, 2, 2, 2)$, $(3, 3, 2, 2)$ and $(3, 2, 3, 2)$, corresponding to these quadrilaterals:



trapezium kite rectangle

hence (B).

20. The sequence begins $1, 2, 4, 7, 10, 13, \ldots$. Ignoring 2, terms of the sequence are precisely those which are one more than a multiple of 3, as argued below. There are $2016 \div 3 = 672$ such numbers less than 2016 (taking care to include 1 instead of 2017), hence there are 673 such numbers in total, including the 2.

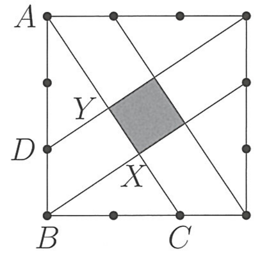
We now prove that the sequence consists of 2, together with all numbers of the form $3k+1$. Suppose that for $n \geq 2$, the first $n+1$ terms of the sequence consist of 2 and all numbers of the form $3k+1$ where $0 \leq k \leq n-1$. This is true for $n = 2$.

Write $m = 3n - 2$ so that $m \geq 4$. Since $1, 2$ and m are in the sequence, the next term is neither $m+1$ or $m+2$.

The number $m+3 = 3n+1$ is one more than a multiple of 3, and is greater than 4. However the possible sums of pairs of numbers are $2+2 = 4$, $2+(3k+1) = 3(k+1)$ and $(3k+1)+(3j+1) = 3(j+k)+2$. Of these, only 4 is one more than a multiple of 3, but it is less than $m+3$. So $m+3$ is not the sum of two terms in the sequence, and $m+3 = 3n+1$ must be the next term. By induction, term $(n+2)$ in the sequence is $3n+1$, for all $n \geq 2$,

hence (E).

21. *Alternative 1*



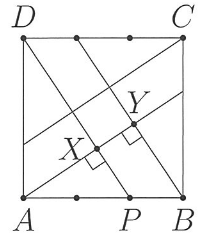
Let the outer square have side $3x$. Then $AC = \sqrt{9x^2 + 4x^2} = x\sqrt{13}$. Triangles $\triangle ABC$, $\triangle AXB$ and $\triangle AYD$ are similar, so $\dfrac{AX}{AB} = \dfrac{AB}{AC}$ and $AX = \dfrac{9x^2}{x\sqrt{13}} = \dfrac{9}{\sqrt{13}}x$.

Similarly $AY = \dfrac{6}{\sqrt{13}}x$ so that $XY = \dfrac{3}{\sqrt{13}}x$.

But $XY = 1$ so that $x = \dfrac{\sqrt{13}}{3}$. Then the side of the large square is $3x = \sqrt{13}$ and the area of the large square is 13 square units,

hence (D).

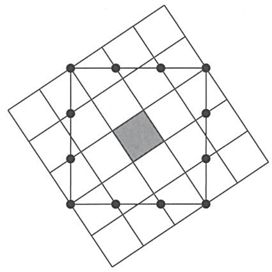
Alternative 2



Triangles $\triangle AXP$ and $\triangle AYB$ are similar with $AX = YB$ and $AP = \frac{2}{3}AB$. Hence $AX = \frac{2}{3}AY$ and $XY = \frac{1}{3}AY$.
But $XY = 1$ so that $AY = 3$ and $AX = 2$.
So $\triangle AYB$ has $AY = 3$ and $YB = 2$, so its area is $\dfrac{3 \times 2}{2} = 3$.
Square $ABCD$ consists of the central square and four triangles congruent to $\triangle AYB$, so its total area is $1 + 4 \times 3 = 13$ square units,

hence (D).

Alternative 3



Continue to draw parallels at equal spacing to the sides of the small square.
Then the grid passes through all the trisection points as shown, so the vertices of the large square lie on grid lines. By counting squares or subdividing areas, the area of the original large square is 13 square units,

hence (D).

22. *Alternative 1*
Let $n^2 + n + 34 = (n + k)^2 = n^2 + 2kn + k^2$ where $k > 0$ is an integer. Then $n = \dfrac{34 - k^2}{2k - 1}$, so if $k > 5$, then $n < 0$. The only values of k that result in integer values of n are 1, 2, 3, and 5. The corresponding values of n are 33, 10, 5, and 1, and the required sum is $33 + 10 + 5 + 1 = 49$,

hence (E).

Alternative 2
The expression $n^2 + n + 34$ is always even, and so suppose $(2a)^2 = n^2 + n + 34$ where $a > 0$. Completing the square,

$$\begin{aligned}
4a^2 &= \left(n + \tfrac{1}{2}\right)^2 - \tfrac{1}{4} + 34 \\
16a^2 &= (2n + 1)^2 + 135 \\
(4a)^2 - (2n + 1)^2 &= 135 \\
(4a + 2n + 1)(4a - 2n - 1) &= 3^3 \times 5
\end{aligned}$$

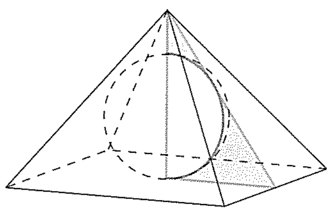
So 135 factorises into a product of two integers $x = 4a + 2n + 1$ and $y = 4a - 2n - 1$. Clearly $x > 0$ and $x > y$, and $y = \dfrac{135}{x} > 0$.
From the prime factorisation of $135 = 3^3 \times 5$, there are four solutions to $xy = 135$ where $x > y > 0$. From these values for $a = \dfrac{x + y}{8}$ and $n = \dfrac{x - y - 2}{4}$ can be found.

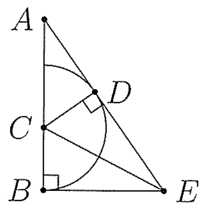
$x = 4a + 2n + 1$	$y = 4a - 2n - 1$	a	n
135	1	17	33
45	3	6	10
27	5	4	5
15	9	3	1
			49

From the table, the sum of the 4 possible values of n is 49,

hence (E).

23. Consider a cross-section made by a triangle through A, the apex, B, the centre of the base and E, the midpoint of one side of the square base. This includes C, the centre of the sphere and D, the point at which the sphere touches AE.





Then $BE = 1$, $AE = \sqrt{3}$ and $AB = \sqrt{2}$. Since EB and ED are both tangents, $EB = ED = 1$ so that $AD = \sqrt{3} - 1$. Since triangles $\triangle ABE$ and $\triangle ADC$ are similar, the radius of the sphere is

$$CD = AD \cdot \frac{BE}{AB} = \frac{\sqrt{3}-1}{\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{2}$$

hence (C).

24. If all 10 cards were green, then each card would have at least one adjacent card with a smaller number. This is not possible for the smallest number of the ten.
However, it is possible to have 9 green cards, and this can be done in many ways.
For instance, number the cards from $n = -4$ to $n = 5$, and write $100 - n^2$ on each card:

$$\boxed{84}\,\boxed{91}\,\boxed{96}\,\boxed{99}\,\boxed{100}\,\boxed{99}\,\boxed{96}\,\boxed{91}\,\boxed{84}\,\boxed{75}$$

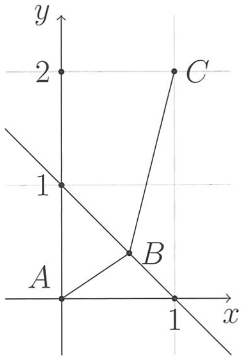
For $-4 \le n \le 4$, the parabola $y = f(n) = 100 - n^2$ is concave down, giving the required inequality. We can check that

$$\frac{f(n-1) + f(n+1)}{2} = \frac{100 - (n+1)^2 + 100 - (n-1)^2}{2} = 99 - n^2 = f(n) - 1$$

to confirm that 9 green cards is the largest possible number of green cards,

hence (E).

25. The value $\sqrt{x^2+(1-x)^2}$ is the distance in the coordinate plane between points $A=(0,0)$ and $B=(x,1-x)$. The value $\sqrt{(1-x)^2+(1+x)^2}$ is the distance between points $B=(x,1-x)$ and $C=(1,2)$.



So the task is to minimise $AB+BC$ where B lies on the line $y=1-x$. The minimum is clearly $AC=\sqrt{5}$, which occurs when ABC is a straight line,

hence (C).

26. (Also I29)

Let the formation have r rows and c columns, so the size of the band is rc.

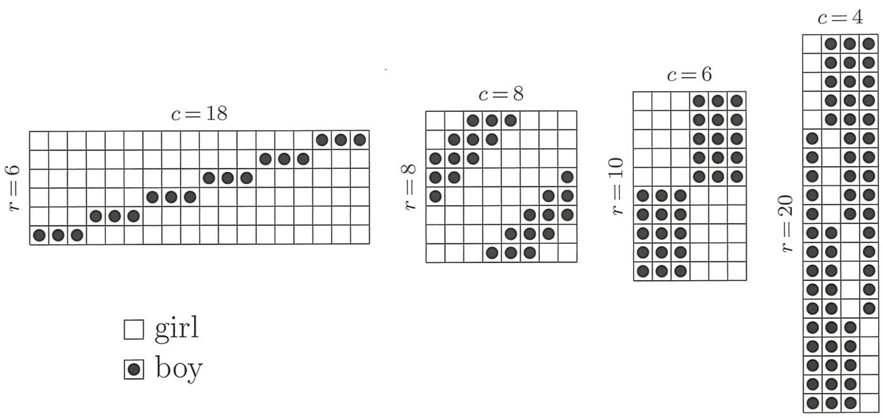
The band contains $3r$ boys and $5c$ girls, so the size of the band is also $3r+5c$.

Then $rc-3r-5c=0$, or equivalently, $(r-5)(c-3)=15$.

The positive integer solutions for the ordered pair $(r-5,c-3)$ are $(1,15)$, $(3,5)$, $(5,3)$, and $(15,1)$. There are negative integer factorisations of 15, but these have $r\le 0$ or $c\le 0$.

The corresponding solutions for (r,c) are $(6,18)$, $(8,8)$, $(10,6)$, and $(20,4)$, and the corresponding values of rc are 108, 64, 60, and 80.

For each of these sizes, there is at least one arrangement of boys and girls:



Therefore the sum of all possible band sizes is $108+64+60+80=312$,

hence (312).

27.

$$f(n)-f(m)=(an^2+bn+c)-(am^2+bm+c)$$
$$2016^2-0=(a(n+m)+b)(n-m)$$

Then $n-m$ is a positive divisor of 2016^2.

Moreover, for each positive divisor d of 2016^2, the values $m=0$, $n=d$, $a=0$, $b=2016^2/d$ show that d is a possible value of $n-m$.

The number of positive divisors of $2016^2 = 2^{10}\,3^4\,7^2$ is $11 \times 5 \times 3 = 165$, and so there are 165 possible values of $n - m$,

hence (165).

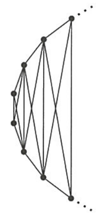
28. Rewriting the equation in index form, we have $a^{\sqrt{b}} = a^{b/2}$. If $a = 1$, then b can take any value from 1 to 100. If $a \neq 1$, then b must satisfy $\sqrt{b} = b/2$ and $b > 0$, so $b = 4$. Then a can take any value from 2 to 100. Hence there are 199 solutions in total,

hence (199).

29. (Also I30)

Alternative 1

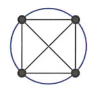
Draw the 64-gon and all 30 diagonals parallel to a fixed side, dividing it into 31 trapeziums. In each trapezium, draw both diagonals. This requires $64 + 30 + 2 \times 31 = 156$ chords. So the maximum number of chords is 156 or more.



In fact, the maximum is 156, as is shown below.

Firstly, for n points on a circle, where n is even, the same argument tells us that if M_n is the maximum number of chords, then $M_n \geq n + \frac{n}{2} - 2 + 2 \times (\frac{n}{2} - 1) = \frac{5}{2}n - 4$.

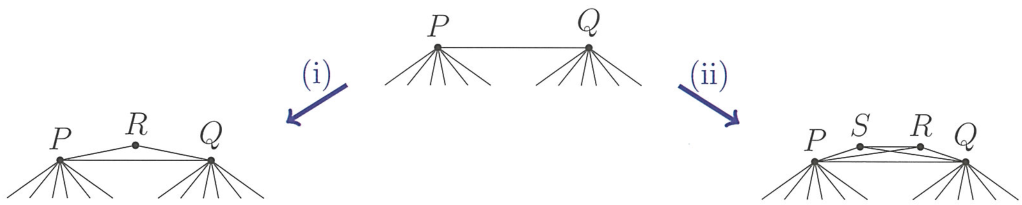
Claim: For all even values of n, $M_n = \frac{5}{2}n - 4$.



Clearly when $n = 4$, $M_n = 6 = \frac{5}{2} \times 4 - 4$, so the claim is true.

For a diagram with a chord PQ on the boundary, consider the following two possible steps:

(i) Between P and Q, add a point R and two chords PR and QR.
(ii) Between P and Q, add two points R, S and five chords PR, PS, QR, QS, RS.

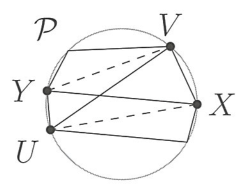


Step (ii) gives more chords per point, so a construction that starts with the 4-point diagram above and builds up using steps (i) and (ii) will have the greatest number of chords if it uses step (ii) as much as possible. For an even number of points, it will only use step (ii). Such a diagram will have $6 + 5 \times \frac{1}{2}(n - 4) = \frac{5}{2}n - 4$ chords.

The only uncertainty is whether a diagram with the maximum number of chords can be built from the $n = 4$ diagram using only steps (i) and (ii).

Suppose the maximum number of chords are drawn. Any chord in the diagram is either an edge of the outer n-gon, a crossed chord or an uncrossed chord. The uncrossed chords divide the n-gon into smaller polygons. If these were triangles and quadrilaterals then the n-gon could be built up using steps (i) and (ii), adding one polygon at a time.

Consider such a polygon $\mathcal{P}$ with each side an uncrossed chord and with 5 or more sides. Some diagonal XY must be a chord in the diagram, since the diagram has the maximum number of chords. Then XY must be crossed by another chord UV, or else the uncrossed chord XY would have split $\mathcal{P}$ into smaller polygons. Any chord passing through any edge of quadrilateral $XUYV$ would cross XY or UV, which would then be



double-crossed, which is forbidden. So every edge of $XUYV$ crosses no chords, so it must be in the diagram (due to maximality) where it will be an uncrossed chord.

Now, since $\mathcal{P}$ has 5 or more sides, $XUYV \neq \mathcal{P}$ so at least one side of $XUYV$, say XU, is not a side of $\mathcal{P}$. Then XU is an uncrossed chord that splits $\mathcal{P}$ into smaller polygons, which cannot happen. Hence such a polygon $\mathcal{P}$ must have 4 or fewer edges.

In conclusion, any diagram with the maximum number of chords can be built from the 4-point diagram using steps (i) and (ii). When n is even, the most chords are obtained using only step (ii), which gives $M_n = \frac{5}{2}n - 4$. Consequently $M_{64} = 160 - 4 = 156$,

hence (156).

Alternative 2

As in the first solution, 156 chords are possible.

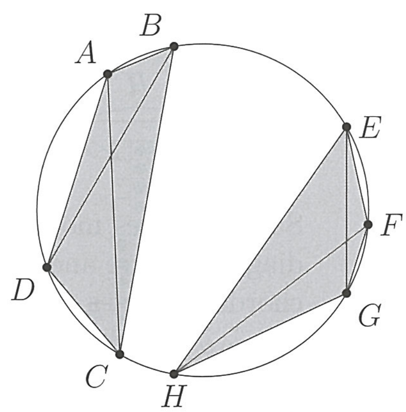
To see that this is the most, we first note a well-known result, known as *triangulation of a polygon*:

> When a polygon with n sides (n-gon) is cut into triangles, where each triangle's vertices are vertices of the original n-gon, there are $n-2$ triangles. Also, there are $n-3$ cuts, each along a diagonal of the n-gon.

A first consequence of triangulation is that the maximum number of non-intersecting diagonals that can be drawn inside an n-gon is $n-3$.

Secondly, for triangles whose vertices are vertices of the original n-gon, the maximum number of non-overlapping triangles that can be drawn inside the n-gon is $n-2$. This is because we can add more triangles to get a triangulation.

Thirdly, for quadrilaterals whose vertices are vertices of the original n-gon, the maximum number of non-overlapping quadrilaterals that can be drawn inside the n-gon is $\frac{n-2}{2} = \frac{n}{2} - 1$. This is because each quadrilateral can be split into two non-overlapping triangles.



Returning to the question, for every pair of crossing chords AC and BD, shade in the quadrilateral $ABCD$. For two shaded quadrilaterals $ABCD$ and $EFGH$, neither diagonal AC or BD intersects EG or FH, so $ABCD$ and $EFGH$ do not overlap.

That is, the shaded quadrilaterals are non-overlapping, and so there are at most $\frac{n}{2} - 1 = 31$ of them. Thus there are at most 31 pairs of crossing chords.

For each pair of crossing chords, remove one chord. There are at most 31 removed chords. The chords remaining have no crossings, and are either sides of the 64-gon (at most 64 of these) or diagonals of the 64-gon (at most 61 of these). Consequently the number of chords originally was at most $64 + 61 + 31 = 156$,

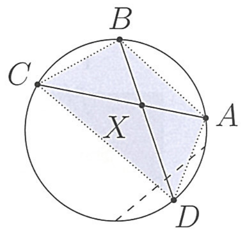
hence (156).

Alternative 3

As in the first solution, 156 chords can be drawn.

To see that 156 is the maximum, suppose the maximum number of chords are drawn—no more can be added. In particular, any possible chord that intersects no other drawn chord must be drawn. This includes all edges of the regular 64-gon. It also means that the only polygons that have all vertices on the circle and no chords inside are triangles, since otherwise a diagonal could be drawn.

When two of the chords AC and BD intersect at an interior point X, there are no other chords intersecting AC and BD, so no other chord will pass inside the quadrilateral $ABCD$. Consequently, the chords AB, BC, CD and DA do not intersect any other chords, so they must be included. Then $ABCD$ appears in the diagram as a *crossed quadrilateral*: all sides and both diagonals are drawn.



So that we can use Euler's formula $f + v = e + 2$, we consider the figure as a planar graph where the vertices include the 64 original points and the intersection points, and the edges include the chords that aren't cut by another chord and the two parts of the chords that are cut by another chord.

There are three types of faces: the exterior of the 64-gon, triangles that are part of a crossed quadrilateral, and triangles that have all vertices on the circle. Suppose there are q crossed quadrilaterals and t triangles. Then

- The number of vertices is $v = 64 + q$, the 64 initial vertices plus one for each crossed quadrilateral.
- The number of faces is $f = 1 + t + 4q$, the outside of the 64-gon, the t triangles, and 4 triangles for each crossed quadrilateral.
- The number of vertices is e where $2e = 64 + 3\,(t + 4q)$. This total is from adding the number of edges on each face, which counts each edge twice.
- The number of chords is $c = e - 2q = 32 + \frac{3}{2}t + 4q$, since for each crossed quadrilateral the number of edges is 2 more than the number of chords.

Then in Euler's formula $f + v = e + 2$:

$$\begin{aligned} 0 = (f + v) - (e + 2) &= (65 + t + 5q) - (34 + \frac{3}{2}t + 6q) \\ &= 31 - \frac{1}{2}t - q \\ c &= 32 + \frac{3}{2}t + 4(31 - \frac{1}{2}t) \\ &= 156 - \frac{1}{2}t \end{aligned}$$

Hence the maximum number of chords drawn is 156, attained when $q = 31$ and $t = 0$, hence (156).

30. *Alternative 1*

Observe that

$$\begin{aligned} f(4n+1) &= 4n+3 \\ f(4n+3) &= f(f(4n+1)) = 2(4n+1) \\ f(2(4n+1)) &= f(f(4n+3)) = 2(4n+3) \\ f(2(4n+3)) &= f(f(2(4n+1))) = 2^2(4n+1) \\ f(2^2(4n+1)) &= f(f(2(4n+3))) = 2^2(4n+3) \\ &\vdots \quad \vdots \\ f(2^k(4n+1)) &= 2^k(4n+3) \\ f(2^k(4n+3)) &= 2^{k+1}(4n+1) \end{aligned}$$

So that $f(2016) = f(63 \times 32) = f((4 \times 15 + 3) \times 2^5) = 61 \times 2^6 = 3904$, hence (904).

Alternative 2

Place the positive integers in the plane and draw arrows between numbers m and $f(m)$. So there are arrows $4n+1 \to 4n+3$ that link pairs of odd numbers, and also whenever $m \to x$, also $x \to 2m$. For instance, with $n = 2$ we get $4n+1 = 9$, $4n+3 = 11$ and then

$$\begin{tikzcd}
\boxed{9} \arrow[d] & \boxed{18} \arrow[d] & \boxed{36} \arrow[d] & \boxed{72} \arrow[d] & \boxed{144} \\
\boxed{11} \arrow[ur] & \boxed{22} \arrow[ur] & \boxed{44} \arrow[ur] & \boxed{88} \arrow[ur] &
\end{tikzcd}$$

Since $2016 = 63 \times 2^5$ where $63 = 2 \times 15 + 3$, we have

$$\begin{tikzcd}
\boxed{61} \arrow[d] & \boxed{122} \arrow[d] & \boxed{244} \arrow[d] & \cdots & \boxed{2^5 61} \arrow[d] & \boxed{2^6 61} \\
\boxed{63} \arrow[ur] & \boxed{126} \arrow[ur] & \boxed{252} & & \boxed{2^5 63} \arrow[ur] &
\end{tikzcd}$$

Then $f(2016) = 2^6 \times 61 = 3904$,

hence (904).

Alternative 3

We have $f(f(f(n))) = f(2n)$ but also $f(f(f(n))) = 2f(n)$, so that $f(2n) = 2f(n)$ and by iterating k times, $f(2^k n) = 2^k f(n)$. Hence $f(2016) = f(2^5\, 63) = 2^5 f(63)$. Also $f(63) = f(f(61)) = 122$ so that $f(2016) = 2^5 \times 122 = 3904$,

hence (904).

Answers

Question	Middle Primary	Upper Primary	Junior	Intermediate	Senior
1	C	D	E	A	A
2	D	D	D	A	E
3	D	C	E	B	B
4	C	B	E	D	C
5	E	C	A	A	B
6	D	D	A	A	E
7	E	E	C	B	A
8	E	D	C	B	B
9	D	A	D	B	B
10	C	E	D	C	B
11	A	A	B	E	C
12	A	D	D	A	D
13	D	A	A	E	D
14	B	D	C	C	E
15	A	E	C	C	D
16	A	B	B	A	A
17	B	E	E	C	A
18	D	C	B	D	E
19	E	A	B	D	B
20	D	B	C	A	E
21	C	B	C	B	D
22	D	B	B	D	E
23	B	C	D	C	C
24	B	D	E	A	E
25	B	E	B	D	C
26	45	14	573	18	312
27	7	32	35	184	165
28	8	251	629	186	199
29	251	35	89	312	156
30	28	832	385	156	904

Notes

CONTACT AMT

AMT EXECUTIVE DIRECTOR
Mike Clapper
Tel: 02 6201 2440
International Tel: +61 2 6201 2440
Fax: 02 6201 5096
International Fax: +61 2 6201 5096

AMT MANAGER
Helen Krig
Tel: 02 6201 2954
International Tel: +61 2 6201 2954
Fax: 02 6201 5052
International Fax: +61 2 6201 5052

POSTAL ADDRESS
Australian Mathematics Competition
University of Canberra
Locked Bag 1
Canberra GPO ACT 2601
AUSTRALIA

Tel: 02 6201 5136
International Tel: +61 2 6201 5136
Fax: 02 6201 5052
International Fax: +61 2 6201 5052
Email: amc@amt.edu.au
Web: www.amt.edu.au/events.html

COMPETITION ENQUIRIES
Tel: 02 6201 5136
International Tel: +61 2 6201 5136
Fax: 02 6201 5052
International Fax: +61 2 6201 5052