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B.Sc. 1st Semester (Honours) Examination-2022-23

MATHEMATICS

Course ID : 12112 Course Code : SH/MTH/102/C-2

Course Title : Algebra (New)

Time : 2 Hours *Full Marks* : 40

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Unit–I

1. Answer **any *five*** questions : 2×5=10

(a) Find the product of all the values of $\left(\sqrt{3}+i\right)^{\frac{3}{5}}$.

(b) Find the dimension of the Subspace S of $\mathbb{R}^4$ defined by $S=\left\{(x,y,z,w)\in\mathbb{R}^4 : x+2y-z=0,\ 2x+y+w=0\right\}$.

(c) Use Euclidean algorithm to find two integers u and v such that gcd $(13,80)=13u+80v$.

(d) Determine a Linear mapping $T:R^3\to R^3$ which maps the basis vectors $\{(0,1,1),(1,0,1)$ and $(1,1,0)\}$ to the vectors $\{(2,1,1),(1,2,1)$ and $(1,1,2)\}$ respectively.

(Turn Over)

(e) If α be an eigen value of an $n \times n$ idempotent matrix A, then show that α is either 0 or 1.

(f) examine if the relation p defined on $N \times N$ by "$(a,b)p(c,d)$ holds if and only if $ad = bc$" is reflexive, symmetric and transitive or not.

(g) Find the rank of the matrix $\begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$

(h) Find the number of real positive and negative roots of the equation $x^3-7x+7=0$, using Strum's theorem.

Unit–II

2. Answer any *four* questions : 5×4=20

(a) (i) Show that the product of any m consecutive integers is divisible by m.

(ii) Let $f, g : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 3x^2 - 5$ and $g(x) = \frac{x}{x^2+1}$. Find fog and gof. Also show that $g^{-1}(g(\{2,3\})) \neq \{2,3\}$. 2+3

(b) Determine the condition for which the system of equations has (i) unique solution (ii) no solution and (iii) many solutions :

$x + 2y + z = 1,\ 2x + y + 3z = b,\ x + ay + 3z = b + 1$

(c) (i) Find arg z where $z = 1 + i\tan\frac{3\pi}{5}$.

(ii) In n be a positive integer and $(1 + z)^n = p_0 + p_1 z + p_2 z^2 + \cdots + p_n z^n$. Prove that $p_0 - p_2 + p_4 - \cdots = 2^{n/2}\cos\frac{n\pi}{4}$ and $p_1 - p_3 - p_5 - \cdots = 2^{n/2}\sin\frac{n\pi}{4}$.

2+3

(d) (i) Let α, β, γ be the roots of the equation $x^3 + ax^2 + bx + c = 0$. Find the value of $\Sigma\alpha^2\beta^2$.

(ii) Solve the equation by Cardan's method : $x^3 - 27x - 54 = 0$. 2+3

(e) (i) Let A be a 3×3 matrix with eigen values are $1, -1, 0$. Find all eigen values of the matrix $I + A^{2022}$.

(ii) Find the algebraic and the geometric multiplicities of each eigen values of the matrix

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$$

1+4

(f) (i) If x, y, z be three unequal positive numbers, then show that $\left(\frac{y+z}{2}\right)^x \left(\frac{z+x}{2}\right)^y \left(\frac{x+y}{2}\right)^z < x^x y^y z^z$.

(ii) Show that $2^{73} + 14^3 \equiv 2 \bmod (11)$ 3+2

Unit–III

3. Answer **any *one*** question : $10\times1=10$

(a) (i) State Caley–Hamilton theorem for a matrix A. Using this theorem find A^{50}, where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

(ii) Show that every integer and its cube leave the same remainder when divided by 6.

(iii) Using the principle of Mathematical Induction, prove that $7^{2n} + 16n - 1$ is divisible by 64, for all $n \in N$. $(1+3)+3+3$

(b) (i) Apply Descarts rule of signs to find the nature of the roots of the equation $x^4 + 2x^4 + 3x - 1 = 0$.

(ii) Use principle of induction to prove that $2{\cdot}7^n + 3{\cdot}5^n - 5$ is divisible by 24 for all $n \in \mathbb{N}$.

(iii) Let the matrix of a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ relative to the ordered bases ((0,1,1), (1,0,1), (1,1,0)) of $\mathbb{R}^3$ be $\begin{pmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{pmatrix}$. Find T. Find the matrix of T relative to the ordered basis (2,1,1), (1,2,1), (1,1,2) of $\mathbb{R}^3$. $2+3+5$

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