Cardon's Method

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Ex – 5F

* Solution for Cubic Equation

10. (i) $x^3 - 27x - 54 = 0$ — (1)

Suppose, $x = u + v$

$\Rightarrow x^3 = (u+v)^3$

$\Rightarrow x^3 = u^3 + v^3 + 3uv(u+v)$

$\Rightarrow x^3 = u^3 + v^3 + 3uvx$

$\Rightarrow x^3 - 3uvx - (u^3+v^3) = 0$ — (2)

Comparing (1) & (2) we get -

$uv = 9$ $u^3 + v^3 = 54$

$\Rightarrow (uv)^3 = 729$

$\Rightarrow u^3 v^3 = 729$

So, u^3 & v^3 be the roots of -

$t^2 - (u^3+v^3)t + u^3v^3 = 0$

$\Rightarrow t^2 - 54t + 729 = 0$

$\Rightarrow t^2 - 2.27t + (27)^2 = 0$

$\Rightarrow (t-27)^2 = 0$

$\Rightarrow t = 27, 27$

$u^3 = 27$, $v^3 = 27$

$\Rightarrow u^3 = 3^3$

$\Rightarrow u^3 - 3^3 = 0$

$\Rightarrow (u-3)(u^2+3u+9) = 0$

$\Rightarrow u = 3$ | $u^2 + 3u + 9 = 0$

$u = \dfrac{-3 \pm \sqrt{9 - 4.1.9}}{2.1}$

$= \dfrac{-3 \pm \sqrt{-27}}{2}$

$= \dfrac{-3 \pm 3\sqrt{3}i}{2} = 3\left(\dfrac{-1 \pm \sqrt{3}i}{2}\right)$

$3 \cdot \dfrac{-1+i\sqrt{3}}{2} = 3\omega$

$3 \cdot \dfrac{-1-i\sqrt{3}}{2} = 3\omega^2$

$u = 3, 3\omega, 3\omega^2$ $\omega^3 = 1$ $1 + \omega + \omega^2 = 0$

$v = 3, 3\omega, 3\omega^2$

$x = u + v$ | As $uv = 9$

when, $u = 3$, $v = 3$ $x = 3 + 3 = 6$

" $u = 3\omega$ $v = 3\omega^2$ $x = 3(\omega + \omega^2)$

When, $u = 3$, $v = 3$ $\quad x = 3 + 3 = 6$

" $u = 3\omega$, $v = 3\omega^2$ $\quad x = 3(\omega + \omega^2) = -3$

" $u = 3\omega^2$, $v = 3\omega$ $\quad x = 3(\omega^2 + \omega) = -3$

So, the 3 roots of the eqn are → $6, -3, -3$

(ii) $x^3 - 9x + 28 = 0$ $\quad (-4, 2 \pm i\sqrt{3})$

(viii) $x^3 + 9x^2 + 15x - 25 = 0$

⇒ Let, put $x = y + h$: then the eqn transform to

$(y+h)^3 + 9(y+h)^2 + 15(y+h) - 25 = 0$

⇒ $y^3 + 3y^2h + 3yh^2 + h^3 + 9y^2 + 18yh + 9h^2 + 15y + 15h - 25 = 0$

⇒ $y^3 + (3h+9)y^2 + (3h^2 + 18h + 15)y + h^3 + 9h^2 + 15h - 25 = 0$

To remove the second term, we put $3h + 9 = 0$

⇒ $h = -3$

So, the eqn becomes -

$y^3 + (3(-3)^2 + 18(-3) + 15)y + (-3)^3 + 9(-3)^2 + 15(-3) - 25 = 0$

⇒ $y^3 - 12y - 16 = 0$

$u^3 = -2 + 2i$ $\qquad v^3 = -2 - 2i$

$r\cos\theta = -2$ $\quad r\sin\theta = +2$

$r^2 = 8$ $\qquad \theta = ?$

$r = 2\sqrt{2}$

$u^3 = r(\cos\alpha + i\sin\alpha)$

$u = r^{1/3}(\cos\alpha + i\sin\alpha)^{1/3}$

(iv) $x^3 - 3x - 2\cos A = 0$

⇒ $x = u + v$

$x^3 - 3uvx - (u^3 + v^3) = 0$

$uv = 1$ $\qquad u^3 + v^3 = 2\cos A$

$uv = 1 \qquad u^3+v^3 = 2\cos A$

$u^3v^3 = 1$

$$t^2 - 2\cos A + 1 = 0$$

$$\Rightarrow t = \frac{2\cos A \pm \sqrt{4\cos^2 A - 4}}{2}$$

$$= \frac{2\cos A \pm \sqrt{4(\cos^2 A - 1)}}{2} \qquad = \sqrt{4(-\sin^2 A)}$$

$$= 2\sin A \cdot i$$

$$= \frac{2\cos A \pm 2i\sin A}{2}$$

$$= \boxed{\cos A \pm i\sin A}$$

$$= x + iy$$

$$\cos\alpha + i\sin\alpha$$

$u^3 = \cos A + i\sin A \qquad v^3 = \cos A - i\sin A$

$$u = (\cos A + i\sin A)^{1/3}$$

$$= \{\cos(2k\pi + A) + i\sin(2k\pi + A)\}^{1/3}$$

$$= \cos\left(\frac{2k\pi + A}{3}\right) + i\sin\left(\frac{2k\pi + A}{3}\right)$$

$k = 0, 1, 2$

$$v = (\cos A - i\sin A)^{1/3} = \cos\frac{2k\pi + A}{3} - i\sin\frac{2k\pi + A}{3} \qquad ; \; k = 0, 1, 2$$

$$u + v = 2\cos\frac{2k\pi + A}{3} \qquad ; \; k = 0, 1, 2$$

$x = 2\cos A/3 \qquad k = 0$

$= 2\cos\frac{2\pi + A}{3} \qquad k = 1$

$= 2\cos\frac{4\pi + A}{3} \qquad k = 2$