$\mathbb{R}^2(\mathbb{R})$, $\mathbb{R}^3(\mathbb{R})$, $\dots$, $\mathbb{R}^n(\mathbb{R})$ v.s

(x_1, x_2) 2-tuple vector; (x_1, x_2, x_3) 3-tuple vector; $\dots$ $(x_1, x_2, \dots, x_n)$ n-tuple vector.

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⊛ Linear dependence & independence :-

$\Rightarrow$ Let, $S = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be a subset of $\mathbb{R}^n$. $\alpha_i \in \mathbb{R}^n$ $i = 1, 2, \dots, n$

S is said to be Linearly dependent if there exists scalars $c_1, c_2, \dots, c_n$ not all zero such that

$$c_1\alpha_1 + c_2\alpha_2 + \dots + c_n\alpha_n = \underbrace{(0, 0, \dots, 0)}_{(n \text{ times})} \quad \text{[null vector] (v.s)}$$

$10 = 2 \times 5$

$\mathbb{R}^2$, $(0,0)$

$\underset{\alpha}{(4,6)} = 2\underset{\beta}{(2,3)}$ $\Rightarrow \alpha = 2\beta$

$\Rightarrow \alpha - 2\beta = \theta = (0,0)$ (zero element)

$\Rightarrow 1\cdot\alpha + (-2)\cdot\beta = (0,0)$ (null vector)

$\mathbb{R}^3$, $\underset{\alpha}{(4,6,8)} = 2\underset{\beta}{(2,3,4)}$

$\alpha = 2\beta$

$\gamma = (5, 7)$

$\Rightarrow 1\cdot\alpha + (-2)\beta + 0\cdot\gamma = (0,0)$

$c_1\alpha + c_2\beta + c_3\gamma = (0,0)$

$S = \{\alpha, \beta, \gamma\}$

$c_1 = 1$, $c_2 = -2$, $c_3 = 0$

c_1, c_2, c_3 not all zero.

α, β, γ is L.D.

If S is not Linearly dependent (L.D) then S is called Linearly independent (L.I)

i.e If S is L.I then

$$c_1\alpha_1 + c_2\alpha_2 + \dots + c_n\alpha_n = (0, 0, 0, \dots, 0)$$

if and only $c_1 = c_2 = \dots = c_n = 0$ $\quad c_i \in \mathbb{R}$

Q-1 Let $\alpha = (1,1,2)$ and $\beta = (2,2,4) \in \mathbb{R}^3$. $S = \{\alpha, \beta\}$ is L.I / L.D ?

$\Rightarrow$ c, d be scalars from $\mathbb{R}$.

$c\alpha + d\beta = (0,0,0)$

$\Rightarrow c(1,1,2) + d(2,2,4) = (0,0,0)$

$\Rightarrow (c, c, 2c) + (2d, 2d, 4d) = (0,0,0)$

$\Rightarrow (c+2d, c+2d, 2c+4d) = (0,0,0)$

$c + 2d = 0$

$c = -2d$

$d \in \mathbb{R}$

$c \in \mathbb{R}$

$2c + 4d = 0$

$c + 2d = 0$

$d = 2 \quad c = -4$

$$\underset{\neq 0}{-4}(1,1,2) + \underset{\neq 0}{2}(2,2,4) = (0,0,0)$$

$$\underset{\neq 0}{-4}(1,1,2) + \underset{\neq 0}{2}(2,2,4) = (0,0,0)$$

S is $\underline{L.D}$

Q-2 Is $S = \{(1,0,0), (0,1,0), (0,0,1)\}$ L.D / L.I ?

$c_1, c_2, c_3 \in \mathbb{R}$ s.t

$c_1(1,0,0) + c_2(0,1,0) + c_3(0,0,1) = (0,0,0)$ //

if $c_1 = c_2 = c_3 = 0$ then S is L.I

c_1, c_2, c_3 not all zero then S is L.D.

$(c_1, c_2, c_3) = (0,0,0) \Rightarrow c_1 = c_2 = c_3 = 0$

S is $\underline{L.I}$

Q-3 $S = \{(0,1,1), (1,0,1), (1,1,0)\}$.

$\Rightarrow$ $c_1(0,1,1) + c_2(1,0,1) + c_3(1,1,0) = (0,0,0)$

$\Rightarrow (c_2 + c_3, c_1 + c_3, c_1 + c_2) = (0,0,0)$

$$\left.\begin{aligned} c_2 + c_3 &= 0 \\ c_1 + c_3 &= 0 \\ c_1 + c_2 &= 0 \end{aligned}\right\} \text{Homogeneous system.}$$

$\underline{(0,0,0)}$ - only ? ✓ / X

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -1(0-1) + 1(1-0) = 1 + 1 = 2 \neq 0$$

So, the system has a unique solⁿ. i.e $(0,0,0)$

So, $c_1 = c_2 = c_3 = 0$ is the only solⁿ.

Hence, S is $\underline{L.I}$

Q-4 $S = \{(2,3,1), (2,1,3), (1,1,1)\} \to$ L.D / L.I

$\Rightarrow$ $c_1(2,3,1) + c_2(2,1,3) + c_3(1,1,1) = (0,0,0)$

$$\begin{aligned} 2c_1 + 2c_2 + c_3 &= 0 \\ 3c_1 + c_2 + c_3 &= 0 \\ c_1 + 3c_2 + c_3 &= 0 \end{aligned}$$

$$\begin{vmatrix} 2 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 2(1-3) - 3(2-3) + 1(2-1)$$
$$= -4 + 3 + 1 = 0$$

So, the system has non-trivial sol^n.

That is c_1, c_2, c_3 not all zero.

Hence, S is L.D.

Q-5 $S = \{(1,2,3), (2,3,1), (3,1,2)\}$ L.D/L.I ? L.I $S \subseteq \mathbb{R}^3(\mathbb{R})$ 3

Q-6 $\{(1,1,1,0), (1,0,1,1), (1,2,1,2), (1,1,1,1)\}$ LD/LI ?

$\Rightarrow$ $c_1(1,1,1,0) + c_2(1,0,1,1) + c_3(1,2,1,2) + c_4(1,1,1,1) = (0,0,0,0)$

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$c_1 + c_2 + c_3 + c_4 = 0$

$c_1 + 2c_3 + c_4 = 0$

$c_1 + c_2 + c_3 + c_4 = 0$

$c_2 + 2c_3 + c_4 = 0$

$A = [\ ]_{4\times 4}$

Rank $A < 4$ = No. of unk.

inf $soln$. $\Rightarrow$ non-zero sol^n, c_1, c_2, c_3, c_4 not all zero.

S . L.D.

Basis :→ Let V be a v.s. over $\mathbb{R}$.

A subset $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ of V is said to be a basis of V, if

(i) S is L.I

(ii) $L(S) = V$.

No. of elements in the basis of a v.s (V) $V(\mathbb{R})$ is called its dimension. It is denoted by $\dim V$.

$\mathbb{R}^3$ $S = \{(1,0,0), (0,1,0), (0,0,1)\}$

(i) $S \to$ L.I

(ii) $L(S) = c_1(1,0,0) + c_2(0,1,0) + c_3(0,0,1)$

$= (c_1, c_2, c_3)$; $c_1, c_2, c_3 \in \mathbb{R}$

$= \mathbb{R}^3$

S is a basis of $\mathbb{R}^3$.

So, $\dim \mathbb{R}^3 = \underline{\underline{3}}$

* In a v.s $V(\mathbb{R})$. $\dim V$ is unique but basis is ~~may~~ not.

$S = \{(1,1,0), (1,0,1), (0,1,1)\} \to$ L.I

$L(S) = \mathbb{R}^3$.

* Let $\dim V = n \in \mathbb{N}$. Then any L.I subset S of V containing n vectors is a basis of V.

$\dim \mathbb{R}^3 = \underline{3}$, $\dim \mathbb{R}^4 = \underline{4}$, $\dim \mathbb{R}^5 = 5$, - -

$\dim \mathbb{R}^n = n$.