B. Sc. Semester II Examinations, 2023-24

Subject: Mathematics

Course ID: 22114 **Course Code: SH/MTH/203/GE-2**

Course Title: Algebra (NEW CBCS Syllabus 2022-25)

Time: 2 Hours. **Full Marks: 40**

The figures in the right-hand margin indicate marks.
Candidates are required to give their answer in their own words as far as practicable.

Notations and Symbols have their usual meaning.

1. Answer any 5 (five) of the following questions:

$(2 \times 5 = 10)$

a) If α, β, γ be the roots of the equation $x^3 + qx + r = 0$ then find the value of $\sum \frac{\alpha}{\beta}$.

b)) Find the value of $(\sqrt{3} - i)^{\frac{1}{7}}$.

c) If $S = a + b + c$, prove that $\frac{s}{s-a} + \frac{s}{s-b} + \frac{s}{s-c} > \frac{9}{2}$ where a, b, c are unequal positive real numbers.

d) Show that the mapping $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x, y) = (e^x, e^y)$ is not linear.

e) Find the rank of the matrix $\begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & 1 & -2 & 8 \\ 0 & 5 & 12 & -10 \end{pmatrix}$.

[Turn Over]

f) If a/b and a/c then prove that $a/(bx + cy)$ for any integers x and y.

g) Let $A = \{1,2, 3\}$. Verify whether the relation $R = \{(1,1),$ $(2,2), (3,3), (2,3), (3,2), (1,2), (2,1)\}$ is transitive.

h) If f: R→R and g: R→R be two mappings, then compute $f \circ g$ with usual notations where

$f(x) = x^2 + 3x + 2$ and $g(x) = 3x - 1 \; \forall \; x \in R$.

UNIT II

2. Answer any 4 (four) of the following questions:

$(5 \times 4 = 20)$

a) State De Moivre's theorem. Prove it for integral values of the index. $1 + 4$

b) If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, then obtain the equation whose roots are $\frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}$. Hence find the value of $\sum \frac{\alpha}{\beta+\gamma}$. $4 + 1$

c) Prove that the mapping $f: A \to B$ is invertible if and only if it is bijective. 5

d) i) Express the vector $(1, 7, -4)$ as a linear combination of the vectors $(1, -3, 2)$ and $(2, -1, 1)$ in the vector space R^3.

ii) For what value of c the set of vectors $\{ (1, 1, 2), (1, c, 3), (2,0, c)\}$ is linearly independent. $3 + 2$

e) Obtain the Sturm's function of the equation $x^3 + 11x^2 - 102x + 181 = 0$. 5

f) i) State the Descartes' rule of signs. Also find the nature of the roots of $x^4 + 15x^2 + 7x - 11 = 0$.

ii) Prove that $3^{2n-1} + 2^{n+1}$ is divisible by 7 $\forall\, n \in \mathbb{N}$. $2 + 3$

UNIT III

3. Answer any 1 (one) of the following questions:

$(10 \times 1 = 10)$

a) (i) If $a \equiv b\ (mod\ m)$ and $c \equiv d\ (mod\ m)$ then prove that $a + c \equiv b + d (mod\ m)$.

(ii) Find the rank of the matrix $\begin{pmatrix} 3 & 1 & 2 \\ 6 & 2 & 4 \\ 3 & 4 & 5 \end{pmatrix}$.

(iii) Determine the conditions for which the system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b$$

has no solution, unique solution & infinite no. of solutions.

$2 + 2 + 6$

b) (i) Find the dimension of the subspace of $S\ of\ \mathbb{R}^4$, where

$S = \{(x, y, z, w) \in \mathbb{R}^4 : x + 2y - z + 2w = 0\}$.

(ii) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$.

(iii) Solve the cubic equation $x^3 - 6x - 9 = 0$ by Cardan's method. $3 + 2 + 5$
