B. Sc. Semester II Examinations, 2023-24

Subject: Mathematics

Course ID: 22116 **Course Code: S/MTH/202/MN-2**

Course Title: Algebra [New Syllabus, NEP]

Time: 2 Hours. **Full Marks: 40**

The figures in the right-hand margin indicate marks.
Candidates are required to give their answer in their own words as far as practicable.

Notations and Symbols have their usual meaning.

Answer all the questions.

UNIT I

1. Answer any 5 (five) of the following questions:

$(2 \times 5 = 10)$

a) Show that $2^{2n+1} > 1 + (2n+1)2^n$, where n is natural number.

b) If the ratio $\frac{z-i}{z-1}$ is purely imaginary then show that the point z lies on the circle whose centre is at the point $\frac{1}{2}(1+i)$ and radius is $\frac{1}{\sqrt{2}}$.

c) If α, β, γ be the roots of $x^3 + px^2 + qx + r = 0$, find the value of $\sum\alpha^2\beta$.

[Turn Over]

d) Let $X = \{a, b, c\}$ be a non-empty set and $P(X)$ be the power set of X. Show that $(P(X), \leq)$ is a poset, where $A \leq B$ if and only if A is a subset of B.

e) Find the rank of the matrix A, where $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 1 & 4 & 4 \end{bmatrix}$.

f) Show that the eigen values of a diagonal matrix are the diagonal elements.

g) Show that $n(n+1)(n+2)$ is always divisible by 6, where n is positive integer.

h) Verify if the mapping $f: \mathbb{R}^3 \to \mathbb{R}^2$ defined by $f(x, y, z) = (3x - 2y + z, x - 3y - 2z)$ is a linear mapping or not.

UNIT II

2. Answer any 4 (four) of the following questions:

$(5 \times 4 = 20)$

a) (i) Show that the union of two equivalence relations is not necessarily an equivalence relation.

(ii) Determine the value of z when $z^6 = \sqrt{3} + i$. [3 + 2]

b) (i) Find the number and position of real roots of the equation $x^5 - 5x + 1 = 0$.

(ii) State the Descartes's rule of sign. [3 + 2]

c) State the Fundamental theorem of Algebra and show that every algebraic equation of degree 5 has 5 roots and no more.

[2 + 3]

d) (i) Solve $35x + 40y = 5$, for $x, y \in \mathbb{Z}$.

(ii) State the First Principle of mathematical induction.

[3 + 2]

e) State Cauchy-Schwarz inequality and apply it to prove that

$$(a_1b_1c_1 + a_2b_2c_2 + \cdots + a_nb_nc_n)^2$$
$$< (a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)(c_1^2 + c_2^2 + \cdots + c_n^2).$$

f) Find the characteristic roots of matrix and verify Cayley Hamilton theorem and hence find A^{-1}, where $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$.

UNIT III

3. Answer any 1 (one) of the following questions:

$(10 \times 1 = 10)$

a) (i) Solve $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$, by Ferrari's method.

(ii) A relation R on Z (set of all integers) is defined by R = {(a, b)∈ Z x Z: a – b is divisible by 7}. Show that R is an equivalence relation. Find all the distinct equivalence classes of the relation R.

[5 + 5]

b) (i) Solve the following system of linear equations, if possible:

$$x + y + z = 1$$
$$2x + y + 2z = 2$$
$$3x + 2y + 3z = 5$$

(ii) Find all the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

[5 + 5]
