BANKURA UNIVERSITY
B. Sc. Semester I (Hons) Examination 2017
MATHEMATICS

Subject Code : 12102 Course Code : UG/SC/102/C-02

Course Title : Algebra

Full Marks : 40 **Time : 2 Hours**

The figures in the right hand side margin indicate marks.

1. Answer <u>any five</u> questions : 2x5=10

a) If the sum of two roots of the equation $x^3+px^2+qx+r=0$ is zero, then show that $pq=r$

b) Express the complex number $-1-i$ in Polar form with Principal argument.

c) z is a variable complex number such that an amplitude of $\frac{z-i}{z+1}$ is $\pi/4$. Show that the point z lies on a circle in the complex plane.

d) Show that for any three real numbers a, b, c

$a^8+b^8+c^8 \geq a^2 b^2 c^2 (ab+bc+ca)$.

e) Let λ be an eigen value of an n x n matrix A. Show that λ^2 is an eigen value of A^2.

f) Is the following transformation linear?

$T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$T(x,y) = (x+y, 0), \forall x, y \in \mathbb{R}$

g) Suppose R and S are two equivalence relations on a nonempty set A. Verify whether $R \cup S$ and $R \cap S$ are equivalence relations on A.

h) Use the theory of congruence to find the remainder when the sum $1^3+2^3+3^3+.....+99^3$ is divided by 3.

2. Answer <u>any four</u> questions : 5x4=20

a) i) If α, β, γ be the roots of the equation $x^3+px^2+qx+r=0$, find the equation whose roots are $\alpha\beta+\beta\gamma,\ \beta\gamma+\gamma\alpha,\ \gamma\alpha+\alpha\beta$. 3

ii) Find the minimum number of complex roots of the equation $x^7-3x^3+x^2=0$. 2

b) Determine the values of a and b for which the system 2+2+1

$$x+y+z=1$$
$$x+2y-z=b$$
$$5x+7y+az=b^2$$

has no solution, only one solution, many solutions respectively.

c) If

$$A=\begin{pmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{pmatrix},$$

then find three eigen vectors of A which are linearly independent. 5

d) i) If a is prime to b then prove that a^2+b^2 is prime to a^2b^2. 2

ii) Prove that the product of k consecutive integers is divisible by k. 3

e) A relation β is defined on $\mathbb{Z}$ by "$x\ \beta\ y$ if and only if x^2-y^2 is divisible by 5" for $x, y, \in \mathbb{Z}$. Prove that β is an equivalence relation on $\mathbb{Z}$ and find the distinct equivalence classes. 5

f) Let V(F) be a vector space and $S \neq \phi$ be a finite subset of V. Then prove the set W of all linear combinations of the elements of S forms a subspace of V. Also show that W is the smallest subspace of V containing S. 3+2

3. Answer either (a) or (b) : 10 x 1 = 10

a) i) Show that the sets $A=\{x\in\mathbb{R} : 0\leq x\leq 1\}$ and $B=\{x\in\mathbb{R} : a\leq x\leq b\}$ have same cardinal number. 3

ii) If $|z|=1$ and amp $z=\theta$, $(0<\theta<\pi)$ then find the modulus and principal amplitude of $\frac{2}{1+z}$. 4

iii) Prove that for $n>3$, the integers n, $n+2$, $n+4$ cannot be all primes. 3

b) i) State and prove the Cauchy - Schwarz inequality. 6

ii) Verify Caley - Hamilton theorem for the matrix 4

$$A=\begin{pmatrix}0 & 0 & 1\\ 3 & 1 & 0\\ -2 & 1 & 4\end{pmatrix}.$$ Hence find A^{-1}.