B.Sc. Semester-II Examination, 2022-23

MATHEMATICS [Honours]

Course ID : 22114 **Course Code : SH/MTH/203/GE-2**

Course Title : Algebra

[NEW SYLLABUS]

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT-I

1. Answer any **five** from the following questions: 2×5=10

   a) Find the number of positive and negative roots of the equation $x^5 - x^4 + x^3 + 8x^2 + 2x - 2 = 0$.

   b) Prove that if $x^2 = e$ for all element of a group G, then G is a commutative.

   c) Z is a Variable complex number such that an amplitude of $\frac{Z-i}{Z+1}$ is $\frac{\pi}{4}$. Show that the point Z lies on a circle in the complex plane.

d) Show that for any three real number a, b, c, $a^8 + b^8 + c^8 \geq a^2b^2c^2(ab + bc + ca)$.

e) Let W be the subspace of $\mathbb{R}^3$ defined by $W = \{(x, y, z) \in \mathbb{R}^3 : 2x + 2y + z = 0, 3x + 3y - z = 0, x + y - 3z = 0\}$. Find the dimension of W.

f) If a is prime to b, prove that a^2 is prime to b and b^2.

g) If α, β and γ be the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$.. Find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$.

h) The eigen values of the matrix A are 5 and 3. Find the eigen values of the matrix $A^2 - 8A + 15I + 30A^{-1}$.

UNIT-II

2. Answer any **four** from the following questions:

5×4=20

a) i) Let $\rho = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid 3a + 4b = 7n,$ for some $n \in \mathbb{Z}\}$. Show that ρ is an equivalence relation.

ii) Use Principle of mathematical induction to prove that $2.7^n + 3.5^n - 5$ is divisible by 24 for all $n \in \mathbb{N}$. 3+2=5

b) i) Let $a_i > -\frac{1}{3}$, $(i = 1, 2, 3)$ and $a + b + c = 1$.

Apply Cauchy-Schwarz inequality to prove that $\sqrt{3a+1} + \sqrt{3b+1} + \sqrt{3c+1} \leq 3\sqrt{2}$.

ii) Find the minimum value of $3x + 2y$ where x,y are positive real numbers satisfying the condition $x^2y^3 = 48$. 3+2=5

c) If α, β, γ be the roots of the equation $x^3 - px^2 + r = 0$, find the equation whose roots are $\frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}, \frac{1}{\beta^2} + \frac{1}{\gamma^2} - \frac{1}{\alpha^2}, \frac{1}{\gamma^2} + \frac{1}{\alpha^2} - \frac{1}{\beta^2}$.

d) If H and K are subgroups of the group $(G, 0)$ then HK is subgroup of G, iff $HK = KH$.

e) Reduce the reciprocal equation $x^5 - 6x^4 + 7x^3 + 7x^2 - 6x + 1 = 0$ to its standard form and solve it.

f) Find the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$ to the vectors (2, 0, 0), (0, 2, 0), (0, 0, 2) of $\mathbb{R}^3$ respectively. Find $Ker\ T$ and $Im\ T$. Show that $\dim Ker\ T + \dim Im\ T = \dim \mathbb{R}^3$. 3+2=5

UNIT-III

3. Answer any **one** of the following questions:

$10\times1=10$

a) i) Let $f:\mathbb{R}\to S=\{x\in\mathbb{R}:1<x<1\}$ defined by $f(x)=\frac{x}{1+|x|}, x\in\mathbb{R}$. Find f^{-1}.

ii) Apply Descartes rule of signs to ascertain the minimum number of complex roots of the equation $x^6-3x^2-2x-3=0$.

iii) Find a matrix P and a diagonal matrix D such that $P^{-1}AP=D$, where $A=\begin{pmatrix}1&1&-2\\-1&2&1\\0&1&-1\end{pmatrix}$.

$3+2+5=10$

b) i) If α,β,γ be the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum\alpha^3\beta^3$.

ii) Show that $Log(1+i)^3\neq3Log(1+i)$.

iii) Show that the ratio of the principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is $\sin(\log2)+i\cos(\log2)$.

$4+2+4=10$