B. Sc. Semester II Examinations, 2023-24

Subject: Mathematics

Course ID: 22118 **Course Code: SP/MTH/201/C1-B**

Course Title: Algebra (NEW CBCS Syllabus)

Time: 2 Hours. **Full Marks: 40**

The figures in the right-hand margin indicate marks.
Candidates are required to give their answer in their own words as far as practicable.

Notations and Symbols have their usual meaning.

1. Answer any 5 (five) of the following questions:

$(2 \times 5 = 10)$

a) Prove that the sum of 99th powers of the roots of the equation $x^7 = 1$ is 0.

b) Find the value of $\emptyset(2401)$.

c) If $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots\cdots + \frac{1}{n}$, then show that $(1+n)S_n > 2n$.

d) A relation R is defined on $\mathbb{Z}$ by aRb iff $ab < 0 \forall a, b \in \mathbb{Z}$. Verify whether R is an equivalence relation.

e) If n is odd then determine the exact number of real roots of the equation $x^n - 1 = 0$.

f) Define linear transformation from one space to another. When is it called zero transformation?

[Turn Over]

g) What is the fundamental theorem of arithmetic?

h) Prove that a set of vectors containing the zero vector can't be linearly independent.

UNIT II

2. Answer any 4 (four) of the following questions:

$(5 \times 4 = 20)$

a) State Cayley-Hamilton's theorem. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$. $2 + 3$

b) For what value of k the system of linear equations

$$\begin{aligned} x + y + z &= 2 \\ 2x + y + 3z &= 1 \\ x + 3y + 2z &= 5 \\ 3x - 2y + z &= k \end{aligned}$$

is solvable and then solve it.

c) Find a basis for R^3 that contains the vectors $(2, 4, 0)$ *and* $(2, 6, 2)$. $2 + 3$

d) Two mappings $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x^2$ and $g(x) = x - 2 \;\forall\, x \in R$. Then verify that $g \circ f \neq f \circ g$ with usual notations. 5

e) Prove that $\sqrt[n]{i} + \sqrt[n]{-i} = 2\cos\frac{\pi}{2n}$ where n is a positive integer. 5

f) Solve $x^3 - 3x + 2 = 0$ by Cardan's method. 5

UNIT III

3. Answer any 1 (one) of the following questions:

$(10 \times 1 = 10)$

a) (i) If $\alpha + i\beta(\beta \neq 0)$ is a root of the equation $x^3 + qx + r = 0$ then find the other roots of it.

(ii) What do you mean by $a \equiv b(mod\ m)$? If $a \equiv b(mod\ m)$ and $b \equiv c(mod\ m)$then prove that $a \equiv c(mod\ m)$.

(iii) Solve the bi-quadratic

$x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ given that the sum of two roots of it is equal to the sum of the other two roots.

$2 + 2 + 6$

b) (i) If V & W are two sub-spaces of R^n and $T: V \to W$ is a linear transformation then prove that $T(\theta_V) = \theta_W$.

(ii) Use method of induction to establish $2n + 1 \leq 2^n\ \forall\ n \geq 3, n \in \mathbb{N}$.

(iii) Reduce the matrix $A = \begin{pmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 6 \end{pmatrix}$ to normal form and hence find $Rank(A)$.

$3 + 2 + 5$
