B.Sc. Semester-II Examination, 2022-23

MATHEMATICS [Programme]

Course ID : 22118 Course Code : SP/MTH/201/C-1B

Course Title : Algebra

[NEW SYLLABUS]

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT-I

1. Answer any **five** from the following questions: 2×5=10

a) Let a, b, c be three arbitrary elements of a group $(G,*)$, If $a*c=b*c$, then show that a = b.

b) Show that $3n(3n+1)^2 > 4((3n)!)^{\frac{1}{n}}$.

c) State the Descartes' rule of signs.

d) Simplify: $(1-i)\left(1-\frac{1}{i}\right)$

e) Solve the equation $x^3-3x^2+4=0$ two of its roots being equal.

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f) If a | c and b | c with gcd(a, b)=1, then prove that ab | c

g) Express -1-i in polar form.

h) Define order of an element in a group. In the group $(Z_6, +)$, find $o(\bar{1}), o(\bar{4})$ and $o(\bar{5})$.

UNIT-II

2. Answer any **four** from the following questions:

$5\times4=20$

a) i) If a, b, c are positive real numbers, then show that $a^3+b^3+c^3 \geq 3abc$

ii) If a_1, a_2, a_3, a_4, a_5 be positive real numbers, then prove that

$$\left(\frac{a_1+a_2+a_3+a_4+a_5}{5}\right)^5 \geq \left(\frac{a_1+a_2}{2}\right)^2\left(\frac{a_3+a_4+a_5}{3}\right)^3$$

2+3

b) i) Show that

$\sinh(x+iy) = \sinh x\cos y + i\cosh x\sin y.$

ii) Expand $\cos^7\theta$ in a series of cosines of multiples of θ. 2+3

c) i) Prove that the intersection of any two sub-groups of a group (G, *) is again a sub-group of (G, *).

ii) Give an example with justification to show that the union of two sub-groups of a group need not be a sub-group of that group.

3+2

d) i) Find the quotient and the remainder when $(3x^7 - x^6 + 31x^4 + 21x + 5)$ is divided by $(x+2)$.

ii) Apply Descartes' rule of signs to find the nature of roots of the equation $x^4 + 16x^2 + 7x - 11 = 0$. 2+3

e) i) If n be any positive integer, then prove that n (n+1)(n+2) is divisible by 6.

ii) Show that the square of an odd integer is of the form (8k+1). 2+3

f) Solve $x^3 - 18x - 35 = 0$ by Cardan's method.

UNIT-III

3. Answer any **one** of the following questions:

10×1=10

a) i) State the fundamental theorem of algebra.

ii) Find the roots of the equation $Z^8 = 1$

iii) If x, y, z are positive real numbers and $x+y+z = 1$, then prove that

$$8xyz \le (1-x)(1-y)(1-z) \le \frac{8}{27}.$$ 1+4+5=10

b) i) Show that $(Z_5, +)$ is a group.

ii) Find the condition that the cubic equation $x^3 - px^2 + qx - r = 0$ should have its roots in G.P.

iii) Find the remainder when $1! + 2! + 3! + \ldots + 100!$ is divisible by 16. $3+3+4=10$
