B.Sc. 3rd Semester (Programme) Examination, 2023-24

MATHEMATICS

Course ID : 32118 **Course Code : SP-MTH-301/C-1C**

Course Title : Algebra

[Syllabus - 2017]

Time : 2 Hours Full Marks : 40

The figures in the right margin indicate marks. Candidates are required to answer in their own words as far as practicable.

UNIT-I

1. Answer ***any five*** from the following questions: $5\times2=10$

a) State fundamental theorem of classical algebra.

b) Prove that $\dfrac{(n+1)^n}{2^n} > n!$

c) Find the values of i^i.

d) Show that the function $f: R \rightarrow R$ defined by $f(x) = |x| + x$ is neither injective nor surjective.

e) Show that the relation p defined on R by the rule "xpy iff $x + y$ is irrational" is not an equivalence relation.

f) Use Cayley-Hamilton theorem to compute A^{-1} where,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

g) Determine the rank of the matrix A^2 where

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 3 & 0 \\ 6 & 2 & 3 \end{bmatrix}$$

h) Find the dimension of the subspace W of R^3 defined by

$$W = \{(x, y, z) \in R^3 : x + y + z = 0\}$$

UNIT-II

2. Answer ***any four*** from the following questions.

$4 \times 5 = 20$

a) If one root of the equation $x^3 + ax + b = 0$ is twice the difference of the other two, then prove that one root is $\dfrac{13b}{3a}$.

b) i) Show that $1!.3!.5!.....(2n-1)! > (n!)^n$.

ii) Prove that number of primes is infinite.

3+2

c) How many different relations can be defined on a set with n elements? How many of these are reflexive? Give reason. 3+2

d) Find a linear operator T on R^3 such that Ker T is the subspace
$U = \{(x, y, z) \in R^3 : 2x + y - z = 0\}$ of R^3.

e) Find two integers u and v satisfying $63u+55v = 1$. 3+2

f) If α be an eigen value of a real orthogonal matrix A, then prove that $\frac{1}{\alpha}$ is also an eigen value of A.

UNIT-III

3. Answer ***any one*** from the following questions: $1\times10=10$

a) i) Prove that the eigen values of a real skew-symmetric matrix are purely imaginary or zero.

ii) Find the least positive residue in $2^{41} \pmod{23}$. 5+5=10

b) i) If $(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots$, prove that

$$a_0 + a_4 + a_8 + \ldots = 2^{n-2} + 2^{\frac{n-2}{2}} \cos\frac{n\pi}{4}$$

ii) A mapping $T : R^3 \to R^3$ is defined by

$$T(x,y,z) = (x+y+z, 2x+y+2z, x+2y+z), (x,y,z) \in R^3$$

Show that T is a linear mapping. Find Ker T and the dimension of Ker T.

5+5=10
