B.Sc. 3rd Semester (Honours) Examination, 2023-24

MATHEMATICS

Course ID : 32114 **Course Code : SH-MTH-304/GE-3**

Course Title : Algebra

[Syllabus - 2017]

Time : 2 Hours Full Marks : 40

The figures in the right margin indicate marks.
Candidates are required to answer in their own words as far as practicable.
Symbols and notations have their usual meaning.

UNIT-I

1. Answer ***any five*** from the following questions: $5\times2=10$

a) Find the values of $(-i)^{\frac{2}{3}}$.

b) Apply Descartes' rule of sign to examine the nature of the roots of the equation

$$x^6+4x^4+2x^2+4x+1=0\cdot$$

c) Construct an equivalence relation on the set $A=\{a, b, c\}$.

d) Show that the function $f:R\to R$ defined by $f(x)=|x|+x$ is neither injective nor surjective.

e) For any non empty set S, prove that $S\times S$ is an equivalence in S.

f) Show that W is a subspace of $\mathbb{R}^4$ defined by $W = \{(x, y, z, w) : x, y, w \in R, 3x + y + z + 2w = 0\}$.

g) Let λ be an eigen value of an idempotent matrix A. Show that λ is 1 or 0.

h) Find the rank of the matrix A^2, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

UNIT-II

2. Answer ***any four*** from the following questions:

$4 \times 5 = 20$

a) If

$\cos\alpha + \cos\beta + \cos\gamma = 0$, $\sin\alpha + \sin\beta + \sin\gamma = 0$

then prove that

(i) $\sin(\alpha+\beta)+\sin(\gamma+\beta)+\sin(\gamma+\alpha) = 0$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$.

3+2=5

b) i) Show that $1!.3!.5!...(2n-1)! > (n!)^n$.

ii) Prove that number of primes is infinite.

3+2=5

c) For what values of a the following system of equations is consistent? Solve in that case.

$$\begin{aligned} x - y + z &= 1 \\ x + 2y + 4z &= a \\ x + 4y + 6z &= a^2 \end{aligned}$$ 5

d) Solve by Cardan's method $x^3 + 3x^2 - 3 = 0$.

e) Use Cayley-Hamilton theorem to compute A^{2023} where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$ 5

f) The matrix of $T : R^3 \to R^3$ relative to the order basis $\{0, 1, 1), (1, 0, 1), (1, 1, 0)\}$ is $\begin{pmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{pmatrix}$. Find the linear transformation T. Find the matrix of T relative to the ordered basis $\{(2, 1, 1), (1, 2, 1), (1, 1, 2)\}$. 3+2

UNIT-III

3. Answer ***any one*** from the following questions: 1×10=10

a) i) Find the least positive residue in 2^{41} (*mod* 23).

ii) A mapping $T : R^3 \to R^3$ is defined by

$T(x, y, z) = (x + y + z, 2x + y + 2z, x + 2y + z), (x, y, z) \in R^3$.

Show that T is a linear mapping. Find Ker T and the dimension of ker T.

5+5=10

b) i) If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find all eigenvalues of A and obtain the all eigenvectors corresponding to all eigenvalues.

ii) If α, β, γ are the roots of the equation $x^3 + 3x^2 + 10 = 0$, then prove that $\alpha^4 + \beta^4 + \gamma^4 = 201$.

iii) Show that $3x^5 - 4x^2 + 6 = 0$ has at least two imaginary roots. 5+3+2

2025/03/03 11:53