B.SC. THIRD SEMESTER (HONOURS) EXAMINATIONS, 2022

Subject: Mathematics **Course ID: 32114**

Course Code: SH/MTH/304/GE-3

Course Title: Algebra

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning.

1. Answer *any five* of the following questions: **2x5=10**

a) If z is any Complex number where $z = x + iy$, then prove that

$|x| + |y| \leq \sqrt{2}\,|z|$.

b) Prove that$2^n > 1 + \sqrt{2^{n-1}}\, n$.

c) Using mathematical induction prove that there are 2^n subsets of a set of n elements.

d) Find the equation whose roots are the squares of the roots of the equation

$x^4 - x^3 + 2x^2 - x + 1 = 0$.

e) How many symmetric relations can be defined on a set with n elements?

f) Two sets A and B contain 5 and 7 elements respectively. Find the total number of mappings from A to B. How many of them are injective?

g) If A is a non-singular matrix of order 4, then determine the rank of the matrix A^4.

h) Find all the real values of x for which the rank of the matrix A is 2, where

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 3 & x \\ 1 & 1 & 6 & x+1 \end{pmatrix}.$$

2. Answer *any four* of the following questions: **5x4=20**

a) Using De Moiver's theorem prove that

$\sin^4\theta\cos^2\theta = \frac{1}{32}[\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2]$.

b) Ifa, b, c are three positive numbers and $a + b + c = 1$, then prove that,

$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2+\left(c+\frac{1}{c}\right)^2 \geq 33\frac{1}{3}$

c) Let $\alpha, \beta, \gamma, \delta$ be the roots of the equation, $x^4+px^3+qx^2+rx+s=0$,

such that $\alpha\beta+\gamma\delta=0$. Then prove that $p^2s+r^2-4qs=0$.

d) Determine the conditions for which the system of equations has (a) only one solution, (b) no solution, (c) many solutions

$$x+y+z=b, \qquad 2x+y+3z=b+1, \qquad 5x+2y+az=b^2.$$

e) If $U = L\{(2,0,1),(3,1,0)\}$ and $W = L\{(2,0,1),(3,1,0)\}$, then find $\dim(U+W)$ and $\dim(U \cap W)$.

f) Find the eigen values and the corresponding eigen vectors of the real matrix

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

3. Answer *any one* of the following questions: **10x1=10**

(a) (i) Find the locus of the point z given by, $amp\left\{\frac{z-1}{z+2}\right\} = \pm\frac{\pi}{2}$.

(ii) Find the greatest number K for which $5x+7y=K$ has exactly 9 solutions in non-negative integers.

(iii) Correct or justify the statement: 'Every relation is a mapping but every mapping is not a relation'. 4+4+2=10

(b) (i) Solve the system of equations

$x+2y+z=1,\ 3x+y+2z=3,\ x+7y+2z=1.$

(ii) If x, y, z are three positive numbers, such that $x^2+y^2+z^2=27$, then prove that $x^3+y^3+z^3 \geq 81$.

(ii) If α, β, γ are the roots of the equation $x^3+3x^2+10=0$, then prove that

$\alpha^4 + \beta^4 + \gamma^4 = 201.$ 4+3+3=10

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