B.SC. FIRST SEMESTER (HONOURS) EXAMINATIONS, 2022

Subject: Mathematics **Course ID: 12112**

Course Code: SH/MTH/102/C-2

Course Title: Algebra

Full Marks: 40 **Time: 2 Hours**

The figures in the margin indicate full marks

Notations and symbols have their usual meaning.

1. Answer any five questions: **2 × 5 = 10**

(a) Let the roots of the equation $x^3 + px^2 + qx + r = 0$ be α, β, γ. Find the equation whose roots are

$\alpha - \frac{\beta\gamma}{\alpha}$, $\beta - \frac{\gamma\alpha}{\beta}$ and $\gamma - \frac{\alpha\beta}{\gamma}$.

(b) If $\{\alpha, \beta, \gamma\}$ forms abasis of a subspace, then show that $\{\alpha + \beta + \gamma, \beta + \gamma, \gamma\}$ also forms a basis of that subspace.

(c) Determine the eigen values of the matrix A=$\begin{pmatrix} a & h & g \\ 0 & b & 0 \\ 0 & c & c \end{pmatrix}$.

(d) Let a function $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 3x - 1 & for\ x > 3 \\ 2x^2 & for - 2 < x \leq 3 \\ 3x^2 - 7 & for\ x \leq -2. \end{cases}$

Find $f^{-1}(5)$.

(e) If f, g, h are three functions from $\mathbb{Z}\ to\ \mathbb{Z}$, such that $(z) = n^2$, $g(n) = n + 1$ and $h(n) = n - 1$. Find $g \circ f \circ h$, $f \circ g \circ h$ and $h \circ f \circ g$.

(f) Solve the equation $x^3 - 7x^2 + 36 = 0$, given that one of its roots is double of another.

(g) Find the smallest positive residue in $2^{41} (mod\ 23)$.

(h) If a, b, c are any three integers such that $gcd(a, c) = 1$ and $gcd(b, c) = 1$, then show that $gcd(ab, c) = 1$.

2. Answer any four questions: **5× 4 = 20**

(a) Find all the values of $(i)^{1/2} + (-i)^{1/2}$.

(b) If α, β, γ are the roots of the equation$x^3 - x^2 + ax + b = 0$ and β, γ, δ are the roots of the equation $x^3 - 4x^2 + mx + n = 0$, and also $\alpha, \beta, \gamma, \delta$ are in A.P. , then show that

$$b = m + n - 3.$$

(c) If a, b, c are three positive real numbers such that $abc = 1$, then prove that

$$\frac{1 + ab}{1 + a} + \frac{1 + bc}{1 + b} + \frac{1 + ca}{1 + c} \geq 3$$

(d) Prove that the product of any three consecutive integers is divisible by 6.

(e) Prove by using Mathematical Induction that $3^{2n} - 8n - 1$ is divisible by 64.

(f) Find all real values of z for which the rank of the matrix $\begin{pmatrix} 1+z & 2 & 3 & 4 \\ 1 & 2+z & 3 & 4 \\ 1 & 2 & 3+z & 4 \\ 1 & 2 & 3 & 4+z \end{pmatrix}$

is less than 4.

3. _Answer any one question:_ **10 x 1 = 10**

(a) (i) Find the general solutions of $\sin z = 2i$.

(ii) Find the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ which transforms the basis vectors $(1,2)$ and $(0,1)$ to $(3,-1,5)$ and $(2,1,-1)$.

(iii) Obtain the characteristic equation of the matrix

$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and verify that A satisfies this equation. Hence find A^{-1}.

(3+2)+5

(b)(i) Express $(1+i)^{-i}$ in the form of $A + iB$.

(ii) For which values of 'k' the following system of equations have a solution?

$$x + y + z = 1, x + 2y + 4z = k, x + 4y + 10z = k^2$$

and solve them completely for each value of 'k'.

5+5

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