B.Sc. 1st Semester (Honours) Examination, 2023-24

MATHEMATICS

Course ID : 12112 **Course Code : SH-MTH-102/C-2**

Course Title : Algebra

[Old Syllabus-2017]

Time : 2 Hours Full Marks : 40

The figures in the right margin indicate marks.

Candidates are required to answer in their own words as far as practicable.

UNIT-I

1. Answer ***any five*** from the following questions: 5×2=10

   a) Show that $-1, i, -i, \frac{1}{2}(\sqrt{2}+i\sqrt{2})$ are concyclic.

   b) Prove that $2^n > 1+\sqrt{2^{n-1}}\,n$.

   c) How many symmetric relations can be defined on a set with n elements?

   d) Find all the real values of x for which the rank of the matrix A is 2, where

$$A=\begin{bmatrix} 2 & 1 & 4 \\ 1 & x & 2 \\ 4 & 0 & x+2 \end{bmatrix}.$$

   e) Show that for any three real number a, b, c $a^8+b^8+c^8 \geq a^2b^2c^2(ab+bc+ca)$.

f) Find mod Z and amp Z when $z = (1 + i)^7$.

g) Prove that $3^{2n-1} + 2^{n+1}$ is divisible by 7, using principle of mathematical induction.

h) Show that the set

$S = \{(x, y, z) : x, y, z \in \mathbb{R}, 2x + y - z = 0\}$ is a subspace of $\mathbb{R}^3$.

UNIT-II

2. Answer ***any four*** from the following questions:

4×5=20

a) If a, b, c are three positive numbers and a+b+c = 1, then prove that,

$$8abc \le (1-a)(1-b)(1-c) \le \frac{8}{27}.$$

b) Let α, β, γ be the roots of the equation, $x^3 + px^2 + qx + r = 0$, Then find the value of $\left(\frac{1}{\beta}+\frac{1}{\gamma}-\frac{1}{\alpha}\right)\left(\frac{1}{\gamma}+\frac{1}{\alpha}-\frac{1}{\beta}\right)\left(\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}\right)$.

c) Determine the conditions for which the system of equations has (a) only one solution, (b) no solution, (c) many solutions.

$$x + y + z = b,\ 2x + y + 3z = b + 1,\ 5x + 2y + az = b^2.$$

d) Find the condition for the equation $X^3 + 3HX + G = 0$ to have three distinct real roots.

e) Stat and Prove the Cauchy-Schwarz inequality.

f) Define rank of a matrix. find the row reduced echelon form of the matrix.

$$\begin{pmatrix} 1 & -1 & 2 & 0 & 4 \\ 2 & 2 & 1 & 5 & 2 \\ 1 & 3 & -1 & 0 & 3 \\ 1 & 7 & -4 & 1 & 1 \end{pmatrix}$$ and find its rank.

UNIT-III

3. Answer ***any one*** from the following questions:

$1 \times 10 = 10$

a) i) Solve the euqation $x^3 - 30x + 133 = 0$, Using cardan's method.

ii) Determine the eigen values and eigen vector of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

iii) Use Cauchy-Schwarz inequality to prove that $(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) > 16$, where a, b, c, d are positive real numbers not all equal. 4+4+2

b) i) Show that $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + y + z, 2x + y + 2z, x + 2y + z)$ is a linear transformation. Find its kernel and image space.

ii) Show that one of the values of

$$(1 + i\sqrt{3})^{\frac{3}{4}} + (1 - i\sqrt{3})^{\frac{3}{4}} \text{ is } \sqrt[4]{32}$$

iii) Prove that $(n^2 + 2)$ is not divisible by 4 for any integer n. 4+3+3
